Cointegration Test of the Monetary Theory of Inflation and Forecasting Accuracy of the Invariate and Vector ARMA Models of Inflation

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Abstract

This study conducts both the cointegration test of the monetary theory of inflation and the Granger-causality test between the variables in the system, and also develops univariate and mutivariate time series models to forecast inflation rates. Quarterly time series data for Pakistan from 1972-2 to 1993-4 is used for empirical investigation. The results suggest no cointegrating or long-run relationship between the variables in the monetary model. Some evidence of Granger-causality running from inflation to output growth is observed. Comparison of out-of-sample quarterly forecasts for the 1988-1 to 1993-4 period are made for univariate and vector ARMA models of inflation. The forecasting accuracy of the multivariate ARMA model is not statistically different from that of the univariate ARMA model.

* We thank Alicia Rambaldi for helpful suggestions.

1. Introduction

Since the seminal work of Box and Jenkins (1970), there have been many applications of time series models to the forecasting of business and economic variables. The simplest time series models are univariate autoregressive moving average (UARMA) models in which forecasts of a variable are made on the basis of its own past history. Often, other variables can be included in the model, without undue costs, in order to improve the forecasts. The transfer function model can be used if one can reasonably assume a one-way causation running from the input variables to the output variable and not vice versa.[1] However, if the input and output variables are interrelated a more sophisticated model is needed.

Vector autoregressive moving average (VARMA) models can be used in these circumstances. They are a type of reduced form model in which each variable is related to lagged values of all variables and disturbance terms in the system, as well as to the current value of its own disturbance term. There are no behavioural constraints imposed on the relationships, so the VARMA model is not subject to the same omitted variable problem which can profoundly affect forecasts from structural econometric models. However, if the variables in the system are co-integrated, a vector autoregressive model in the differences of the data will be misspecified for omitting the 'error-correction' term. Hence, an error correction representation is appropriate for a cointegrating system (Engel and Granger, 1987).

The purpose of this study is to apply the cointegration tests on the monetary theory of inflation, to test the Granger-causality between the variables[2] and to forecast inflation

[1] See Shamsuddin, *et al* (1985), and Holmes and Shamsuddin (1993) for applications of the transfer function model in the context of inflation and employment respectively.

[2] X Granger-causes Y if the past history of X can be used to predict Y more accurately than if one simply uses the past history of Y alone.

rates in Pakistan using appropriate time series models. Previous literature provide overwhelming support for the monetary theory of inflation for countries with high inflation rates; however, no firm conclusion can be drawn for countries which experienced moderate to low inflation rates (Saini, 1982). Except the periods of OPEC oil shocks, Pakistan experienced moderate to low inflation rates in the last twenty five years compared to other developing countries (see Figure 1). The value of this study comes from the fact that it is the first attempt to apply these models in the context of a country such as Pakistan with a moderate to low rate of inflation, and also because it explores the issue of causality and forecasting efficiency of alternative models within a well structured empirical framework.

Section 2 of this paper presents the results of cointegration tests. In Section 3 an appropriate multivariate model is developed based on the cointegration test and employed to test Granger-causality. Section 4 presents the estimation results for univariate and vector ARMA models and section 5 evaluates the out-of-sample forecasting performance of two models. The univariate ARMA model is used to provide benchmark forecasts for the evaluation of the forecasts obtained from the multivariate model. The final section provides some concluding remarks.

2. Cointegration Test of the Monetary Theory of Inflation

The recent developments in the literature suggest that in modelling economic time series, it is important to investigate first, the cointegrating properties of the data, and then choose either a pure VARMA or an error correction model based on the results obtained. The cointegration in the variables simply reveals the existence of a long-run relationship among the variables. Following the monetary theory of inflation we take into account three variables: consumer price index, money supply, and real output. Recognising the pegged nature of interest rates in a LDC such as Pakistan, the potential effect of interest rates on the inflation rate can be ignored. If the monetarist theory of inflation is true, the following relationships should hold in the long run:

(1) $$\log P_t = \beta_o + \beta_1 \log M_t + \beta_2 \log Y_t + e_t$$

where, P = consumer price index, M = money supply , Y = real output.[3] The constant term, β_o can be interpreted as the (logarithm of the) income velocity of money. From the above equation one can obtain the basic inflation equation:

$$\Delta \log P_t = \beta_1 \Delta \log M_t + \beta_2 \Delta \log Y_t + (e_t - e_{t-1})$$

In the empirical estimation of a monetarist inflation equation, researchers often augment this basic model by including, as explanatory variables, the expected cost of holding money, lagged money supply and the growth rate of import price (see Saini, 1982; Ryan and Milne, 1994). None of the above modifications of the model is required in the present context. First, the interest rate or the expected rate of inflation are often used to measure the cost of holding money. Pakistan had a pegged exchange rate regime over the sample range under consideration. Moreover, due to insufficient monetisation of the economy and the presence of a segmented capital market, the interest rate existing in the formal capital market may not be a good measure of the cost of holding money. As an alternative to the inflation rate, some researchers use the expected rate of inflation measured in terms of the one-period lagged inflation rate. Our study uses multivariate time series models where the lag length of the inflation rate is determined empirically. Second, lagged money supply growth is often included to address the issue of partial adjustment of the price level to changes in money supply. However, inclusion of an adhoc lag structure is no longer necessary under our empirical framework. Finally, a typical consumption bundle includes both

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[3] The quarterly inflation rate is computed from the quarterly CPI data obtained from the IMF. Traditionally, real GNP is used to measure aggregate activity in an economy. However, the quarterly GNP data is not available for Pakistan. Bhattacharya (1974, 1976) has emphasized the importance of using monetised income instead of national income in the context of developing countries. Consequently, we use industrial production defined to include mining and quarrying, manufacturing, construction, electricity, gas and water supply, which is considered a good proxy for real income in the monetised sector. Following Baumol (1952), we use the narrow definition of money which includes currency in circulation and demand deposits. All data are obtained from the computerized data bank of IMF.

imported and domestic commodities. Therefore, the relation between CPI and prices of imported goods is an accounting rather than a causal relationship. Hence, we choose not to include import price inflation as an explanatory variable in the consumer price inflation equation.

Before turning to the estimation of an inflation equation, we wish to focus on the long run relationship in levels of the variables. In long-run equilibrium the error term, e_t in equation (1) should be zero. A non-zero error term in any period reveals the deviation of the price level from its long-run equilibrium level. Hence, e_t is interpreted as the equilibrium error:

(2) $$e_t = \log P_t - \beta_o - \beta_1 \log M_t - \beta_2 \log Y_t$$

This long-run relationship will exhibit stability if e_t is stationary. Macroeconomic time series such as $\log P_t$, $\log M_t$ and $\log Y_t$ are found to be integrated of order 1 in many studies. In general a linear combination of I (1) series is integrated of order 1. However, there exists a special case where the linear combination of I (1) series can be I (0). Hence if $\log P_t$, $\log M_t$ and $\log Y_t$ are I (1) and the linear combination of these variables (i.e., e_t) is I (0), the system is said to be cointegrated. Broadly speaking, there exist two procedures for the cointegration test: the Augmented Dickey-Fuller (ADF) test (1979, 1981) and the Johansen test (1988).

The ADF test is simple to implement and based on the intuitively appealing concept of the stationary 'equilibrium error' due to Engle and Granger (1987). The first step is to conduct the unit root test on each time series. If the null hypothesis of unit root cannot be rejected for each series, the procedure is to estimate the cointegrating regression (1) using the OLS estimator and then test the null hypothesis of the non-stationarity of the OLS residuals $\hat{e}_t$. We apply two alternative ADF regression equations to test unit root in a time series X:

(3) $$\Delta \log X_t = \alpha_o + \alpha_1 \log X_{t-1} + \sum_{i=1}^{p} \delta_i \Delta \log X_{t-i} + u_t$$

(4) $$\Delta \log X_t = \alpha_0 + \alpha_1 \log X_{t-1} + \alpha_2 t + \sum_{j=1}^{p} \delta_j \Delta \log X_{t-j} + u_t$$

The lag length, p is chosen to ensure that u_t is a white noise series. Table 1 depicts the results on the unit root test. The augmented Dickey-Fuller (ADF) test suggests that the null hypothesis of unit root cannot be rejected for any time series in the system. This means $\log X_t$ is non-stationary but the first difference, $\Delta \log X_t$ is a stationary process. Next, we estimate the cointegrating regression (1) with and without trend using the OLS estimator and conduct a Dickey-Fuller unit root test on the residuals. A test for cointegration is simply a test for the stationarity of the residual series $\hat{e}_t$. More specifically, the cointegration test is based on the following regression equation:

(5) $$\Delta \hat{e}_t = \omega \hat{e}_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta \hat{e}_{t-1} + v_t$$

Table 2 presents the results of the ADF test for cointegration. The null hypothesis of unit root in the residuals is not rejected regardless of whether or not we include a trend variable in the cointegrating regression. This means that $\hat{e}_t$ follows a non-stationary process and there exists no cointegrating or long-run equilibrium relationship between the variables in the system:

(Insert Table 1 & 2)

The ADF test for cointegration is based on the Engel-Granger cointegration procedure and this test has some limitations. First, the choice of the dependent variable in the cointegrating regression is arbitrary. We could obtain three possible sequences of residual series, e_P, e_M, e_Y, based on which variable is considered as the dependent variable. In a small sample, a cointegration test based on $\hat{e}_p$ may not be equivalent to that of $\hat{e}_M$. The second problem with the ADF test is that it cannot be used to identify the existence of multiple cointegrating vectors. The Johansen procedure overcomes these limitations and it can be viewed as a multivariate generalisation of the ADF test. The Johansen test is based on the following equation system:

(6) $\mathbf{x}_t = A_1\mathbf{x}_{t-1} + \varepsilon_t$

where, $\mathbf{x}_t$ is a 3 x 1 vector of variables $[\log P_t, \log M_t, \log Y_t]'$, A_1 is the 3 x 3 matrix of parameters and e_t is the 3 x 1 vector of errors, $[\varepsilon_P, \varepsilon_M, \varepsilon_Y]'$. Subtracting $\mathbf{x}_{t-1}$ from both sides of (6) and denoting $\pi = (A_1 - I)$ we obtain:

(7) $\Delta\mathbf{x}_t = \pi\mathbf{x}_{t-1} + \varepsilon_t$

The Johansen test involves testing the rank (r) of π. The rank of π gives the number of independent cointegrating vectors and the following test-statistic is used to determine the rank:

$$\lambda_{trace}(r) = -T\sum_{i=r+1}^{n}\ln(1-\hat{\lambda}_m)$$

where, $\hat{\lambda}_m$ = estimated eigenvalues of the $\hat{\pi}$ matrix and T = the effective number of observations. The results on the Johansen trace test for cointegration are presented in Table 3. The null hypothesis of r = 0 cannot be rejected which means that there exists no cointegrating relationship between the variables in the system. Thus in the present context, the Johansen test complements our earlier finding from the ADF test. The economic implication of this result is that the data do not exhibit a long-run stable relationship between the price level, money supply and real output level in the case of Pakistan. Put differently, the monetary model of inflation may not be adequate to explain the long-run movements of the general price level in Pakistan. This finding is consistent with the previous empirical evidence that the monetarist theory of inflation is more applicable to countries with high inflation experiences than to countries with moderate or low inflation rates (Saini, 1982). Hence, it is appropriate to use a pure VARMA model in differences of the variables rather than an error-correction model. The next section presents the VARMA model and tests for Granger-causality.

(Insert Table 3)

3. Granger-causality Test

The identification of a VARMA model of inflation is based on economic theory as well as on empirical relationships in the data. Economic theory is needed in the choice of variables to be included in the system, while other issues of model specification such as exclusion restrictions and dynamic characteristics are determined empirically. Our choice of variables to be included in the VARMA model comes from the monetarist theory of inflation, as discussed in the last section. The general form of the VARMA model can be expressed as:

$$\phi (B)\mathbf{x}_t = C + \theta (B) \varepsilon_t$$

where, $\mathbf{x}$ = [logP, logM, logY]′, C is a 3xl vector of constants, $\varepsilon = [\varepsilon_P, \varepsilon_M, \varepsilon_Y]'$, $\phi(B) = I - \phi_1 B - \ldots - \phi_p B^p$, $\theta(B) = I - \theta_1 B - \ldots - \theta_q B^q$, θ_i are 3 x 3 matrices of MA coefficients and ϕ_j are 3 x 3 matrices of AR coefficients. In developing a VARMA model we follow the procedures proposed by Tiao and Box (1981). The identification of the model is based on examination of the cross correlation (CCM) and partial autoregression (PAR) matrices and the criteria of parsimony and white noise residuals. We choose two tentative VARMA models for investigation.

1. VARMA (4, 0, 0) ie, a pure VAR(4) model
2. $VARMA_C$ (4, 0, 1) with the constraint $\phi_1 = \phi_2 = \phi_3 = 0$.

The estimated parameters of the unconstrained model, VARMA (4, 0, 1) do not satisfy the stationarity and invertibility conditions. Hence, we conduct the Granger causality test under models 1 and 2 because both models satisfy the diagnostic test on estimated parameters and the residuals. The following likelihood ratio test is used (for details see Enders, 1995, pp. 312-316):

$$LR = (T - k)(\ln|\Sigma_R| - \ln|\Sigma_U|)$$

where, T = effective number of observations, k = number of parameters estimated in each equation of the unrestricted system, and $|\Sigma_R|$ and $|\Sigma_U|$ are the determinants of the residual variance-covariance matrix of the restricted and unrestricted system respectively. The LR statistic follows a χ^2 distribution with r degrees of freedom, where r is the number of restrictions imposed. The null hypothesis of Granger-noncausality is rejected if LR > Critical χ^2. The results are given in Table 4. After taking into account the effects of lagged inflation and output growth rates, money supply growth does not affect the inflation rate. Put differently, the growth rate of money supply does not Granger-cause the inflation rate. However, the inflation rate causes money supply growth to decrease when VAR(4) model is used. This is possibly a manifestation of the Pakistani government's contractionary monetary policy in response to high inflation. Finally, inflation rate Granger-causes output growth under the $\text{VARMA}_C(4, 0,1)$ but not under the VAR(4) model. No evidence can be found in support of reverse causality. Put differently, the results provide weak support for the notion that higher inflation is detrimental to output growth. The result is consistent with the earlier study (Glezakos, 1978) based on cross-country data. However, the absence of a cointegrating relationship between the variables in the system unambiguously suggest that no long-term stable relationship exists between the levels of general price and real output.

(Insert Table 4)

4. Estimation Results

The Univariate ARMA model

The UARMA model for inflation is based on procedures previously described in Holmes and Shamsuddin (1988). The quarterly time series, $\log P_t$ is found to be non-stationary but the first difference of this series, $\Delta \log P_t$ is stationary (see Figure 1) so that the general form of our model becomes:

$$\phi(B)\Delta \log P_t = c + \theta(B)\varepsilon_t$$

where, $\phi(B)$ = autoregressive polynomial, $\Delta \log P_t$ = quarterly inflation rate, c = a constant, $\theta(B)$ = moving average polynomial and ε_t = white noise error. Identification of the model is based on the estimated ACF and PACF for the inflation series, the principle of parsimony of the model and the diagnostic test of the residuals. The empirical investigation of the data leads to the following constrained univariate ARMA (1, 4) model for inflation:

$$(8) \qquad (1 - \phi_1 B)\Delta \log P_t = C + (1 - \theta_4 B^4)\varepsilon_t$$

The parameters are estimated by the exact maximum likelihood technique due to Hillmer and Tiao (1979). The initial estimation period is 1972-2 to 1987-4. The estimates are also obtained for successively longer estimation periods so that the out-of-sample forecasting accuracy of the model can be evaluated. Table 5 shows the various parameter estimates which reveal both stability and the satisfaction of both stationarity and invertibility conditions. The Ljung-Box Q-statistic suggests that residuals follow a white noise process for every estimation period. Contrary to our a priori expectation, the constant term in the UARMA inflation equation is significantly different from zero and the value of this term remains around 0.017 over the 25 estimation periods. This result signifies that the inflation series contains a deterministic component which is invariant to time. The autoregressive parameter obtains a value of 0.29 which means that last quarter's inflation rate has a moderate positive effect on the current inflation rate. The moving average parameter obtains a value of - 0.40 which implies that an inflation shock or unanticipated inflation in quarter t - 4 leads to a decrease in the current inflation rate. Although it is impossible to provide any unique economic interpretation to this finding, we outline some plausible reasons. Perhaps the anti-inflationary measures adopted, in response to a inflationary shock, by the Pakistani government affect the actual inflation rate with a lag length of four quarters. Furthermore, an inflationary shock may lead to excess aggregate supply of final consumption goods. Real wages of a vast majority of workers might decrease due to the low bargaining power of non-unionised workers of the urban informal and rural sector. Thus, the real income effect will depress aggregate demand. On the other hand, aggregate supply might increase if adjustment in input prices in response to output price shock is incomplete. An increase in aggregate supply together with a fall in aggregate demand will lead to a decrease in the current inflation rate.

(Insert Table 5)

The VARMA model Results

As noted earlier, both unconstrained VAR (4) and constrained $VARMA_C$ (4, 0, 1) models satisfy the diagnostic test on residuals. In the VAR (4) model, only 6 out of 36 AR parameters are found to be statistically significant. The inclusion of a large number of parameters with high standard errors considerably increases the forecast error variance. Hence, we choose the constrained $VARMA_C$ (4, 0, 1) model for forecasting for the sake of parsimony. The maximum likelihood estimates of the selected forecasting model for the estimation period, 1972-2 to 1987-4 is presented below. The same estimation periods as for the UARMA model are employed and this VARMA model is found to be adequate throughout all estimation periods running from 1972-2 to 1987-4 to 1972-2 to 1993-4.

$$\begin{bmatrix} \Delta \log P_t \\ \Delta \log M_t \\ \Delta \log Y_t \end{bmatrix} = \begin{bmatrix} 0.012^* \\ 0.017^* \\ 0.026 \end{bmatrix} + \begin{bmatrix} 0.506^* & -0.017 & 0.012 \\ -0.218 & 0.704^* & -0.030 \\ -0.755^* & 0.290 & 0.658^* \end{bmatrix} \begin{bmatrix} \Delta \log P_{t-4} \\ \Delta \log M_{t-4} \\ \Delta \log Y_{t-4} \end{bmatrix} - \begin{bmatrix} -0.386^* & -0.080 & 0.014 \\ 0.458^* & 0.113 & 0.078 \\ 0.948^* & 0.335 & 0.690^* \end{bmatrix} \begin{bmatrix} \varepsilon_{P,t-1} \\ \varepsilon_{M,t-1} \\ \varepsilon_{Y,t-1} \end{bmatrix}$$

Log likelihood at final estimates = 490.6
Tiao-Box (1981) M-statistic (p = 12) = 11.12
* t-statistics exceed 2

The VARMA model adequately captures the short-term dynamic relationship between the variables in the system.[4] The Tiao-Box (1981) M-statistic suggests that the residuals are white noise implying that historical information on each series is used efficiently.[5] The results suggest that lagged money supply growth has no statistically significant effect on the inflation rate, after taking into account the effects of lagged inflation and output growth. This finding is in line with our Granger-causality test, and it suggests the absence of non-contemporaneous causation running from money supply growth to the inflation rate. Both the actual inflation rate, P_{t-4} and the inflationary shock, $\varepsilon_{P,t-1}$ negatively affect the current quarter output growth. However, only the effect of the inflationary shock is statistically significant at the 5% level. One possible interpretation of this result is that an inflationary shock creates uncertainty in the economy which in turn adversely affect investment and eventually the growth rate of output. The results contradict the Phillips curve relationship and is consistent with the papers by (Glezakos, 1978) and Nugent and Glezakos (1982). Sheehey (1986), however, found the existence of a Phillips curve relationship in the context of Latin American countries.[6] We must emphasise that the earlier studies are based on the Lucas supply function where either inflation or output growth is treated as the endogenous variable. Hence, no direct comparison between the present and the earlier studies is feasible due to methodological differences.

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[4] The results for VAR(4) model of inflation is available from the authors on request.

[5] The M-statistic is distributed as a χ^2 distribution with k^2 degrees of freedom where k is the number of time series.

[6] In a survey article on Asia-Pacific developing countries Treadgold (1990) notes that "Inquiry into the possibility of a Phillips-type relationship between inflation and unemployment has had mixed results."

5. Forecasting accuracy of the two models

The selected UARMA and VARMA models have been used to forecast the quarterly inflation rate of Pakistan for the 1988-1 to 1993-4 period. Our initial estimation period runs from 1972-2 to 1987-4. These initial estimates of the two models yield separate forecasts for the four quarter forecasts running from 1988-1 to 1988-4. In the next step of our forecasting exercise we include an additional quarter in the estimation period which then runs from 1972-2 to 1988-1. The second four quarter out-of-sample forecasts then run from 1988-2 to 1989-1. By repeating this procedure 24 times, we obtain, for each model, 24 out-of-sample forecasts, the last of which run from 1993-4 to 1994-3. Since the actual data at the time of our empirical work extend to 1993-4, we have 24 one quarter ahead forecasts that we can evaluate by comparing forecast to actual values. Further, we lose one observation for each quarter added to the forecast horizon so that we have 23 of the two quarter ahead forecasts, 22 of the three quarter ahead, and 21 of the four quarter ahead forecasts that we can evaluate for each model. This enables us to assess forecasting accuracy by both the forecasting model and the forecast horizon.

To quantify the forecasting error, we have employed two widely used statistics: mean absolute error (MAE) and root mean squared error (RMSE). The computed forecasting errors are reported in Table 6 which shows the Univariate ARMA model to have slightly lower MAE and RMSE at all forecast horizons. To test the statistical significance of the difference in the forecasting accuracy in moving from univariate to multivariate ARMA model, we apply the Williams-Kloot (WK) testing procedure, discussed in Williams (1959) and Howrey (1994). Let f_u and f_v denote forecasts of inflation from the univariate and vector ARMA models respectively, and $\dot{P}$ the actual inflation rate. The WK test involves the following regression:

$$\dot{P} - (f_u + f_v)/2 = \psi_0 + \psi_1(f_u - f_v) + \eta$$

where, η is a random error term. The null hypothesis of no difference in forecasting accuracy is rejected if a t-test suggests that ψ_1 is significantly different from zero. The test results are presented in Table 7. For every forecast horizon we find that no statistically significant difference in the forecasting accuracy when we move from the univariate to multivariate ARMA model.

(Insert Table 6 and 7)

6. Summary and Conclusions

This study conducts the cointegration test of the monetary theory of inflation and the Granger-causality test between the variables in the system. In addition, we evaluate the forecasting accuracy of univariate and mutivariate models of inflation. Using quarterly time series data for 1972_1 to 1987_4 we find no cointegrating relation among CPI, money supply and output in the case of Pakistan. This means the monetary theory of inflation does not adequately explain the long-run changes in the general price level of Pakistan. Our Granger-causality tests based on multivariate models also suggest that money supply growth does not lead to inflation. However, we must emphasise that Granger-non causality does not imply that money supply growth has no contemporaneous effect on inflation. To address the issue of contemporaneous relationship one should use structural, rather than reduced form VAR model. As noted earlier, the previous literature provides overwhelming support for the monetary theory of inflation for countries with high inflation rates, rather than countries with moderate to low inflation rates. Pakistan belongs to the latter category and our finding is consistent with the literature. Our results show that for both models, forecasting accuracy generally decreases as the forecast horizon lengthens, and that the forecasting accuracy of the multivariate ARMA model is not statistically different from that of the univariate ARMA model. This finding suggests that univariate rather than vector ARMA models should be used for forecasting inflation in countries such as Pakistan. Utilising information on money supply and real output does not provide a more accurate inflation forecast than basing the forecast solely on the past history of inflation.

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Figure 1(a): Logarithm of Quarterly CPI in Pakistan

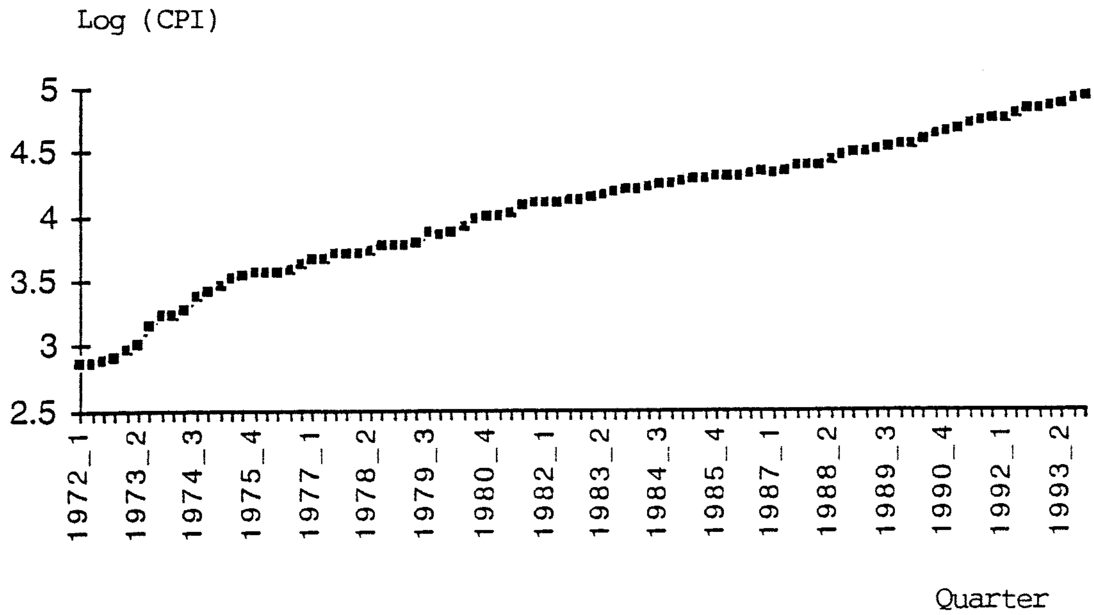

Figure 1(b): Quarterly Inflation Rate in Pakistan

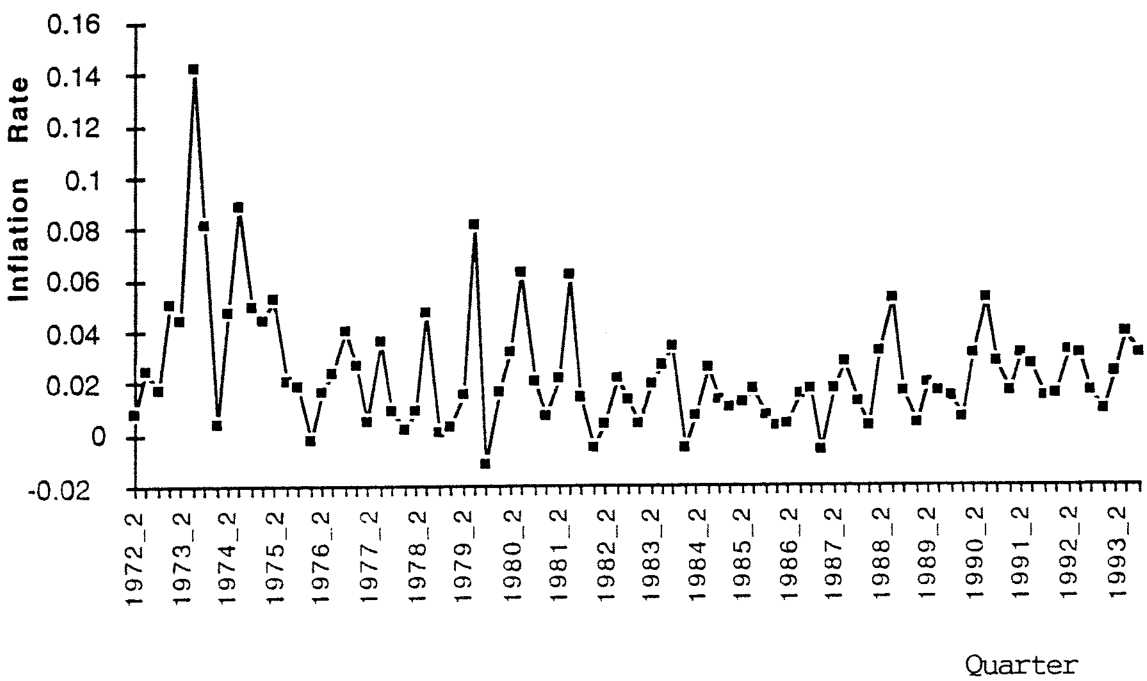

Table 1: Augmented Dickey-Fuller Unit Root Test

		ADF Test Statistic*	
Variable	Null Hypothesis	Full sample	Estimation period
$\log(P_t)$	$\alpha_1 = 0$	-3.07	-2.37
	$\alpha_1 = \alpha_2 = 0$	4.75	3.36
$\log(M_t)$	$\alpha_1 = 0$	-2.14	-2.20
	$\alpha_1 = \alpha_2 = 0$	2.66	2.45
$\log(Y_t)$	$\alpha_1 = 0$	-1.73	-0.34
	$\alpha_1 = \alpha_2 = 0$	3.70	2.91

Notes: * The asymptotic critical value of the t-test for the null hypothesis $\alpha_1 = 0$ is -3.41 at the 5 per cent level of significance. The asymptotic critical value of the F-test for the null hypothesis $\alpha_1 = \alpha_2 = 0$ is 6.25 at the 5 per cent level of significance.

Table 2: Augmented Dickey-Fuller Test for Cointegration

Cointegrating Regression	ADF Test Statistic	
	Full Sample	Estimation period
$\log(P_t) = \beta_o + \beta_1 \log(M_t) + \beta_2 \log(Y_t) + e_t$	-3.41	-2.82
$\log(P_t) = \beta_o + \beta_1 \log(M_t) + \beta_2 \log(Y_t) + \beta_3 t + e_t$	-2.29	-2.08

Notes: The asymptotic critical values of the t-test at the 5 per cent level of significance for a unit root in the residuals are -3.74 and -4.12 for the first and second equations respectively.

Table 3: The Johansen Trace test for Cointegration

Null Hypothesis	Alternative Hypothesis	λ_{trace} Statistic		
		Full sample	Estimation period	90% critical value*
$r = 0$	$r > 1$	24.19	28.51	32.09
$r \leq 1$	$r > 1$	7.12	7.17	17.96
$r \leq 2$	$r > 2$	0.0006	0.91	7.56

* Source: Johansen and Juselius (1990, Table A3).

Table 4: The LR test for Granger causality

Null Hypothesis (H_0)	VAR (4)	$VARMA_C$ (4, 0, 1)
Money supply growth does not cause inflation	Accept H_0	Accept H_0
Inflation does not cause money supply growth	Reject H_0	Accept H_0
Output growth does not cause inflation	Accept H_0	Accept H_0
Inflation does not cause output growth	Accept H_0	Reject H_0

Table 5: Univariate ARMA model results by estimation period

Estimation Period	$\hat{c}$	$\hat{\phi}_1$	$-\hat{\theta}_4$	$\hat{\sigma}$	Ljung-Box Q- statistic
1972-2 to 1987-4	0.017	0.290	0.413	0.0229	9.8
1972-2 to 1988-1	0.017	0.292	0.417	0.0227	9.4
1972-2 to 1988-2	0.017	0.289	0.421	0.0226	9.6
1972-2 to 1988-3	0.018	0.293	0.425	0.0226	9.8
1972-2 to 1988-4	0.018	0.285	0.429	0.0225	9.9
1972-2 to 1989-1	0.017	0.286	0.431	0.0224	10.3
1972-2 to 1989-2	0.017	0.288	0.431	0.0222	10.3
1972-2 to 1989-3	0.017	0.291	0.425	0.0221	9.8
1972-2 to 1989-4	0.017	0.292	0.424	0.0220	10.0
1972-2 to 1990-1	0.017	0.294	0.426	0.0219	10.2
1972-2 to 1990-2	0.017	0.290	0.425	0.0218	10.6
1972-2 to 1990-3	0.017	0.297	0.406	0.0218	11.6
1972-2 to 1990-4	0.017	0.296	0.405	0.0217	12.1
1972-2 to 1991-1	0.017	0.296	0.406	0.0215	12.3
1972-2 to 1991-2	0.017	0.295	0.394	0.0215	12.1
1972-2 to 1991-3	0.017	0.297	0.395	0.0213	12.3
1972-2 to 1991-4	0.017	0.299	0.395	0.0212	12.4
1972-2 to 1992-1	0.017	0.299	0.395	0.0212	12.4
1972-2 to 1992-2	0.017	0.297	0.395	0.0211	12.8
1972-2 to 1992-3	0.017	0.298	0.391	0.0210	13.3
1972-2 to 1992-4	0.017	0.297	0.391	0.0288	13.5
1972-2 to 1993-1	0.017	0.297	0.392	0.0208	14.2
1972-2 to 1993-2	0.017	0.297	0.392	0.0207	14.3
1972-2 to 1993-3	0.017	0.297	0.397	0.0206	14.4
1972-2 to 1993-4	0.017	0.298	0.397	0.0205	14.5

Note: 1. $\hat{\sigma}$ stands for estimated error variance. Q-statistic has a chi-square distribution with 12 degrees of freedom under the null hypothesis that the residuals from the estimated model are white noise. The critical value at the 95% confidence level is 21.03. For details about the Q statistic see Ljung and Box (1978).

2. T-statistic of both AR and MA coefficients exceed 2 in all sample ranges.

Table 6

MAE and RMSE for the 4-Quarter Out-Sample Forecasts Using Univariate and Vector ARMA Models by Forecast Horizon

Forecast Horizon	Number of Forecasts	MAE		RMSE	
		UARMA	VARMA	UARMA	VARMA
1	24	0.00926	0.00967	0.01195	0.01199
2	23	0.01025	0.01106	0.01283	0.01352
3	22	0.00996	0.01090	0.01257	0.01343
4	21	0.00936	0.01011	0.01170	0.01244

Table 7

The William-Kloot Test for the Difference Between Forecasting Accuracy of the Univariate and Vector ARMA Models

	Forecast Horizon			
	1	2	3	4
t-statistic for the parameter ψ_1	0.09	0.93	1.39	1.14

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