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Abstract

Some recent studies have examined the impact of government spending on production trade when all factors of production are fully employed and conditions of increasing returns to scale prevail in one sector of the economy. This paper attempts to extend the existing literature by examining the relationship between (i) government spending on a public good and pattern of international trade and (ii) government spending on a public good and international capital mobility when labour is underemployed and conditions of variable returns to scale prevail in one sector of the economy.

1. Introduction

Some existing studies [e.g., McMillan (1978), Manning and McMillan (1979), Khan (1983), Okamoto (1985), Abe (1990), Clarida and Findlay (1991), and Anwar (1992 & 1994)] have considered the relationship between government spending on a public input and the pattern of trade. However, the relationship between government spending on public goods and the pattern of international trade has not received much attention in the available literature. In addition, almost all relevant studies are based on the assumption that constant returns to scale prevail in all industries.[1]

Anwar (1995) has considered the relationship between government spending and pattern of trade in the presence of increasing returns to scale. The framework of this study is limited in certain respects. For example labour is the only primary factor of production which is assumed to be fully employed. The present study examines the relationship between government spending on a public good and the pattern of trade in the presence of variable returns to scale and unemployment. In addition, the relationship between government spending on a public good and international capital mobility is also considered.

A simple general equilibrium model of a closed economy is developed in the next section. The private sector of the

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[1] Melvin (1969) has considered the role of increasing returns in determining the pattern of trade between countries. However his paper does not include a public good or public input.

economy produces one final consumption good by means of capital, labour and an intermediate good. The intermediate good, which is not available for direct consumption is produced by capital and labour. The government provides a public consumption good which is produced by capital and labour. There are constant returns to scale in the production of the consumption good and the public good. On the other hand, there are variable returns to scale in the production of the intermediate good. Variable returns are due to Marshallian external economies or diseconomies.

The model is used to examine the relationship between the supply of a public good and autarky relative prices. The direction of the relationship between the supply of a public good and relative prices in a closed economy is used to predict the pattern of trade and capital mobility in a two-country world. The two countries have identical technologies and factor supplies.

The paper shows that when labour is underemployed and both primary factors (i.e., capital and labour) are internationally immobile then the country that produces more public good imports (exports) the intermediate good in exchange for the final good; if the intermediate good is produced under increasing (decreasing) returns to scale. On the other hand, when (i) labour is underemployed and internationally immobile and (ii) the intermediate (final) good is non-traded and the final (intermediate) good is traded then the country that produces more public good exports the

final (intermediate) good and imports capital; if the intermediate good is produced under decreasing returns to scale. In addition, it is also shown that in the presence of variable returns to scale in one sector of the economy, Samuelson's rule for the optimal provision of public good does not hold.

The relationship between autarky relative prices and the supply of a public good in the presence of variable returns is examined in section two. Section three contains concluding remarks.

2. A Simple General Equilibrium Model

Consider a closed economy that produces one final consumption good (Y) by means of capital, labour and an intermediate good (X). The intermediate good is produced by means of capital and labour. There are constant returns to scale in the production of Y. On the other hand, the production technology for X exhibits increasing returns to scale. Increasing returns to scale in the production of X are due to Marshallian external economies.

The government provides a public consumption good which is produced by means of capital and labour. Both capital and labour are fully mobile within the economy. Capital is fully employed but labour is underemployed.

Underemployment of labour in the present study is assumed to be due to economy wide rigid wages. Several alternative reasons can be found for the downward real wage rigidity: the

real wage may be indexed institutionally; the efficiency wage theory, as expounded by Shapiro and Stiglitz (1984), Wiess (1980) and Yellen (1984) provides a mechanism under which the real wage becomes downward rigid. This assumption is often adapted in the literature on international trade (see Bhagwati and Srinivasan (1983); and Itoh and Negishi (1989) for an elegant survey). The present study follows the literature in adopting this assumption.

The firms in each industry are identical. The production functions of the ith firm in industry X and a representative firm in industry Y are given below:[2]

$$X_i = f(X)\ k(K_{xi},\ L_{xi}); \quad X = n_x X_i \ (i = 1,\ 2,\ \ldots n_x)$$

$$Y = h(X_y,\ K_y,\ L_y)$$

where X_i is the output of firm i in industry X; n_x is the number of firms in industry X; L_{xi} and L_y are labour used in the production of X_i and Y; K_{xi} and K_y are capital used in the production of X_i and Y; X_y is the amount of X used in the production of Y.

f(X) is concave and the absolute value of the elasticity of f(.) with respect to X lies in the range [0,1]. In the present case when increasing returns to scale prevail in the

[2] The setup of the model is similar to Kemp and Ohyama (1978) and Faini (1984) However, these studies do not consider public goods or inputs.

intermediate good industry, the elasticity of f(X) with respect to X is positive. On the other hand, when decreasing returns to scale prevail in the intermediate good industry, the elasticity of f(X) with respect to X is negative. The technology for the production of the public good is the following, where K_g and L_g respectively are the capital and labour used in the production of G.

$$G = g(K_g, L_g)$$

There are constant returns to scale in the production of G. Y is the numéraire and all markets are competitive. Zero profit conditions for industry Y and X are as follows:

$$1 = c(w_0, r, p) \tag{1}$$

$$p = e(w_0, r)/f(X) \tag{2}$$

where w_0 is the minimum wage which is fixed; r is the rate of return on capital; p is the price of the intermediate good; $c(w_0, r, p)$, $e(w_0, r)/f(.)$ are unit cost functions for Y and X respectively.

The factor market clearing conditions are as follows where d(.) is the unit cost function for the public good:

$$Yc_r(w_0, r, p) + Xe_r(w_0, r)/f(X) + Gd_r(w_0, r) = K_0 \tag{3}$$

$$Yc_p(w_0, r, p) = X \qquad (4)$$

$$Yc_w(w_0, r, p) + Xe_w(w_0, r)/f(X) + Gd_w(w_0, r) = L \qquad (5)$$

where $Yc_r(.)$, $Xe_r(.)/f(.)$ and $Gd_r(.)$ are capital used in the production of Y, X and G respectively; K_0 is the supply of capital which is assumed to be fixed; $Yc_w(.)$, $Xe_w(.)/f(.)$ and $Gd_w(.)$ are labour used in the production of Y, X and G respectively; L is the labour employed in all industries which is less than labour supply (L_0) and; $c_p(.) = X_y/Y$.

(3) is the capital market clearing condition. (4) is the market clearing condition for the intermediate good which shows that the intermediate good is not available for direct consumption. (5) is the labour market clearing condition which shows that labour is underemployed in the closed economy.

The cost of the public good is financed by means of a proportional income tax. The budget constraint of the government is as follows, where t is the income tax rate:

$$t(w_0L + rK_0) = Gd(w_0, r)$$

The right hand side of the above equation is the total cost of public good; whereas the left hand side is government tax revenue. This completes the discussion of the production side.

On the demand side, the problem of the aggregate consumer

is the following, where u(.) is the utility function and C_y is the consumption of the final good.

$$\text{Max } U = u(C_y, G) \text{ subject to } C_y = (1-t)(w_0L + rK_0)$$

By using the government budget constraint the indirect utility function can be written as following:

$$U = u[((1-t)(w_0L + rK_o), G] = u[(w_0L + rK_o) - Gd(w_0L + rK_0), G]$$

Although the private sector takes the supply of the public good as given, for the economy as a whole, G is endogenous. The optimal supply of the public good can be determined by maximising the above indirect utility function with respect to G.

For a given G, equilibrium of the private sector can be characterised by (1) to (5). In five equations there are five endogenous variables; Y, X, r, p and L.

Pattern of Trade and Capital Mobility

The relationship between government spending on a public good and autarky price of the intermediate good can be examined by differentiating (1) to (5) with respect to G as follows:

$$(\partial p/\partial G)(G/p) = -\lambda_{fx}[K_gX_y\{rc_r(.)/c(.)\}]/\Delta \qquad (6)$$

where λ_{fx} is the elasticity of f(X) with respect to X and Δ is the determinant of the matrix of the relevant coefficients.

The sign of (6) depends on the sign of λ_{fx} and Δ. Δ is negative provided that the equilibrium is stable.[3] When increasing (decreasing) returns to scale prevail in the X industry then λ_{fx} is positive (negative). If X is produced under constant returns to scale then λ_{fx} is zero. (6) shows that the autarky relative price of the intermediate good is positively related to the supply of the public good as long as the intermediate good is produced under increasing returns to scale and the equilibrium is stable. In the context of a two-country world, this implies that if two countries have (i) identical technology and factor supplies (ii) capital and labour are internationally immobile and labour is underemployed (iii) the intermediate good is produced under increasing returns to scale then the country that produces more public good can produce the final good relatively cheaply. Consequently, the country that produces more public good will export the final good produced under conditions of constant returns to scale and import the intermediate good produced under conditions of increasing returns to scale. On the other hand, if the intermediate good is produced under decreasing returns to scale then the country which produces more public good will import the final good and export the intermediate good. (6) also shows that when the intermediate

[3] Stability condition is discussed in the appendix, Chang (1981) and Okamoto (1985) have derived a similar condition.

good is produced under constant returns to scale then differences in the supply of the public good alone can not determine the pattern of international trade.

The relationship between the supply of the public good and the equilibrium rate of return on capital is discussed in the following:

$$(\partial r/\partial G)(G/r) = \lambda_{fx}[K_g X_y\{pc_p(.)/c(.)\}]/\Delta \qquad (7)$$

(7) shows that when X is produced under increasing returns to scale and the equilibrium is stable then an increase in the production of the public good leads to a decrease in the equilibrium rate of return on capital. The rate of return on capital is measured in terms of the final good consumption good. In the context of a two-country world where (i) labour is underemployed and internationally immobile, and (ii) the final good is traded and the intermediate good is non-traded, (7) implies that the country that produces more public good will import (export) the final good in exchange for domestic (foreign) capital as long as the intermediate good is produced under increasing (decreasing) returns to scale. If the intermediate good is produced under constant returns to scale then differences in the supply of the public good have no influence on the pattern of trade and factor movements.

In the present study Y is the numéraire. When the intermediate good X is the numéraire, it can easily be shown

that if (i) labour is underemployed and internationally immobile and (ii) the final good is non-traded and the intermediate good is traded, then the country that produces more public good will import (export) the intermediate good and export (import) capital as long as the intermediate good is produced under increasing (decreasing) returns to scale. In simple words, if two countries have identical technologies and factor supplies, then the country that produces more public good will be the country where the rate of return on capital is relatively lower.

As long as conditions of variable returns to scale prevail in the production of the intermediate good, $\partial r/\partial G$ is non-zero. In such a case, by differentiating the indirect utility function, it can be shown that the first best rule for the provision of public good does not hold. The first best rule, also known as Samuelson's rule, requires that the marginal willingness to pay must equal the marginal cost of the public good.

The relationship between the supply of the public good and the production of the final and the intermediate good is discussed below:

$$(\partial X/\partial G)(G/X) = K_g X_y\{[rc_{rr}(.)/c_r(.)] - [pc_{pp}(.)/c_p(.)][re_{rr}(.)/e_r(.)]\}/\Delta < 0 \qquad (8)$$

$$(\partial Y/\partial G)(Y/G) = K_g\{[rc_{rr}(.)/c_r(.)] - [pc_{pp}(.)/c_p(.)][re_{rr}(.)/e_r(.)] -$$

$$\lambda_{fx}\{[pc_p(.)/c(.)][rc_{pr}(.)/c_p(.)] -$$
$$[rc_r(.)/c(.)][pc_{pp}(.)/c_p(.)]\}/\Delta \ ? \qquad (9)$$

An increase in the supply of the public good decreases the output of X whereas its impact on the production of the final good is not clear. The sign of (8) does not depend on the presence of variable returns in the production of the intermediate good. However the sign of (9) is negative if the intermediate good is produced under constant returns to scale which implies that an increase in the supply of the public good decreases the production of the final good.

In the present study, the presence of variable returns to scale plays an important role. Notice that if λ_{fx} is zero then $(\partial p/\partial G)(G/p) = (\partial r/\partial G)(G/r) = 0$. In other words, in the absence of variable returns to scale, differences in the production of the public good alone cannot determine the pattern of trade and factor movements. The results of the present study are derived on the basis of the assumption that Y is produced under constant returns and X is produced under variable returns to scale. It can easily be shown that the results of the present study will not be affected if Y is produced under variable returns and X is produced under constant returns to scale.

3. Concluding Remarks

This paper considers the relationship between government spending on households and autarky relative prices in a representative economy which produces one final good by means of capital, labour and an intermediate good. The intermediate good, which is not available for consumption, is produced by means of capital and labour. The production of the intermediate good is subject to Marshallian external economies or diseconomies. Capital is fully employed but due to rigid wages labour is underemployed. The direction of the relationship between government spending and autarky relative prices is used to predict the pattern of trade in a two-country world.

It is shown that, when there is no factor mobility, the country that produces more public good imports (exports) the output of decreasing (constant) cost industry as long as the intermediate good is produced under increasing (decreasing) returns to scale. When capital is internationally mobile and the intermediate (final) good is non-traded, the country that produces more public good imports the final (intermediate) good and exports the services of labour as long as the intermediate good is produced under increasing (decreasing) returns to scale. In addition, Samuelson' rule for the optimal provision of public good does not hold in the presence of variable returns to scale.

It is worth pointing out that the results derived in this paper also hold in the context of a standard Heckscher-Ohlin

model with a public good as long as (i) the public and the private sectors are equally capital intensive, (ii) the utility function is separable in the public and the private goods and (iii) the tastes are identical and homothetic. However, the algebra is much more simple in the present setting.

Appendix

Equations (1) to (4) can also be used to derive the Routh-Hurwitz stability conditions. The postulated dynamic adjustment process is described by means of the following equations, where the left hand side is the time derivative of the relevant variable.

$$dY/dt = \theta_y\{1 - c_w(.)\}$$

$$dX/dt = \theta_x\{p - e(.)/f(X)\}$$

$$dr/dt = \theta_r\{Yc_r(.) + Xe_r(.)/f(X) + Gd_r(.) - K_0\}$$

$$dp/dt = \theta_p\{Yc_p(.) - X\}$$

$$dL/dt = \theta_l\{Yc_w(.) + Xe_w(.)/f(X) + Gd_w(.) - L\}$$

where the relevant speeds of adjustment (θ_y, θ_x, θ_p, θ_r and θ_l) are assumed to be positive constants.

The economic meanings of the above equations are obvious,

therefore the interpretation is not included. The relevant Jacobian matrix, denoted by J is the following:

$$J = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix}$$

where

$a_{11} = 0;\ a_{12} = 0;\ a_{13} = -\ c_r(.);\ a_{14} = -c_p(.);\ a_{15} = 0;$

$a_{21} = 0;\ a_{22} = e(.)f'(.)/[f(.)]^2;\ a_{23} = -e_r(.)/f(X);\ a_{24} = 1;$

$a_{25} = 0;\ a_{31} = c_p(.);\ a_{32} = e_r[f(.)-Xf'(.)]/[f(.)]^2;$

$a_{33} = Yc_{rr}(.) + Xe_{rr}(.)/f(X) + Gd_{rr}(.);\ a_{34} = Yc_{rp}(.);\ a_{35} = 0;$

$a_{41} = c_p(.);\ a_{42} = -1;\ a_{43} = Yc_{pr}(.);\ a_{44} = Yc_{pp}(.);\ a_{45} = 0$

$a_{51} = c_w(.);\ a_{52} = e_w(.);\ a_{53} = Yc_{wr}(.) + Xe_{wr}(.)/f(X) + Gd_{wr}(.);$

$a_{54} = Yc_{wp}(.);\ a_{55} = -1$

where $f'(.) = \partial f(X)/\partial X$ and $c_{ij}(.)$ is the cross partial derivative of the unit cost function.

One of the Routh-Hurwitz stability conditions requires that $(-1)^5|J| > 0$, where "| |" stands for the determinant. The determinant of the above Jacobian matrix in the present case is, except for a positive scalar, is identical to Δ. Clearly, the determinant condition is satisfied if Δ is negative.

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