EFFECTS OF TERMS OF TRADE CHANGES IN A PUBLIC INPUT ECONOMY

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Abstract

This paper develops a simple three-period model of a small open economy which produces two final goods by means of private inputs and a public input. The allocation of resources between the private sector (which produces the final goods) and public sector (which produces the public intermediate good) is endogenous. The model is used to investigate the effect of termsof-trade changes, in either period, on the private and public sectors.

1. Introduction

Most open economy studies {for example Bhagwati and Srinivasan (1983), Dixit and Norman (1980), and Woodland (1982)} do not take government spending into account and consider the private economy exclusively. These studies provide an excellent survey of the alternative theories of international trade but do not explicitly consider either public goods or public inputs. However, these studies appear to be consistent with the assumption that the resource allocation problems of the private and public sectors are independent. In fact, despite separate management, the private and public sectors are highly interdependent in all mixed economies.

The assumed independence of resource allocation problems of the private and public sectors implies that neither domestic nor foreign shocks, which affect the private sector directly, influence the supply of public goods or public inputs in an open include (exogenous) these shocks Examples of economy. technological progress in the domestic private sector; an increase in the price of imported raw material; and an improvement in the terms-of-trade.

Governments spend large sums of monies on various services provided to the firms and households in all mixed economies. Nevertheless, the theoretical studies which take government spending into account often assume that such spending enters into household utility functions but not into production functions [see for example; Devereux (1988), Durlauf and Staiger (1990),

Frenkel and Razin (1987), and Svensson (1987).¹ These studies also assume that the utility functions are strongly separable in the public and private goods. The present study considers government spending on public inputs exclusively. Examples of these inputs include government financed scientific research whereby information on new production techniques is made available to all firms simultaneously, and production infrastructure.

Most available open economy studies which include government spending on public inputs do not take into account the lag between the production and supply of these inputs. For example, government financed scientific research conducted in the present is expected to benefit future users; production infrastructure utilised by firms in the present was built in the past. Very often significant repairs of the existing infrastructure involve a long period of time. Therefore, the lags between the production and supply of public goods cannot be ignored. These lags can only be taken into account in a multi-period setting.

The existing literature which explicitly includes government spending on industries and where the allocation of resources between the private and public sectors is endogenously determined is almost entirely static in nature.² In addition, these studies

¹ The studies which explicitly include a public input include Abe (1990), Anwar (1991 and 1993), Altenburg (1987), Ishizawa (1988), Okamoto (1985), and Tawada and Abe (1984).

² McMillan (1978) has utilised a continuous time model of a small open economy whereas Anwar (1991) has considered a twocountry, two-period model. However these studies have not considered the role of terms-of-trade changes.

do not examine the impact of either tariffs or terms-of-trade changes on the provision of public inputs. It is well-known that the terms-of-trade changes directly affect the private sector. However, in a mixed economy, the terms-of-trade changes also affect the provision of public inputs which affects the private sector indirectly.

The purpose of this paper is to develop a simple threeperiod perfect foresight model. The small open economy under consideration produces two final goods by means of private inputs and a public input. The public input is produced by means of private inputs. The final good producers take the supply of public input as given but in the full general equilibrium the supply of public input is endogenous. The allocation of resources between the private and public sectors in the small open economy is therefore endogenous. The model also takes into account the lags between the production and supply of the public input. By means of a comparative static exercise, the present study examines the impact of terms-of-trade changes on the provision of the public input and hence the production of final goods.

The paper is organised as follows. Section two develops a simple model of a small open economy. The effect of terms-oftrade changes on the private and public sectors is examined in the third section. Section four contains a summary and concluding remarks.

2. A Three-Period Model

The purpose of this section is to develop a simple framework where the allocation of resources between the private and public

sectors is endogenous and which allows an investigation of the effects of terms-of-trade changes on the two sectors. The present study explicitly deals with government spending on a pure public input. However, after some minor modifications, the results presented in this study can also be extended to include an impure public input.

There are three periods, indexed t = 1, 2 and 3, which can be interpreted as the past, the present, and the future respectively.³ In each period, the private sector produces two final goods by means of labour, a public input, and other specific factors. The government provides the public input, produced by means of labour, free of charge to the private sector. The pure public input under consideration is utilised by the producers of both final goods. Examples of such a public input, which is non-congestible both within an industry and across industries, include information on improved production management techniques.

In the first period, private producers use the pre-existing stock of public input, whereas the government produces a public input which is made available to the private sector in the second period. In the second period, the government produces a public input which is made available to the private sector in the third period. The public input can be used for only one period. In other words, the public input is durable but its rate of

³ The results presented in the present study can be generalised to any finite number of periods. However, a threeperiod setting captures the important role played by the lag in the production and eventual provision of public input without much mathematical complexity.

depreciation is 100%. This implies that the private sector utilises only the most recent information on production management techniques.

There is no private investment. The purpose of this assumption is to bring the role of lags in the supply of public input into sharp focus. The supply of labour in each period is fixed and there is perfect labour mobility between the private and public sectors.

In each period, the two final goods (X and Y) are traded at relative prices set by the rest of the world. The open economy under consideration can also borrow and lend from the rest of the world at a fixed rate of interest. In other words, the economy under consideration is small in both goods and credit markets. Demand conditions therefore have no role to play in determining prices in the present study. Good X is numeraire and the public input is also measured in its units. The economy under consideration exports good Y.

As indicated earlier, this study explicitly deals with a pure public input, the final good production functions are therefore assumed to exhibit constant returns to scale for a given level of public input. The private and public good production functions for the economy under consideration are given below, where the specific factors are not explicitly included.⁴

$$X_1 = G_0^{\alpha 1} F_1 (L_{1x}, T_{1x})$$

⁴ The subscripts 1, 2, and 3 refer to the first, the second, and the third period respectively.

$$Y_{1} = G_{0}^{\beta 1}H_{1}(L_{1y}, T_{1y})$$

$$G_{1} = \gamma_{1g} L_{1g}$$

$$X_{2} = G_{1}^{\alpha 2}F_{2}(L_{2x}, T_{2x})$$

$$Y_{2} = G_{1}^{\beta 2}H_{2}(L_{2y}, T_{2y})$$

$$G_{2} = \gamma_{2g} L_{2g}$$

$$X_{3} = G_{2}^{\alpha 3}F_{3}(L_{3x}, T_{3x})$$

 $Y_3 = G_2^{\beta 3} H_3 (L_{3y}, T_{3y})$

where X_t and Y_t are respectively production of exportable and importable good in period t; G_t is public input produced in period t; L_{tx} , L_{ty} and L_{tg} are respectively labour employed in the production of X, Y and G; T is the supply of sector specific input; α_t and β_t are the elasticity of X and Y with respect to G in period t respectively.⁵

The functional form of the above production functions implies that the public input is cooperative with private inputs in the production of both final goods. The labour-market clearing conditions are given below where N_t refers to the supply of labour in period t:

⁵ Specific factors are assumed to be fully employed therefore the relevant market clearing conditions can be ignored.

(1)
$$L_{1g} + L_{1x} + L_{1y} = N_1$$

(2)
$$L_{2g} + L_{2x} + L_{2y} = N_2$$

(3)
$$L_{3x} + L_{3y} = N_3$$

Equations (1) to (3) indicate that the entire labour force is fully utilised in each period. Efficiency conditions for the small open economy under consideration can be derived by maximising the present value of the final goods produced by the private sector. For the sake of simplicity, the relevant intertemporal discount factors are assumed to be unity. In other words, the rate of interest on foreign borrowing and lending is assumed to be zero.⁶ In addition, for simplicity γ_{tg} and G_0 are assumed to be unity.⁷

The relevant efficiency conditions are the following:

(4)
$$F_{1L}(N_1 - L_{1y} - G_1, T_{1x}) = \alpha_1 G_1^{\alpha_1 - 1} F_2(N_2 - L_{2y} - G_2, T_{2x}) + P_2 \beta_1 G_1^{\beta_1 - 1} H_2(L_{2y}, T_{2y})$$

(5)
$$G_{1}^{\alpha 1-1} F_{2L} (N_{2} - L_{2y} - G_{1}, T_{2x}) = \alpha_{2} G_{2}^{\alpha 2-1} F_{3} (N_{3} - L_{3y}, T_{3x}) + P_{3} \beta_{2} G_{2}^{\beta 2-1} H_{2} (L_{3y}, T_{3y})$$

(6) $F_{1L}(N_1 - L_{1y} - G_1, T_{1x}) = P_1 H_{1L}(L_{1y}, T_{1y})$

⁷ An increase in value of these parameters from unity can be interpreted as technological progress.

⁶ Because the open economy under consideration is small, the results do not depend on this assumption.

(7)
$$G_1^{\alpha 1-\beta 1}F_{2L}(N_2-L_{2y}-G_2, T_{2x}) = P_2 H_{2L}(L_{2y}, T_{2y})$$

(8)
$$G_2^{\alpha 2 - \beta 2} F_{3L} (N_3 - L_{3y}, T_{3x}) = P_3 H_{3L} (L_{3y}, T_{3y})$$

where $F_{tL}(.)$ and $H_{tL}(.)$ are respectively marginal product of labour in the production of X and Y in period t; $\alpha t G_t^{\alpha t-1} F_t(.)$ and $\beta t G_t^{\beta t-1} H_t(.)$ are respectively the marginal product of pure public input in the production of X and Y in period t.

The economy described by equations (4) to (8) is a public input economy. There are five efficiency conditions [equations (4) to (8)] in five endogenous variables: G_1 , G_2 , L_{1y} , L_{2y} , and L_{3y} . Equations (4) and (5) are the conditions for the optimal provision of public input in period two, and three respectively. The right-hand side of these equations is the present value of marginal benefits to the final good producers from an additional unit of public input, whereas the left-hand side is its marginal cost.⁸ Because labour is fully mobile within the small open economy, its marginal productivity in private and public sectors is identical. Equations (6) to (8) simply indicate this fact. Equations (4) and (5) in conjunction with (6) and (7) also demonstrate the implications of perfect labour mobility between the private and public sectors in the first and second period: the wage rate in within the private sector is identical.

The equilibrium can be interpreted as a perfect foresight equilibrium over time. The present study explicitly assumes that

⁸ These conditions are similar to those derived by Kaizuka (1965), and Sandmo (1972) in a single period setting.

the producers of final goods do not pay for the use of pure public input. The reward of public input is captured by the private factors. The government uses a flat rate income tax to recover the cost of public input. Since all agents and the government have access to the world credit market, the timing of these taxes does not matter. The government can also use a perunit output tax to finance the public production but the tax rate must be identical across industries and time. In other words, the findings of the model is robust to a number of non-distortionary taxation regimes.

3. Comparative Statics

The purpose of this section is to investigate the impact of temporary terms-of-trade changes on the private and public sectors. A temporary improvement in the (temporal) terms-of-trade means that dP_t is positive for only one t = {1, 2, 3}. The impact of technological improvement in the production of final goods is also considered.

In the present study, the private and public sectors are linked through perfect labour mobility in period one and two. The public input and labour are cooperative in the production of both final goods. The comparative statics results presented in this paper can be signed unambiguously only if both industries benefit equally from the supply of public input.

The impact of terms-of-trade changes in period one is discussed in the following. The results are derived by differentiating equations (4) to (8) with respect to P_1 .

Terms-of-Trade Changes in Period one

The following equations describe the impact of a temporary change in the terms-of-trade (in period one) on the optimal G_1 , G_2 , L_{1y} , L_{2y} , and L_{3y} :

(9)
$$\partial G_1 / \partial P_1 = H_{1L}(.) F_{1LL}(.) [P_2 H_{2LL}(.) F_{2LL}(.)$$

+ $(\beta_2 - 1) G_2^{-1} G_1^{\alpha 1} F_{2L}(.) \{ P_2 H_{2LL}(.) + F_{2LL}(.) \}] / H < 0$

(10)
$$\partial G_2 / \partial P_1 = \alpha_1 G_1^{\alpha_1 - 1} [F_{2L}(.) F_{1LL}(.)] [P_2 H_{2LL}(.) + F_{2LL}(.)] / H > 0$$

(11)
$$\frac{\partial L_{1y}}{\partial P_1} = -[H_{1L}(.) + F_{1LL}(.) \partial G_1 / \partial P_1]$$
$$/[F_{1LL}(.) + P_1 H_{1LL}(.)] > 0$$

(12)
$$\partial L_{2v} / \partial P_1 = -[F_{2LL}(.) / [F_{2LL}(.) + P_2 H_{2LL}(.)] [\partial G_2 / \partial P_1] < 0$$

$$\partial L_{3v} / \partial P_1 = 0$$

The equilibrium is stable when H is positive.⁹ Equation (9) shows that an improvement in the terms-of-trade in period one leads to a decrease in the production of public input. This result can be explained by using efficiency conditions (4) and (6). According to equation (6), an improvement in the terms-of-trade in period one increases the wage rate in the private sector. Due to perfect labour mobility between the private and

⁹ H is the determinant of the relevant Jacobian matrix. The jacobian matrix contains the second-order derivatives of equations (4) to (8).

public sectors, an improvement in the terms-of-trade leads to labour outflow from the public sector which increases the marginal cost of the public input above its marginal benefits in the second period. According to the efficiency condition (4), the production of public input in period one must fall.

Equation (10) shows that an improvement in the terms-oftrade in period one results in a increase in the output of public input in period two. This result follows from efficiency condition (5), an improvement in the terms-of-trade in period two decreases the output of public input in period one and therefore decreases its marginal cost in period two [i.e., $G_1^{\alpha l-1}F_{2L}(.)$] below its marginal benefits. Therefore, the optimal output of public input in the second period must increase.

Equation (11) shows that the presence of a public input in the model strengthens the expected result.¹⁰ Equation (12) shows that an improvement in the terms-of-trade in period one lowers the employment in the production of Y_2 . This follows from the fact that an improvement in the terms-of-trade increases the output of public input in period two which is produced by means of labour. Consequently, fewer workers will be available for employment in the private sector. Equation (13) can be explained in the following way. Public input is not produced in period three therefore all workers are available for employment in the private sector. A change in the terms-of-trade therefore does not affect the employment within the private sector.

¹⁰ If the allocation of resources between the private and public sectors was exogenous, $\partial G_1 / \partial P_t$, and $\partial G_2 / \partial P_t$ would be zero for all t = {1, 2, 3}.

The impact of terms-of-trade changes in period one on the output of the private sector is discussed in the following:

(14)
$$\partial X_1 / \partial P_1 = F_{1L}(.) [H_{1L}(.) - P_1 H_{1LL}(.) \partial G_1 / \partial P_1] / [P_1 H_{1LL}(.) + F_{1LL}(.)]$$
?

(15)
$$\partial Y_1 / \partial P_1 = H_{1L}(.) [\partial L_{1y} / \partial P_1] > 0$$

(16)

$$\partial X_2 / \partial P_1 = \alpha_1 G_1^{\alpha_1 - 1} F_2(.) [\partial G_1 / dp_1]$$

 $- G_1^{\alpha_1} [P_2 H_{2LL}(.) F_{2L}(.) / [P_2 H_{2LL}(.)]$
 $+ F_{2LL}(.)] [\partial G_2 / \partial P_1] < 0$

(17)
$$\frac{\partial Y_2}{\partial P_1} = \beta_1 G_1^{\beta_1 - 1} H_2(.) [\partial G_1 / \partial P_1]$$
$$+ G_1^{\beta_1} H_{2L}(.) [\partial L_{2y} / \partial P_1] < 0$$

(18)
$$\partial X_3 / \partial P_1 = \alpha_2 G_2^{\alpha_2 - 1} F_3(.) [\partial G_2 / \partial P_1] > 0$$

(19)
$$\partial Y_3 / \partial P_1 = \beta_2 G_2^{\beta_2 - 1} H_3(.) [\partial G_2 / \partial P_1] > 0$$

Equation (14) shows that the impact of an improvement in P_1 on the production of X_1 is ambiguous because the sign of $\partial L_{1x}/\partial P_1$ is ambiguous. If the public input were absent from the model or if it were produced by sector-specific labour, then the sign of equation (14) would be unambiguously negative. But in the present case the production of public input decreases in response to an improvement in the terms-of-trade. Labour is released from the production of public input, but it is not clear if all of this labour finds employment in the production of Y_1 . If in the new

equilibrium all the labour released by the public sector is absorbed in the production of Y_1 , then there will be no change in the production of X_1 . Equation (15) shows that the incorporation of public spending in the production function strengthens the expected result: an improvement in the terms-of-trade in period one leads to a increase in the production of Y_1 .

Equations (16) to (19) show that the impact of an improvement in the terms-of-trade in period one is transmitted to period two and three through its effect on the supply of public input. According to equations (16) and (17), its effect on the production of private sector in period two is negative whereas equations (18) and (19) indicate that the private sector in period three benefits. It is noticeable that if the public input was not included in the model then an improvement in the terms-of-trade in period one will not affect the output of the private sector in period two and three.

An improvement in the terms-of-trade in period one leads to a decrease in the output of private sector in period two because it results in a decrease in the production of public input in period one (i.e., G_1) which is used by the private sector in period two. However, it leads to an increase in the production of public input in period two (i.e., G_2) which is used by the private sector in period three. This implies that less labour will be available to the private sector in period two which reinforces the effect of a decrease in the supply of public input in period two.

The effect of an improvement in the terms-of-trade in period one on the output of the private sector in period three is

positive because it results in an increase in the supply of public input without affecting the supply of labour to the private sector.

Terms-of-Trade Changes in Period Two

The model developed in the previous section is a perfect foresight model. The following equations describe the impact of a change in the terms-of-trade, in period two, on the optimal G_1 , G_2 , L_{1y} , L_{2y} , and L_{3y} . This change was anticipated in the beginning of period one:

(20)
$$\partial G_{1} / \partial P_{2} = -\alpha_{1} G_{1}^{\alpha 1 - 1} [F_{1LL}(.) + P_{1} H_{1LL}(.)]$$
$$[P_{2} H_{2LL}(.) F_{2LL}(.) H_{2}(.) + (\alpha_{2} - 1) G_{2}^{-1} G_{1}^{\alpha 1} F_{2L}(.) \{P_{2} H_{2LL}(.) \}$$
$$+ F_{2LL}(.) \} - G_{1}^{\alpha 1} F_{2L}(.) H_{2L}(.) F_{2LL}(.)] / H > 0$$

$$(21) \qquad \partial G_2 / \partial P_2 = G_1^{\alpha 1} F_{2LL}(.) H_{2L}(.) [F_{1LL}(.) P_1 H_{1LL}(.) + G_1^{\alpha 1-1} \{F_{1LL}(.) + P_1 H_{1LL}(.)\} [(\alpha_1 - 1) F_{1L}(.) - \alpha_1 G_1^{-1} F_{2L}(.) H_2(.) \{P_2 H_{2LL}(.) + F_{2LL}(.)\}] / H < 0$$

(22)
$$\partial L_{1y} / \partial P_2 = -\{F_{1LL}(.) / [F_{1LL}(.) + P_1 H_{1LL}(.)]\}$$
$$\{\partial G_1 / \partial P_2\} < 0$$

(23)
$$\partial L_{2y} / \partial P_2 = -[H_{2L}(.) + F_{2LL}(.) \partial G_2 / \partial P_2]$$
$$/[F_{2LL}(.) + P_2 H_{2LL}(.)] > 0$$

$$\frac{\partial L_{3v}}{\partial P_2} = 0$$

Equations (20) and (21) indicate that an anticipated

improvement in the terms-of-trade in period two increases the production of public input in period one but it results in a decrease in the production of public input in period two. An anticipated improvement in the terms-of-trade in period two increases the anticipated marginal cost of the production of public input in period two (i.e., G_2) above the present value of its marginal benefits in period three. The optimal production of public input in period two therefore decreases. This however increases the marginal benefits of the public input supplied in period two (i.e., G_1) above its marginal in period one. An anticipated improvement in the terms-of-trade in period two therefore leads to a increase in the production of public input in period two the production of public input in period two the terms-of-trade in period two therefore leads to a increase in the production of public input in period one.

An anticipated improvement in the terms-of-trade in period one also affects the demand for labour in the production of final goods in period one and two. Equation (22) shows that the demand for labour in the production of Y_1 decreases, whereas the demand for labour in the production of Y_2 increases (see equation (23)). Equation (24) shows that the demand for labour in the production of X_3 and Y_3 is unaffected by anticipated changes in the termsof-trade in period two.

The following discussion pertains to the impact of an anticipated improvement in the terms-of-trade in period two on the production of private sector:

(25)
$$\partial X_1 / \partial P_2 = -[F_{1L}(.)P_1H_{1LL}(.)] / [P_1H_{1LL}(.)] + F_{1LL}(.)] [\partial G_1 / \partial P_2] < 0$$

(26)
$$\partial Y_1 / \partial P_2 = H_{1L}(.) [\partial L_{1y} / \partial P_2] < 0$$

(27)
$$\partial X_2 / \partial P_2 = G_1^{\alpha_1} F_{2L}(.) \{ [H_{2L}(.) - P_2 H_{2LL}(.) \partial G_2 / \partial P_2] / [P_2 H_{2LL}(.) + F_{2LL}(.) \} + [\alpha_1 G_1^{\alpha_1 - 1} F_2(.)] [\partial G_1 / \partial P_2] ?$$

(28)
$$\frac{\partial Y_2}{\partial P_2} = [G_1^{\beta 1} H_{2L}(.)] [\partial L_{2y} / \partial P_2] \\ + [\beta_1 G_1^{\beta 1 - 1} H_2(.)] [\partial G_1 / \partial P_2] > 0$$

(29)
$$\partial X_3 / \partial P_2 = \alpha_2 G_2^{\alpha 2 - 1} F_3(.) [\partial G_2 / \partial P_2] < 0$$

$$(30) \qquad \qquad \partial Y_3 / \partial P_2 = \beta_2 G_1^{\beta_2 - 1} H_3(.) \quad [\partial G_2 / \partial P_2] < 0$$

According to equations (25), (26), (29), and (30), an anticipated improvement in the terms-of-trade in period two leads to a decrease in the output of both final goods in period one and period three. The effect on the output of X_2 is ambiguous whereas the output of Y_2 increases.

Terms-of-Trade Changes in Period Three

The following equations describe the impact of an improvement in the terms-of-trade, in period three, on the optimal G_1 , G_2 , L_{1y} , L_{2y} , and L_{3y} . This improvement was anticipated in the beginning of period one:

(31)
$$\partial G_{1} / \partial P_{3} = [-\alpha_{1} \alpha_{2} G_{2}^{\alpha 2 - 1} G_{1}^{\alpha 1 - 1} F_{2L}(.) H_{3}(.)]$$
$$[P_{2} H_{2LL}(.) + F_{2LL}(.)] [P_{1} H_{1LL}(.) + F_{1LL}(.)] / H < 0$$

 $(32) \quad \partial G_2 / \partial P_3 = \{ [-\alpha_2 G_2^{\alpha_2 - 1} H_3(.)] [P_2 H_{2LL}(.) + F_{2LL}(.)] \}$

$$[(\alpha_{1}-1)G_{1}^{-1}F_{1L}(.)][P_{1}H_{1LL}(.)+F_{1LL}(.)]/H > 0$$

(33)
$$\partial L_{1y} / \partial P_3 = -\{F_{1LL}(.) / [F_{1LL}(.) + P_1 H_{1LL}(.)]\}$$

 $\{\partial G_1 / \partial P_3\} > 0$

(34)
$$\partial L_{2y} / \partial P_3 = - \{F_{2LL}(.) / [F_{2LL}(.) + P_2 H_{2LL}(.)]\}$$

 $\{\partial G_2 / \partial P_2\} < 0$

(35)
$$\partial L_{3v} / \partial P_3 = - \{H_{3L}(.) / [F_{3LL}(.) + P_3 H_{3LL}(.)]\} > 0$$

Equation (32) shows that an anticipated improvement in the terms-of-trade, in period three, leads to a increase in the public input produced in period two. This result can be explained by using equation (5) which shows that an anticipated improvement in the terms-of-trade in period three directly increases the marginal benefits of public input above its marginal cost. Consequently, the output of public input in period two (i.e., G_2) increases. An anticipated increase in the production of public input in period two leads to a decrease in the marginal benefits of the public input produced in period one below its marginal cost [see the right-hand side of equation (4)], the output of the public input in period one (i.e., G_1) therefore decreases.

Equation (33) shows that an anticipated improvement in the terms-of-trade in period one, due to its negative effect on the optimal G_1 , leads to an increase in labour employed in the production of Y_1 . On the other hand, the employment of labour in the production of Y_2 decreases, as indicated by equation (34). Equation (35) is independent of the supply of public input, it

shows that the demand for labour in the production of Y_3 increases, which is not surprising.

The impact of an anticipated improvement in the terms-oftrade in period three on the production of private sector is discussed in the following:

(36)
$$\partial X_1 / \partial P_3 = -\{F_{1L}(.)P_1H_{1LL}(.)/[P_1H_{1LL}(.)+F_{1LL}(.)]\}$$

 $\{\partial G_1 / \partial P_3\} > 0$

$$(37) \qquad \partial Y_1 / \partial P_3 = H_{1L}(.) \quad [\partial L_{1y} / \partial P_3] > 0$$

(38)
$$\partial X_2 / \partial P_3 = -[G_1^{\alpha 1} F_2(.)] \{F_{2L}(.) P_2 H_{2LL} / [P_2 H_{2LL}(.) + F_{2LL}(.)] \}$$

 $\{\partial G_2 / \partial P_3\} + [\alpha_1 G_1^{\alpha 1 - 1} F_2(.)] [\partial G_1 / \partial P_3] < 0$

(39)
$$\frac{\partial Y_2}{\partial P_3} = [G_1^{\beta_1}H_{2L}(.)] [\partial L_{2y}/\partial P_3] \\ + [\beta_1 G_1^{\beta_1 - 1}H_2(.)] [\partial G_1/\partial P_3] < 0$$

(40)
$$\partial X_3 / \partial P_3 = [\alpha_2 G_2^{\alpha_2 - 1} F_3(.)] [\partial G_2 / \partial P_3]$$
$$- [G_2^{\alpha_2} F_{3L}(.)] [\partial L_{3y} / \partial P_3] ?$$

(41)
$$\frac{\partial Y_3}{\partial P_3} = [\beta_2 G_2^{\beta_2 - 1} H_3(.)] [\partial G_2 / \partial P_3] + [G_2^{\beta_2} H_{3L}(.)] [\partial L_{3y} / \partial P_3] > 0$$

Equations (36) and (37) indicate that an anticipated improvement in the (temporal) terms-of-trade in period three results in an increase in the output of both final goods produced in period one. On the other hand, equations (38) and (39) indicate that the output of both final goods in the second period decreases. The above results depend entirely on the response of public sector, i.e., the sign of $\partial G_t / \partial P_3$. Equation (40) shows that the presence of a public input in the model strengthens the expected result, whereas the impact on the output of X_3 is ambiguous.

The results presented in this section clearly indicate the importance of the timing of terms-of-trade changes. These results also demonstrate the important role played by lags in the production and supply of public inputs. The public and private sectors are linked through unrestricted labour mobility. In other words, the allocation of resources between the two sectors is endogenous. Accordingly, terms-of-trade changes influence the private sector directly as well as indirectly; through their impact on the supply of public input. The main results presented in this paper are summarised in Table 1.

The purpose of using a three period model is to spell-out the pattern of the transmission of terms-of-trade changes from one period to another. In an n-period setting, it can be shown that the sign of $\partial G_t / \partial P_1$ will be negative for t = 1, 3, 5, 7, ... and positive for other values of t. The effect on the output of X_1 will be ambiguous whereas the output of Y_1 will increase. The output of both final goods will decrease (increase) for t = 2, 4, 6, (t = 3, 5, 7,...).

4. Concluding Remarks

The present study develops a three-period perfect foresight model of a small open economy. The model is used to demonstrates that the terms-of-trade changes in either period affect both

private and public sectors of an economy, in that period, and the lags in the production and supply of a public input transmit these effects to the other periods.

The economy under consideration produces two final goods by means of a public input, labour and other fixed factors. The public input is produced by means of labour which is fully mobile between the private and public sectors. The allocation of resources between the private and public sectors is therefore endogenous. The public input produced in period t is made available to the final good producers in period t+1. In other words, the lags in the production and supply of public inputs are explicitly taken into account.

The model is used to investigate the impact of terms-oftrade changes, in either period, on the private and public sectors. The results presented in section four demonstrate the important role played by the timing of a change in the terms-oftrade. It has been shown that an improvement in the terms-oftrade in period one leads to a decrease (increase) in the production of public input in period one (two). On the other hand, an anticipated improvement in the terms-of-trade in period two leads to a increase (decrease) in the production of public input in period one (two). Finally, an anticipated improvement in the terms-of-trade in period three leads to a decrease (increase) in the production of public input in period one (two).

Due to a lag in the production and supply of public input, the effect of terms-of-trade changes in either period is transmitted to the other periods. An improvement in the terms-oftrade in period one results in a decrease (increase) in the

production of both final goods in period two (three). On the other hand, an anticipated improvement in the terms-of-trade in period two results in a decrease in the production of both final goods in period one and three. Finally, an anticipated improvement in the terms-of-trade in period three results in an increase (decrease) in the production of both final goods in period one (two).

Table 1

	dG1	dG₂	dY1	dY2	dY3	dx,	dX2	dx₃
dP1	-	+	+	-	+	?	-	+
dP ₂	+	-	-	+	-	-	?	-
dP3	-	+	+	-	+	+	-	?

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