CONSTRUCTION OF COST-OF-LIVING INDEX NUMBERS
- A UNIFIED APPROACH

D.S. Prasada Rao

No.5 - April 1979

ISSN 0157-0188
ISBN 0 85834 235 9

CONSTRUCTION OF COST-OF-LIVING INDEX NUMBERS - A UNIFIED APPROACH

D.S. PRASADA RAO
University of New England, Armidale, Australia.

There are a number of alternative index number formulae available in literature that can be used to measure changes in general price level. Generally these index numbers can be used only for binary comparisons, i.e., comparison of two price vectors prevailing in two situations. These indices then can be chained to obtain price comparisons when more than two price vectors are involved in comparisons.

Since there is a multitude of these index number formulae, generally choice of an 'ideal' index number formula turns out to be a formidable problem. All the usual tests to separate the good formulae from the rest are generally not of much use either because there is generally a large subset of these formulae which still satisfy these tests or because these tests themselves turn out to be arbitrary and one may find it hard to reject an index number formula just because it does not satisfy these tests.

An alternative approach to this problem is to look at all these index number formulae as special cases of a much general structure. In such a case each index number formula can be judged according to the special restrictions necessary to derive the formula from the general structure. Then the choice problem can be viewed as one of evaluating these special conditions. This seems to be a logical way of tackling the problem of choice.

In what follows, in section 1, we discuss two such general approaches to construction of index numbers which are already in vogue. In section 2, we present a new unified approach to the index number construction. Then we derive some of the well-known index number formulae such as Laspeyres and Paasche index numbers. In this section we also consider some possible ways of generalizing these index number formulae for multilateral comparisons.

In the last section we discuss briefly a more general system involving basic micro-economic theoretic ideas.

SECTION 1. Two known approaches

We shall discuss these two approaches and rest of this paper with following notation. Let (p_1,q_1), $(p_2,q_2)\ldots(p_M,q_M)$ represent M pairs of price and quantity vectors of dimension N, the number of commodities. Then p_{ij} and q_{ij} stand for price and quantity, respectively, of ith commodity in jth pair of price and quantity vectors. These M pairs may represent price and quantity observations over time or space. For purposes of exposition we consider these as observations over time. The problem is one of binary comparisons if $M = 2$ and $M \geq 3$ concerns multilateral comparisons. As noted earlier, one can always construct indices for multilateral comparisons using the chain method and index numbers for binary comparisons.

Let us briefly look at the two general approaches which are discussed usually in literature. We shall call them statistical and welfare approaches to construction of cost-of-living index numbers.

1.1 Statistical approach: This approach makes use of the idea of measuring price change for one commodity. For any commodity, say ith, price change can be measured by the ratio of prices in two periods, say 1 and 2, which is given by p_{i2}/p_{i1}. This ratio measures the price change from period 1 to period 2, for ith commodity. This is known as a price relative. Thus we have N such price relatives p_{i2}/p_{i1}, for $i = 1, 2, \ldots, N$. Now the price index number construction can be posed as a problem of combining these N ratios into a single numerical value. This can be viewed as a statistical problem of obtaining a suitable 'measure of location (central tendency)'. Now most of the known index number formulae can be shown to be different measures of central tendency. One can think of arithmetic, geometric or harmonic mean of these price relatives to be appropriate means of calculating location parameter. Then there is choice of weighted or unweighted means and one can think of some suitable functions of these basic index numbers too.

Laspeyres, Paasche, Edgeworth-Marshall (see Fisher (1922) Geary-Khamis (Geary (1958), Khamis (1970), Braithwaite (1968)) fixed weight index numbers can be looked at as special cases of weighted arithmetic means of the price relatives, which is given by $\sum_{i=1}^{N} \frac{p_{i2}}{p_{i1}} . w_i$ where w_i's are such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{N} w_i = 1$. Then

Laspeyres index $= \frac{\Sigma p_{i2} q_{i1}}{\Sigma p_{i1} q_{i1}}$ can be derived with $w_i = \frac{p_{i1} q_{i1}}{\Sigma p_{i1} q_{i1}}$

Paasche index $= \frac{\Sigma p_{i2} q_{i2}}{\Sigma p_{i1} q_{i2}}$ can be derived with $w_i = \frac{p_{i1} q_{i2}}{\Sigma p_{i1} q_{i2}}$

Edgeworth-Marshall index $= \frac{\Sigma p_{i2}(q_{i1}+q_{i2})}{\Sigma p_{i1}(q_{i1}+q_{i2})}$ with $w_i = \frac{p_{i1}(q_{i1}+q_{i2})}{\Sigma p_{i1}(q_{i1}+q_{i2})}$

Geary-Khamis index $= \dfrac{\Sigma p_{i2} \dfrac{q_{i1} q_{i2}}{q_{i1}+q_{i2}}}{\Sigma p_{i1} \dfrac{q_{i1} q_{i2}}{q_{i1}+q_{i2}}}$ with $w_i = \dfrac{p_{i1} \dfrac{q_{i1} q_{i2}}{q_{i1}+q_{i2}}}{\Sigma p_{i1} \dfrac{q_{i1} q_{i2}}{q_{i1}+q_{i2}}}$

Braithwaite fixed index $= \frac{\Sigma p_{i2} q_{ia}}{\Sigma p_{i1} q_{ia}}$ with $w_i = \frac{p_{i1} q_{ia}}{\Sigma p_{i1} q_{ia}}$

where q_a is an arbitrarily chosen quantity vector.

Similarly Kloek-Theil (1965), Theil (1973) and Prasada Rao (1972) log-change index numbers, in a multiplicative form, can be viewed as weighted geometric means.

$$\text{Kloek-Theil index} = \prod_{i=1}^{N} \left[\frac{p_{i2}}{p_{i1}}\right]^{w_i} \qquad \text{where } w_i = \frac{w_{i1}+w_{i2}}{2}$$

$$\text{Theil index} = \prod_{i=1}^{N} \left[\frac{p_{i2}}{p_{i1}}\right]^{w_i} \qquad \text{where } w_i = \frac{\left[\left(\frac{w_{i1}+w_{i2}}{2}\right) w_{i1}w_{i2}\right]^{1/3}}{\sum_i \left[\left(\frac{w_{i1}+w_{i2}}{2}\right) w_{i1}w_{i2}\right]^{1/3}}$$

$$\text{Prasada Rao index} = \prod_{i=1}^{N} \left[\frac{p_{i2}}{p_{i1}}\right]^{w_i} \qquad \text{where } w_i = \frac{\frac{w_{i1}w_{i2}}{w_{i1}+w_{i2}}}{\sum \left(\frac{w_{i1}w_{i2}}{w_{i1}+w_{i2}}\right)}$$

$$\text{and} \quad w_{i1} = \frac{p_{i1}q_{i1}}{\Sigma p_{i1}q_{i1}} \qquad \text{and} \quad w_{i2} = \frac{p_{i2}q_{i2}}{\Sigma p_{i2}q_{i2}}$$

Finally there is a class of index numbers formulae where each number is a function of one or more of the index numbers which are measures of location of the N price relatives. Famous examples are

$$\text{Fisher's index} = \sqrt{L_{12}.P_{12}}$$

$$\text{Drobisch index} = \frac{L_{12}+P_{12}}{2}$$

$$\text{Stuvel's index} = \frac{L_{12}-L^q_{12}}{2} + \sqrt{\left(\frac{L_{12}-L^q_{12}}{2}\right)^2 + \frac{\Sigma p_{i2}q_{i2}}{\Sigma p_{i1}q_{i1}}}$$

where L_{12} and P_{12} stand for Laspeyres and Paasche price index numbers respectively, and L^q_{12} stands for Laspeyres quantity index number (see Banerjee, 1961). Thus statistical approach can be viewed as a fairly useful approach encompassing most of the index formulae in the literature.

1.2 Welfare approach: This approach draws heavily from the micro-economic theory concerning a consumer and generally useful in clarifying what a cost-of-living index number should measure. This approach is also useful in making price comparisons after allowance is made for changes in personal tastes and quality of goods consumed, which cannot be incorporated into the index numbers through statistical approach.

This approach starts with the assumption that there is a 'well behaved' utility function, or equivalently a proper preference structure, guiding individual's decisions regarding consumption. Let $U(q)$ represent such a utility function, then corresponding to any level U_* , one can evaluate the cost of achieving this utility at a given level of prices say p, by minimizing the expenditure over all the commodity vectors which keep the individual on the same level of indiffernce as U_* . Let C represent the cost function then C is given by

$$C(p, U_*) = \text{minimum} \sum_{i=1}^{n} p_i q_i \quad \text{over } \{q: U(q) = U_*\}$$

$$= \min_{\{q:\ U(q) = U_*\}} p'q$$

Then welfare approach defines the cost-of-living index as the ratio of costs under two different price situations p_1 and p_2

$$I_{12} = \frac{C(p_2, U_*)}{C(p_1, U_*)} \tag{1.2.1}$$

where U_* is a pre-specified level of utility. One may specify U_* through a commodity bundle q_* as $U_* = U(q_*)$.

Now actual construction of index numbers would involve specification of utility function as well as the utility level at which the index is defined and this will enable one to determine the costs involved in (1.2.1). Many index numbers can be derived by identifying the implied functions U and level of utility U_* used in (1.2.1). Popular Laspeyres index can be seen to be using a fixed coefficient utility function and U_* is $U(q_1)$.

Evidently we assume that the observed pairs (p_1,q_1) and (p_2,q_2) are optimal in situations 1 and 2. This leads to an index

$$I_{12} = \frac{C(p_2,U_*)}{C(p_1,U_*)} = \frac{C(p_2,U(q_1))}{C(p_1,U(q_1))} = \frac{C(p_2,U(q_1))}{\Sigma p_1 q_1} = \frac{\Sigma p_2 q_1}{\Sigma p_1 q_1} .$$

Similarly Paasche index can be derived using $U_* = U(q_2)$ and a fixed coefficient utility function. This gives

$$I_{12} = \frac{C(p_2,U_*)}{C(p_1,U_*)} = \frac{C(p_2,U(q_2))}{C(p_1,U(q_2))} = \frac{\Sigma p_2 q_2}{\Sigma p_1 q_2} .$$

Both of these follow from the fact that $\Sigma p_1 q_1$ and $\Sigma p_2 q_2$ are optimal in periods 1 and 2 respectively.

This can be easily extended to other index numbers, but with a difference. If we use $U_* = U(q_*)$ where q_* differs from observed q_1 or q_2 then the index defined (1.2.1) requires both the costs to be computed, which was not the case with Laspeyres or Paasche index numbers. This result is true since q_1 and q_2 are optimal in periods 1 and 2 and the utility function is of a fixed coefficient type. In other cases, we need to know the exact ratios involved in the fixed coefficient function to be able to determine the two costs involved in (1.2.1). In such cases, those indexes can be used as approximations to the true index number in (1.2.1). For example, the Edgeworth-Marshall index can be considered as

$$I_{12} = \frac{\Sigma p_2(q_1+q_2)}{\Sigma p_1(q_1+q_2)} \simeq \frac{C(p_2,\ U(q_1+q_2))}{C(p_1,\ U(q_1+q_2))} .$$

There are many recent studies, like Samuelson and Swamy (1974), Fisher and Shell (1973), Theil (1975), which go into many other aspects of construction of true-cost-of-living numbers.

SECTION 2. A new Approach

This approach draws its basic concepts from a study by Geary (1958) which were examined and studied by Khamis (1970), Prasada Rao (1970) and U.N. Study (1973). Geary in his study introduces two concepts, 'exchange rate' and 'average price' and uses them to derive an index number formula, which is now known as Geary-Khamis index number. We develop this new approach by observing that what Geary-Khamis index number uses in the definition is only one way of looking at these concepts and many other possible definitions and interpretations exist and some of them are more meaningful than the definitions used by Geary and Khamis. We have a brief discussion of these concepts below.

2.1 Basic Concepts: The concept of 'exchange rate' seems to be relevant whenever two situations with differential price levels are considered. Even if the same currency unit is used in the two situations, very common in inter-temporal comparisons and inter-regional comparisons within a country, the purchasing power of the currency unit is different and hence they should be treated differently. This is to say that sums of money expressed in same currency units over time are not strictly additive. In such cases, the only way to add them is to establish an 'exchange rate' between these currencies so that all the sums of money can be translated into sums which are additive. This idea is similar to the idea of exchange rates for currencies of different nations. So the 'exchange rate' concept is a logical off-shoot of the fact that the price levels in different time periods are different and this results in differential purchasing powers of money. The idea of exchange rate is implicit in all price comparisons and index number construction. Obviously the price index I_{12} would be the ratio of exchange rates.

The concept of 'average price' of a commodity over all the situations crops up as a corollary. If the exchange rates are known, we can translate all the prices into a common currency unit which allows us to define average price of a commodity, average over all observations. This would not be meaningful if the exchange rates were not used to make price observations comparable.

These two concepts are of a fairly general nature and they can be used for construction of price index numbers for binary as well as multilateral comparisons. It seems that these two concepts have a much wider applicability and conceptually more powerful than what was realized in the definitions used by Geary and Khamis. In the next few subsections we consider comparison of general price levels in M situations with (p_1,q_1), $(p_2,q_2)\ldots(p_M,q_M)$ representing the price and quantity data. Since we make distinction between money units corresponding to each pair (p_j,q_j) we refer to the money unit as currency unit. Let R_1, $R_2,\ldots,R_M$ denote the exchange rates for different currency units and P_1, P_2, ..., P_N denote the average prices of different commodities, stated in a common currency unit, average over all the M sets of observations. We briefly state the Geary-Khamis system of index numbers and then demonstrate how a few other index numbers can be derived from these concepts. The price index for kth vector with jth vector as base I_{jk} is given by R_j/R_k.

2.2 Geary-Khamis system of index numbers: In this subsection we briefly summarize the Geary-Khamis system of index numbers and indicate possible ways of using this basic structure to derive other index number systems. Geary (1958) defines the exchange rates and average prices using the following simultaneous system of linear equations. The exchange rate R_j for jth currency, $j = 1,2,\ldots, M$, and the average price P_i of ith commodity,

$i = 1,2,\ldots,N$, are given by

$$\left.\begin{aligned} R_j &= \frac{\sum_{i=1}^{N} P_i q_{ij}}{\sum_{i=1}^{N} p_{ij} q_{ij}} \\ \\ P_i &= \sum_{j=1}^{M} R_j p_{ij} q_{ij} \Big/ \sum_{j=1}^{M} q_{ij} \end{aligned}\right\} \qquad (2.2.1)$$

Average price of each commodity is defined using the total value of ith commodity, expressed in a common currency unit, and the total quantity of ith commodity, total obtained over all M situations. This definition makes use of the implicit price component in a pair of value and quantity observations. Then the exchange rate for jth currency unit R_j is defined by comparing the price vector p_j and the average prices P_i, $i = 1,\ldots,N$. This is done by comparing the expenditure sufficient to buy the quantity vector q_j at these two price vectors. Thus the equation system (2.2.1) seems to be a system derived from intuitive understanding of the concepts of exchange rates and average prices.

Usefulness of equation system (2.2.1) for price and quantity comparisons depends upon whether meaningful solution of R_j's and P_i's emerges from this for any set of , plausible, price and quantity vectors.

This property has been established in Prasada Rao (1970) where necessary and sufficient conditions for the existence of unique (up to scalar multiplication) positive solutions for R_j's and P_i's were derived.

Solution for R_j's for the case M = 2 is given by

$$R_1 = 1 \text{ and } R_2 = \frac{\sum_{i=1}^{N} p_{i1} \dfrac{q_{i2} q_{i1}}{q_{i2}+q_{i1}}}{\sum_{i=1}^{N} p_{i2} \dfrac{q_{i2} q_{i1}}{q_{i2}+q_{i1}}}$$

and the index I_{12}, given by R_1/R_2 would be

$$I_{12} = \frac{\sum_{i=1}^{N} p_{i2} \dfrac{q_{i2}q_{i1}}{q_{i2}+q_{i1}}}{\sum_{i=1}^{N} p_{i1} \dfrac{q_{i2}q_{i1}}{q_{i2}+q_{i1}}}$$

this index was already listed in Section 1.

Now we derive some of the other index numbers known in the literature. Before we do this let us look at equation system (2.2.1). It is easy to see that the exchange rate R_j is defined as a weighted average of (P_i/p_{ij}), $i = 1,\dots,N$, with weights $p_{ij}q_{ij}/\sum_{i=1}^{N} p_{ij}q_{ij}$ and P_i is defined as a weighted average of $(R_j\ P_{ij})$, $j = 1,\dots,M$, with weights $q_{ij}/\sum_{j=1}^{M} q_{ij}$. Now it is evident that (2.2.1) is not a unique definition for system of index numbers since we can obtain different systems by using different averaging procedures and weighing schedules.

2.3 Laspeyres, Paasche and Fisher Index Numbers:

Consider the following set of equations defining R_j's and P_i's

$$\left.\begin{aligned} R_j &= \frac{\Sigma P_i q_{ij}}{\Sigma P_{ij} q_{ij}} && \text{for } j = 1,\dots,M \\ P_i &= \frac{1}{M}\ \Sigma R_j p_{ij} && \text{for } i = 1,\dots,N \end{aligned}\right\} \qquad (2.3.1)$$

In the system (2.3.1), the exchange rates are defined in the same manner as in (2.2.1), but the average price of each commodity is defined as simple average of the ith commodity prices in M situations transformed into a common currency. Let us look at the solution for

R_j's from (2.3.1) with M = 2 since we are interested in Laspeyres, Paasche and Fisher indices which are defined for binary comparisons. Substituting the values of P_i's in R_j's, we have for k = 1, 2.

$$R_k = \frac{\sum_{i=1}^{N}\left[\frac{1}{M}\sum_{j=1}^{2} R_j p_{ij}\right] q_{ik}}{\sum_{i=1}^{N} p_{ik} q_{ik}}$$

$$= \frac{1}{M}\frac{\sum_{i=1}^{N}\sum_{j=1}^{2} R_j p_{ij} q_{ik}}{\sum_{i=1}^{N} p_{ik}q_{ik}} = \frac{1}{M}\frac{R_1 \sum_{i=1}^{N} p_{i1}q_{ik}}{\sum_{i=1}^{N} p_{ik}q_{ik}} + \frac{1}{M}\frac{R_2 \sum_{i=1}^{N} p_{i2}q_{ik}}{\sum_{i=1}^{N} p_{ik}q_{ik}}$$

Thus we have $R_1(1-\frac{1}{M}) - R_2\frac{1}{M}\frac{\Sigma p_{i2}q_{i1}}{\Sigma p_{i1}q_{i1}} = 0$

$$-R_1\frac{1}{M}\frac{\Sigma p_{i1}q_{i2}}{\Sigma p_{i2}q_{i2}} + R_2(1-\frac{1}{M}) = 0$$

or, since M = 2,

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2}\frac{\Sigma p_{i2}q_{i1}}{\Sigma p_{i1}q_{i1}} \\ -\frac{1}{2}\frac{\Sigma p_{i1}q_{i2}}{\Sigma p_{i2}q_{i2}} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Obviously this system has only a trivial solution. But to obtain meaningful price comparisons we require positive solutions. This can be done by ignoring one of the equations in (2.3.2) and solve for R_1 and R_2.

Let us look at this alternative more closely. Obviously ignoring one of the equations would imply ignoring the quantities in one of the equations are dropped. Now if we are interested in an index I_{jk} then we ignore the current quantity vector q_k by dropping the kth equation from the system and define I_{jk} as R_k/R_j where R_k and R_j are obtained as solutions from the system without kth equation. Similarly Paasche index can be defined. Analagous to the case M = 2, we can derive a new set of exchange rates using all the different exchange rate vectors by taking the geometric mean. These exchange rates give generalized Fisher's index numbers. Interpreting and deriving these most often used index numbers using the exchange rate and average prices can lead to a straight forward and meaningful way of generalizing these index numbers for multi-lateral comparisons without using the idea of chain index numbers.

2.4 ECLA or Fixed Weight Index Numbers:

Fixed weight index numbers are the simplest index numbers for binary or multilateral comparison. Let q_a be an arbitrary vector of quantities, may or may not depend on the observed quantity vectors q_j, $j = 1,\ldots, M$. There are many interesting special cases of these fixed weight index numbers. For the case M = 2, $q_{ia} = \frac{q_{i1}+q_{i2}}{2}$ for each i, leads to the Edgeworth-Marshall index

$$I_{12} = \frac{\Sigma p_{i2}(q_{i1}+q_{i2})}{\Sigma p_{i1}(q_{i1}+q_{i2})} .$$

Similarly, $q_{ia} = \sqrt{q_{i2}q_{i1}}$ give rise to Drobisch index

$$I_{12} = \frac{\Sigma p_{i2}\sqrt{q_{i2}q_{i1}}}{\Sigma p_{i1}\sqrt{q_{i2}q_{i1}}} .$$

The index number used to compute prices indices for Latin American countries, Braithwaite (1968) uses $q_{ia} = \frac{1}{M}\sum_{j=1}^{M} q_{ij} = \bar{q}_i$ and the indices are given by

$$I_{jk} = \frac{\Sigma p_{ik} q_{ia}}{\Sigma p_{ij} q_{ia}} = \frac{\Sigma p_{ik} \bar{q}_i}{\Sigma p_{ij} \bar{q}_i} \qquad (2.4.1)$$

These indices can also be derived using the exchange rate framework. The following system of equations leads to the ECLA indices

$$\left.\begin{aligned} R_j &= \frac{\sum_{i=1}^{N} P_i \bar{q}_i}{\sum_{i=1}^{N} p_{ij} \bar{q}_i} \\ P_i &= \frac{\sum_{j=1}^{M} R_j p_{ij} q_{ij}}{\sum_{j=1}^{M} q_{ij}} \end{aligned}\right\} \qquad (2.4.2)$$

This system obviously gives index number I_{jk} identical to the index in (2.4.1). However even if we replace P_i by any other definition still the indices that result in would be identical to the one in (2.4.2) and (2.4.1). As a corollory, Edgeworth-Marshall and Drobisch indices can be derived from (2.4.2) by using appropriate $\bar{q}_i$.

2.5 Kloek-Theil Index Numbers:

So far we have been looking at only index numbers which are in an additive form. Now we look into some index numbers which involve products and geometric means. Kloek-Theil (1965) used the following index number for international price comparison with M = 2. This binary index I_{12} is defined as

$$I_{12} = \prod_{i=1}^{N} \left[\frac{p_{i2}}{p_{i1}}\right]^{v_{i1}+v_{i2}/2}$$

where v_{ij}'s are the expenditure shares and $v_{ij} = \dfrac{p_{ij} q_{ij}}{\sum_{i=1}^{N} p_{ij} q_{ij}}$

for i = 1,..., N and j = 1,2. This is obviously a weighted geometric mean of the price relatives where the weight for each item is the average of the expenditure shares.

We will derive this system of index numbers using exchange rate and average price concepts with M = 2. Consider

$$\left.\begin{aligned} R_j &= \prod_{i=1}^{N} \left[\frac{P_i}{p_{ij}}\right]^{v_{ij}} && j = 1,\dots, M \\ P_i &= \prod_{j=1}^{M} [R_j p_{ij}]^{\frac{1}{M}} && i = 1,\dots, N \end{aligned}\right\} \tag{2.5.1}$$

It is easy to see that (2.5.1) is infact a multiplicative version of (2.3.1), which gave Laspeyres and Paasche index numbers. This gives an implicit relationship between Laspeyres, Paasche, Fisher and Theil-Kloek index numbers.

Transforming (2.5.1) into logarithms we have for M = 2,

$$\log R_j = \sum_{i=1}^{N} (\log P_i - \log p_{ij})\, v_{ij} \qquad j = 1,2$$

$$\log P_i = \frac{1}{M}\ (\log R_j + \log p_{ij}) \qquad i = 1,2,\dots, N.$$

This yields a linear non-homogeneous equation system in log R_j's. By eliminating P_i's we have

$$\begin{bmatrix} \tfrac{1}{2} & -\tfrac{1}{2} \\ -\tfrac{1}{2} & \tfrac{1}{2} \end{bmatrix} \begin{bmatrix} \log R_1 \\ \log R_2 \end{bmatrix} = \begin{bmatrix} \tfrac{1}{2} \sum_{j=1}^{2} \sum_{i=1}^{N} (\log p_{ij} - \log p_{ij})\, v_{i1} \\ \tfrac{1}{2} \sum_{j=1}^{2} \sum_{i=1}^{N} (\log p_{ij} - \log p_{2j})\, v_{i2} \end{bmatrix} \tag{2.5.2}$$

In this section we present a system which is consistent and yields index numbers which are consistent for multilateral comparisons. This is achieved by a slight modification of the system (2.5.1). We propose a new system which makes use of a weighted geometric average of $R_j p_{ij}$'s, weights depending on the value ratios. Consider the system

$$\left.\begin{aligned} R_j &= \prod_{i=1}^{N} \left[\frac{P_i}{p_{ij}}\right]^{v_{ij}} & j = 1,\ldots,M \\ \text{and} \\ P_i &= \prod_{j=1}^{M} [R_j p_{ij}]^{v^*_{ij}} & i = 1,\ldots,N \end{aligned}\right\} \qquad (2.6.1)$$

where $$v_{ij} = \frac{p_{ij}q_{ij}}{\sum_{i=1}^{N} p_{ij}q_{ij}} \quad \text{and} \quad v^*_{ij} = \frac{v_{ij}}{\sum_{j=1}^{M} v_{ij}} .$$

Definition of average prices used in (2.6.1) is the multiplicative version of

$$P_i = \sum_{j=1}^{M} R_j p_{ij} v_{ij} / \sum_{j=1}^{M} v_{ij}$$

where each $R_j\, p_{ij}$ is weighted by the relative importance of the expenditure share.

If we transform system (2.6.1) into a linear form using logarithm and eliminating P_i's we get for $K = 1,2,\ldots, M$

$$\log R^*_k = \sum_{j=1}^{M} \sum_{i=1}^{N} R^*_j\, v^*_{ij}\, v_{ik} + \sum_{j=1}^{M} \sum_{i=1}^{N} [\log p_{ij} - \log p_{ik}]\, v_{ik}\, v^*_{ij}$$

$$\text{where} \quad R^*_k = R_k \sum_{i=1}^{N} p_{ik}q_{ik} \quad \text{for each } k. \qquad (2.6.2)$$

This is a set of M linear non-homogeneous equations in as many unknowns. Prasada Rao (1972) proves that this system is consistent, i.e., there exists a solution for R_k^*'s and hence R_k's for this equation system and that the solution is unique up to scalar multiplication, given that price and quantity data satisfy same regularity conditions as required for Geary-Khamis method.

Thus we have a viable system of index numbers from yet another set of definitions of exchange rates and average prices. This system seems to have two natural advantages over other systems considered so far including the Geary-Khamis system. Since this system uses weighted geometric means this is more robust towards the presence of illbehaved quantity and price vectors. Such vectors can indeed produce unacceptable results when we use Geary-Khamis and other consistent methods. Further average prices in (2.6.1) are defined with weights depending on the expenditure shares rather than the quantity shares. So the average prices (2.6.1) fluctuate less violently than the systems which depend on quantity weights.

Let us look at the resulting index numbers in the case M = 2. The solution for R_1 and R_2 is given by

$$R_1 = 1 \quad \text{and} \quad R_2 = \prod_{i=1}^{N} \left[\frac{p_{i1}}{p_{i2}}\right]^{v_{i1}v_{i2}^* / \sum_{i=1}^{N} v_{i1}v_{i2}^*}$$

and the price index is

$$I_{12} = \prod_{i=1}^{N} \left[\frac{p_{i2}}{p_{i1}}\right]^{v_{i1}v_{i2}^* / \sum_{i=1}^{N} v_{i1}v_{i2}^*}$$

This index can be seen to be a weighted geometric mean of the price relatives with harmonic mean of the expenditure shares v_{i1} and v_{i2} as weights where as the Kloek-Theil index uses arithmetic mean of expenditure shares as weights.

REFERENCES

Banerjee, K.S. (1961): 'A unified statistical approach to the index number problem', Econometrica, Vol.29.

Braithwaite, S.N. (1968): 'Real Income levels in Latin America', Review of Income and Wealth, Series 14, No. 2.

Fisher, I. (1922): 'The making of Index Numbers', Houghton Miffin, Boston, Mass.

Fisher, F.M. and Shell, K. (1973): 'Economic Theory of Price Indexes' Academic Press.

Geary, R.C. (1958): 'A note on the comparison of exchange rates and purchasing powers between countries', J.R.S.S., Vol. 121.

Khamis, S.H. (1972): 'A new system of Index Numbers for National and International purposes', J.R.S.S. Series A.

Kloek, T. and Theil, H. (1965): 'International comparisons of prices and quantities consumed', Econometrica, Vol. 33.

Prasada Rao, D.S. (1970) 'Existence and uniqueness of a new class of index numbers', Sankhya, Series B, Vol. 33.

Prasada Rao, D.S. (1972): Contributions to Methodology of construction of Consistent index numbers. Unpublished Ph.D. Dissertation, Indian Statistical Institute.

Prasada Rao, D.S. (1976): 'Existence and Uniqueness of a system of Consistent Index Numbers', Economic Review, (Japan), Vol. 7.

Samuelson, P. and Swamy, S. (1974): 'Invariant Economic Index Numbers and canonical duality. Survey and Synthesis' American Economic Review.

Theil, H. (1973): 'A new index number formula', Review of Economics and Statistics.

Theil, H. (1975): 'Theory and Measurement of consumer demand Vol. 1', North-Holland Publishing company.

United Nations (1973): International Price comparisons project methods for International Product and Purchasing Power comparisons. Statistical Office of U.N.