FINITE SAMPLE PROPERTIES OF STOCHASTIC FRONTIER ESTIMATORS AND ASSOCIATED TEST STATISTICS

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Finite Sample Properties of Stochastic Frontier Estimators and Associated Test Statistics*

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ABSTRACT

The majority of applications of the stochastic frontier production function, that have appeared in the last decade, have used the method of maximum likelihood to estimate the unknown parameters of the half-normal model of Aigner, Lovell and Schmidt (1977) and have illustrated the superiority (or otherwise) of the stochastic frontier over the average production function using a Wald t-test. This paper uses Monte Carlo experimentation to investigate the finite sample properties of the maximum likelihood estimator (MLE) and the Wald test. A comparison is made with the corrected ordinary least squares (COLS) estimator and with four alternative test statistics, namely the likelihood ratio test, the one-sided likelihood ratio test and two tests of the normality of the ordinary least squares residuals. The results indicate substantial bias in the estimates of the variance ratio parameter when its true value is near zero, and that overall MLE and COLS do not appear to differ substantially. The hypothesis test results suggest that a simple asymptotic t-test of the significance of the third moment of the OLS residuals has equivalent power to the one-sided likelihood ratio test, and that the Wald test has very poor size properties.

Keywords: Stochastic Frontier, Monte Carlo, Estimation, Tests.

1. INTRODUCTION

The stochastic frontier production function was first proposed by Aigner, Lovell and Schmidt (1977) (hereafter referred to as ALS) and Meussen and van den Broeck (1977). It may be defined, for a sample of firms, as

$$y_i = x_i\beta + V_i - U_i \quad , \; i=1,2,\ldots,N, \tag{1}$$

where y_i is output; x_i is a vector of inputs and β is an unknown vector of parameters. Note that, if y_i and x_i were logarithms of output and the inputs, respectively, then this would be a Cobb-Douglas production function. The stochastic frontier is characterised by an error term which has two components, a non-negative error term to account for technical inefficiency (U_i) and a symmetric error term to account for other random effects (V_i). Thus the stochastic frontier can be viewed as a combination of the traditional (stochastic) ordinary least squares (OLS) model and the deterministic frontiers, suggested by Aigner and Chu (1968) and others.

The many applications of this stochastic frontier which have been published over the past 16 years have differed, not only in the data that they analyse, but also in the methods used to estimate the parameters of the stochastic frontier, and in the hypothesis testing procedures used to determine whether the stochastic frontier model is a significant improvement over the traditional OLS model. The choices that are made with respect to estimation and hypothesis testing are usually governed by the computer software that is available or by the personal preferences of the researchers involved.

The purpose of this paper is to use Monte Carlo experimentation to attempt to shed some light on the finite sample properties of a number of different estimators and test statistics. Few Monte Carlo analyses of frontier models have appeared in the literature. This is most likely a consequence of the amount of computer time required to obtain maximum likelihood estimates of the parameters of the stochastic frontier model. The first Monte Carlo experiment involving frontiers was a limited investigation of the finite sample performance of MLE in ALS. This was next followed by a comparison of COLS and MLE in Olsen, Schmidt and Waldman (1980) (hereafter referred to as OSW). The COLS estimator was developed following the suggestion of ALS, regarding the adaption of the deterministic COLS estimator of Richmond (1974) to the stochastic frontier. The OSW Monte Carlo experiment found little difference in the finite sample performance of MLE and COLS. Given the computational complexity of MLE relative to COLS, one would expect this to encourage the use of COLS over MLE, yet MLE has been used in more applications than COLS since the publication of the OSW paper.

The only other applications of Monte Carlo experimentation to stochastic frontiers which could be found were Banker et al (1988) and Gong and Sickles (1989, 1990). These three papers consider the influence of method selection upon the technical efficiency predictors. The first considers the performance of Data Envelopment Analysis (DEA) versus stochastic frontiers, while the latter two papers use panel data and compare stochastic frontiers with dummy variable, error components and DEA methods. Though of interest to all users of frontier methods, the latter three papers are not of relevance to this paper, where the issue of technical efficiency prediction is not of direct interest.

This paper is divided into six sections. The following two sections consider some of the choices that are available in estimation and hypothesis testing, respectively. Section 4 details the design of the Monte Carlo experiment, while the results are presented in Section 5. The final section contains concluding comments.

2. METHODS OF ESTIMATION

The two most commonly used methods of estimating the parameters of a stochastic frontier are maximum likelihood estimation (MLE) and corrected ordinary least squares (COLS). This paper compares the performance of these methods.

The analysis is confined to the half-normal error specification of ALS, as it has been assumed in the majority of applications to date. ALS derive the log-likelihood function for the model defined by Equation 1 with U_i assumed to be iid truncations at zero of a $N(0,\sigma_U^2)$ random variable, independent of the V_i which are assumed iid $N(0,\sigma_V^2)$. These two variance parameters are replaced by $\sigma^2 = \sigma_U^2 + \sigma_V^2$ and $\lambda = \sigma_U/\sigma_V$ before estimation by maximum likelihood. Battese and Corra (1977) replace λ with $\gamma = \sigma_U^2/\sigma^2$, because γ must take a value between zero and one. This parameterization has advantages during estimation, because the parameter space of γ can be searched for a suitable starting value for an iterative maximization algorithm. Battese and Corra (1977) show the log-likelihood under this parameterization is equal to

$$\log(L) = -\frac{N}{2}\log(\pi/2) - \frac{N}{2}\log(\sigma^2) + \sum_{i=1}^{N} \log[1-\Phi(z_i)] - \frac{1}{2\sigma^2}\sum_{i=1}^{N} (y_i - x_i\beta)^2, \qquad (2)$$

where

$$z_i = \frac{(y_i - x_i\beta)}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}$$

and $\Phi(.)$ is the distribution function of a standard normal random variable.

The maximum likelihood estimates of β, σ^2 and γ are obtained by finding the maximum of the log-likelihood defined in Equation 2. The maximum likelihood estimates are consistent and asymptotically efficient.

The computer program FRONTIER Version 2.0 (see Coelli, 1992) is used to obtain the maximum likelihood estimates reported later in this paper. This program uses a three step estimation procedure. The first step involves calculation of OLS estimates of β and σ^2. These estimates are unbiased estimators of the parameters in Equation 1, with the exception of the intercept, β_0, and σ^2. In the second step, the likelihood function is evaluated for a number of values of γ between zero and one.[1] The final step uses the best estimates (that is, those corresponding to the largest log-likelihood value) from the second step as starting values in a Davidon-Fletcher-Powell maximization routine which produces the final maximum likelihood estimates.

With the exception of the above program and the LIMDEP econometrics package (see Greene, 1992), there are very few computer programs that have been written to estimate frontier models. Prior to the availability of these programs, many researchers were required to write their own programs to obtain maximum likelihood estimates. The corrected ordinary least squares (COLS) method requires much less computation, and for this reason it is an attractive option. The method uses the moments of the OLS residuals to calculate an estimate of γ and then uses this value to adjust the OLS estimates of β_0 and

[1]In these calculations, the OLS estimates of β_0 and σ^2 are adjusted by

$$\hat{\sigma}^2 = \hat{\sigma}^2(\text{OLS}).\frac{T-K}{T}.\frac{\pi}{\pi-2\hat{\gamma}}$$

and Equations 5 (see below), respectively. The OLS estimates are used for the remaining parameters in β.

σ^2. The COLS estimates of the remainder of the β parameters are set equal to the OLS estimates, as they are unbiased (as noted by ALS).

OSW present expressions for the COLS estimator for the ALS parameterization of the half-normal frontier model, and observe that the estimates will be consistent but not asymptotically efficient. The COLS estimators of the Battese and Corra (1977) paramaterization of the half-normal frontier model can be obtained by straight forward manipulation of the expressions in OSW. The COLS estimators of σ^2, γ and β_0 can be shown to be equal to

$$\hat{\sigma}^2 = m_2 + \frac{2}{\pi}\left[\sqrt{\frac{\pi}{2}} . \frac{\pi}{(\pi-4)} m_3\right]^{2/3} \quad (3)$$

$$\hat{\gamma} = \hat{\sigma}^{-2}\left[\sqrt{\frac{\pi}{2}} . \frac{\pi}{(\pi-4)} m_3\right]^{2/3} \quad (4)$$

$$\hat{\beta}_0 = \hat{\beta}_0(\text{OLS}) + \sqrt{\frac{2\hat{\gamma}\hat{\sigma}^2}{\pi}} \quad (5)$$

where m_2 and m_3 are the second and third sample moments of the OLS residuals.

OSW note that the COLS estimator is prone to failure when the λ parameter approaches zero or infinity. With our parameterization, this is equivalent to γ approaching zero or one. These failures will occur when m_3 is positive or when m_2 is less than $\frac{(\pi-2)}{\pi}\left[\sqrt{\frac{\pi}{2}} . \frac{\pi}{(\pi-4)} m_3\right]^{2/3}$, respectively. OSW labels these failures Type I and Type II, respectively. A positive value of m_3 causes the expression inside the square brackets in Equation 3 to be negative and hence prevents the calculation of an estimate. Thus we have set m_3 to zero whenever it is positive. A Type II error will not prevent estimates from being calculated, but will provide an estimate of γ which exceeds one. This can only occur if one of σ_V^2 or σ_U^2 are negative, which is not theoretically possible. Some COLS estimates of γ slightly larger than one in value are obtained in this experiment when the true value of γ is set near to or equal to one. In these instances the COLS estimate of γ is set to one.

The lack of acceptance of the COLS estimator in the applied literature could be partly due to its failure to provide an estimate of λ when a Type II error occurs. The use of the Battese and Corra (1977) parameterization, which uses γ instead of λ, does not suffer from this problem, as γ can be set to one in these instances. The unpopularity of the COLS estimator may also be due to access to estimated standard errors. OSW provide an outline of the derivation of the asymptotic standard errors of the COLS estimators of β_0, σ_V^2 and σ_U^2, but they do not provide final expressions for them, nor do they provide any expressions for σ^2 or λ. A derivation of the standard errors of the COLS estimators of β_0, σ^2 and γ, as defined in Equations 3 to 5, is presented in Appendix 1. To further facilitate utilization of these results, SHAZAM code for the calculation of COLS estimates and their standard errors is listed in Appendix 2. Note that the SHAZAM computer package (White, 1993) is but one of many computer packages which could be used to calculate COLS estimates.

3. HYPOTHESIS TESTS

Consider the frontier model defined by Equation 1 with the assumption that U_i is half-normal. A test of the hypothesis that the frontier model is a significant improvement over the OLS model can be achieved by testing the significance of γ (or λ or σ_U^2). This can be done in a number of ways. One method is to calculate the ratio of the estimate of γ to its estimated standard error

$$W = \hat{\gamma}/S_{\hat{\gamma}}. \tag{7}$$

This Wald statistic is asymptotically distributed as a standard normal random variable. The test is best performed as a one-sided test as γ cannot take negative values. The estimated standard errors of the maximum likelihood estimates are taken from the square-roots of the diagonal elements of the direction matrix in the final iteration of the Davidon-Fletcher-Powell (DFP) algorithm. These estimates can sometimes be poor if the DFP algorithm does not run for a sufficient number of iterations. As shall be seen in the results section of this paper, this test will also perform badly if the estimate of γ is biased.

An alternative test is the likelihood ratio test. It requires estimation of the model under both the null and alternate hypotheses. It is calculated as

$$LR = -2[\log(L_0)-\log(L_1)] \tag{8}$$

where $\log(L_0)$ is the log-likelihood value under the null hypothesis (i.e. OLS) and $\log(L_1)$ is the log-likelihood value assuming the null is false (i.e. $\gamma \neq 0$). This ratio is asymptotically distributed as a chi-square with one degree of freedom.

The fact that the Wald test is one-sided while the likelihood ratio test is two-sided suggests the Wald test should have better power as it incorporates the information that γ cannot take a negative value. In a similar case involving the α parameter in the panel data error components model, Honda (1985) constructs a one-sided version of the Breusch-Pagan Lagrangian multiplier test of the null hypothesis that α was zero. The similarities between that α paramater, which is the ratio of the variance of the firm effects to the total variance, and the γ paramater in the stochastic frontier are obvious. In this paper a one-sided likelihood ratio test is proposed to test the hypothesis that γ is zero versus the alternative that it is positive. The approach is simple and intuitive. One calculates the likelihood ratio statistic as specified in Equation 8 and the decision rule for a test of size α is "reject H_0 if LR > $\chi_1^2(2\alpha)$". Thus one would expect the one-sided test to reject the null hypothesis more often than the two-sided test, as the critical value for a 5% test is reduced from 3.84 to 2.71. The above simple approach only applies for single restriction hypotheses. The extension to multiple restrictions is much more complicated, where the underlying mixed chi-square distribution of the one-sided test statistics must be utilized in constructing a multi-option decision rule. The interested reader is advised to consult Gourieroux, Holly and Monfort (1982) for more on this issue. For example, this information would be of interest in applications of frontiers where the truncated normal distribution suggested by Stevenson (1980) is assumed and tests of the joint hypothesis that γ and μ are both zero are considered.

The three tests considered thus far all require the estimation of the parameters of the unrestricted model, that is, the stochastic frontier model. If the null hypothesis is accepted, the maximum likelihood estimates are of no further value. Schmidt and Lin (1984) search for a test which can be constructed using the restricted (OLS) estimates only. They begin by attempting to specify a Lagrangian multiplier (LM) statistic for the hypothesis that λ is equal to zero. They show that an LM test cannot be constructed because, under the null hypothesis, the vector of first partial derivatives of the log-likelihood function is a zero vector and the information matrix is singular. These two results are derived in Waldman (1982).

Having reached this conclusion, Schmidt and Lin (1984) look to other alternatives. They note that the question of whether the frontier is to be preferred to OLS is essentially a test of whether (V_i-U_i) is normally distributed. They observe that a number of tests for normality of regression residuals exist, such as the Kolmogorov-Smirnov and Shapiro-Wilk tests, but note that the sum of a normal plus a half-normal should suffer from skewness (lack of symmetry) in particular, and therefore propose the use of a test of whether the skewness ($\sqrt{b_1}=m_3/m_2^{2/3}$) in the OLS residuals is significant. The distribution of $\sqrt{b_1}$ is discussed and tabulated in a number of papers, such as D'Agostino and Pearson (1973), who tabulate values of δ and λ such that $\delta[\sinh^{-1}(\sqrt{b_1}/\lambda)]$ is approximately standard normal. Schmidt and Lin (1984) apply this test to data employed in Schmidt and Lovell (1979) and observe that it comes to the same conclusion as the Wald test conducted in that paper.

Two different tests of the normality of the OLS residuals are considered in this paper. The first is due to Bera and Jarque (1981) who propose the calculation of

$$BJ = N[m_3^2/6m_2^3 + (m_4/m_2^2 - 3)^2/24] \tag{9}$$

where m_4 refers to the fourth sample moment of the OLS residuals and all other notation is as defined earlier. This statistic is asymptotically distributed as a chi-square with two degrees of freedom. Bera and Jarque (1981) observe that this test statistic is easy to compute and, through a Monte Carlo experiment, show that it has good power in finite samples, relative to six other tests for normality.

The second test of normality of the OLS residuals considered here, attempts to focus upon the issue of skewness, in particular, the negative skewness likely to occur in (V_i-U_i). The BJ test considered above is a simultaneous test of skewness and kurtosis. As negative skewness will occur when the third moment is negative, a test of whether the third moment is greater than or equal to zero should be appropriate (following a similar argument to that made by Schmidt and Lin to support their test of the skewness parameter). The third moment of OLS residuals is asymptotically distributed as a normal random variable with mean 0 and variance $6m_2^3/N$ (Pagan and Hall, 1983). Thus the following test statistic

$$M3T = m_3/\sqrt{6m_2^3/N} \qquad (10)$$

is asymptotically distributed as a standard normal random variable. This test has been selected in preference to the $\sqrt{b_1}$ test proposed by Schmidt and Lin (1984) to avoid the necessity to consult tables which are not routinely reproduced in econometrics texts. This test statistic is very easy to calculate, as is evident in the SHAZAM code in Appendix 2.

Justification for the above five tests is based upon asymptotic theory. The finite sample properties of the tests are unknown. The subsequent Monte Carlo experiment attempts to help us in this regard.

4. DESIGN OF THE MONTE CARLO EXPERIMENT

The sample space in the experiment is β, σ^2, γ, N and X, where $X=(x_1 \ldots x_N)'$. We utilise the invariance results noted in OSW to set $\sigma^2=1$ and $\beta=1$. Furthermore, we will follow ALS and OSW and consider the instance where X only contains a vector of ones. That is, the model has a constant term, but no regressors. These restrictions reduce our sample space to γ and N. This last restriction is not unreasonable, as the model in Equation 1 assumes neutral technical inefficiency effects. This is the property which ensures the unbiasedness of the OLS estimates of the production elasticities. It should be noted, however, that a number of applied studies, for example see Son, Coelli and Fleming (1993), have obtained both OLS and maximum likelihood estimates of the stochastic frontier, and observe that the production elasticities differ substantially between the methods. In these instances, the neutrality of the technical inefficiency effects, and hence the traditional definition of the stochastic frontier, may be questioned. This question is beyond the scope of this study, but is a point which should be kept in mind when considering the applicability of the Monte Carlo results.

The experiment involves nine values of γ and four values of N. This gives a total of 36 combinations, for which 200 replications are made. A larger number of replications would be preferred, but the computational cost was too great to consider at this time. The nine values of γ considered were 0.0, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95 and 1.0 and the four sample sizes were 50, 100, 400 and 800.

In each of the 200 replications, two vectors, each of length 800, of pseudo standard normal random numbers were generated. The experiment was conducted using DEC Fortran Version 3.1 on a Digital DECsystem5500 running Ultrix. This Fortran compiler has the intrinsic function RAN() which is a multiplicative congruential generator of uniform random numbers on the range between zero and one. The following approximate central limit result, taken from Hendry (1984, p948), was used to derive standard normal random numbers (z_i) from the uniform random numbers (w_j)

$$z_i = \sum_{j=1}^{12} w_j - 6. \tag{11}$$

These two vectors of standard normal random numbers were then used to construct the vectors of the U_i and the V_i for the different values of γ. The V_i were formed by multiplying the first vector of random numbers by the value of σ_V implied by the particular value of γ, while the absolute values of the second vector of standard normal random numbers were obtained before they were multiplied by the appropriate value of σ_U to form the vector of U_i's.

The V_i and U_i were then used to construct the y_i as defined in Equation 1. This was done for each of the 200 replications. In each replication the first 50 of the 800 y_i observations were used when N=50, the first 100 were used when N=100, and so on. This approach has the effect of using common random numbers whenever possible. This has the advantage of reducing experimental variability between different pairs of γ and N, but has the disadvantage of requiring more involved programming.

The MLE and COLS estimates were calculated for each of the above 7200 generated data sets. Furthermore, the five hypothesis tests detailed in the previous section were conducted for each of the data sets.

Before discussing the results, it would be useful to briefly consider the similarities and differences between this experimental design and that used by OSW, as comparisons will be made when assessing the performance of the rival estimators. The common aspects include the assumptions that $\sigma^2=1$, $\beta=1$ and that X is a vector of 1's. Thus the parameter spaces are similar, but not the same, as OSW considered the paramaterization of ALS, involving λ, while this experiment assumes the parameterization of Battese and Corra (1977), which involves γ. The γ parameter can take values between zero and one inclusive. These end-points equate to two restricted forms of the stochastic frontier, those of the OLS model and the deterministic frontier, respectively. The corresponding values for λ are zero and infinity. The experiment in this paper includes the end-points of the parameter space (γ=0,1) while the OSW experiment did not. To be fair to OSW, the consideration of $\lambda=\infty$ was obvioulsy not possible given the parameterization of their model. OSW hold N fixed at 50 and consider nine values of λ, then hold λ fixed at one and consider six values of N. The experiment in this paper considers all combinations of nine values of γ and 4 values of N and conducts 200 replications of each. In the OSW experiments the number of replications were either 50, 100 or 200.[2]

5. **RESULTS**

The results of the experiment are presented in Tables 1 to 3. Table 1 contains the bias, variance and mean square errors (MSE) of the maximum likelihood estimates for the 36 different combinations of γ and N. Initially, the discussion will concentrate upon the influence of γ by fucusing upon the results for N=100. The influence of sample size is considered shortly. The third column of Table 1 contains measures of the bias in the estimates of γ. A pattern emerges, with positive bias observed for γ less than 0.5 and a negative bias for values greater than or equal to 0.5. Furthermore, the bias tends to diminish as γ approaches one. The positive bias when γ is close to or equal to zero could be explained by the observation that γ cannot take a negative value. The MSE of $\hat{\gamma}$, listed in the last column of Table 1, reduces as γ increases from zero to one. This decline in MSE is as much due to a decrease in variance (refer to column six) as to the decrease in bias.

The patterns observed for $\hat{\gamma}$ appear to be similar in the estimates of β_0 and σ^2. They both exhibit an upward bias when γ is small and a negative bias

[2]The different sizes of these two experiments is a reflection of the advances that have been made in computer technology over the past 13 years, and should not be viewed as an indication of different degrees of effort.

for larger values of γ, which diminishes as γ approaches one. The variance and MSE of these two parameters also follow that pattern observed for $\hat{\gamma}$, of a decline as γ varies from zero to one.

The above results do not have much in common with the limited Monte Carlo experiment presented in ALS but do resemble the general results of OSW. One difference between the results discussed above and the corresponding results in OSW is that, in their study, the bias and variance of σ^2 tend to increase for the larger values of λ considered. Noting that a large value of λ corresponds to a value of γ near one, this appears to conflict with our results. This difference is likely to be a function of the alternative parameterizations considered, but a precise explanation is yet to be found.

The results for the COLS estimators are presented in Table 2. The similarity of these results with the MLE results in Table 1 is evident. The patterns in bias, variance and MSE are almost the same. A detailed comparison of the two tables suggests that the COLS estimator may be preferred for values of γ less than or equal to 0.75 while the MLE estimator appears a little better for values of γ greater than 0.75. However, it should be stressed that the differences are generally quite small. Similar conclusions are made in OSW.

The preceeding discussion of the numbers in Tables 1 and 2, has considered the influence of the value of γ upon the performance of the estimators for a sample size of 100. If attention is focussed upon sample size, we note that an increase in N does not appear to have any substantial influence upon our conclusions, other than to produce the expected decline in bias, variance and MSE. We do note, however, that the value of γ at which the COLS estimator has the lower MSE declines from 0.75 to 0.5 when sample sizes of 400 and 800 are considered. This is one indication of a slight improvement in the performance of MLE relative to COLS when sample size is increased. A careful inspection of Tables 1 and 2 confirms this.

The final set of results relate to the five hypothesis tests considered in Section 4. The percentage of rejections of the null hypothesis (that OLS is sufficient) are listed in Table 3. All hypothesis tests have been conducted at the five percent level of significance. To begin with, consider the results when the true value of γ is zero. These indicate that the one-sided likelihood ratio and third moment tests both have correct size. That is, the observed sizes are no more than two standard deviations either side of the required size of 5%. The Bera/Jarque and two-sided likelihood ratio appear to be undersized. These results are not surprising given the discussion in Section 4. The Wald test appears to have very poor size. For example, when sample size is 100, it rejects the null hypothesis 24% of times instead of the required 5%. The positive bias in the MLE estimates of γ, when γ is small, would obviously be contributing to this result.

The number of rejections when γ is not equal to zero, provide an indication of the finite sample power of these tests. The Wald test has equivalent or better power than the other four tests for all combinations of γ and N. However, given its poor size, we will reject it and discuss the relative power of the remaining four tests. These four tests all have rather poor power for small values of γ and N, which improves as both γ and N become larger. It is evident that the two-sided likelihood ratio and Bera/Jarque tests have lower power than the other two tests. This is not surprising, given their smaller size. The one-sided likelihood ratio and third moment tests have power functions which are almost identical, with a suggestion that the one-sided likelihood ratio test may perform slightly better in the smaller samples.

6. CONCLUSIONS

The MLE and COLS estimators appear to perform in a similar manner for a range of values of γ and N. The COLS estimator is slightly better when γ is small or around 0.5, while MLE could be preferred when γ is large. Both estimators provide biased estimates of γ, β_0 and σ^2. This bias is positive when γ is small and negative when γ is large. The bias in γ diminishes as γ approaches one. The bias, variance and MSE of both estimators declined as sample size increases and the relative performance of MLE improved slightly as sample size increases.

The hypothesis test results suggest that the two most commonly used tests of the hypothesis that γ is equal to zero, the Wald test and the two-sided likelihood ratio test, have incorrect size. The two-sided likelihood ratio test is slightly under-sized because it does not incorporate the knowledge that γ can only be positive, while the Wald test is badly over-sized because of the bias in the estimator of γ when the true value of γ is small. The incorrect size of the two-sided likelihood ratio test can be avoided by using a one-sided likelihood ratio test. This test has correct size and superior power. Another test which has equivalent size and power to the one-sided likelihood ratio test, is the test of the significance of the third moment of the OLS residuals. This latter test has the advantage of not requiring the calculation of the maximum likelihood estimates of the stochastic frontier.

These results are great news for 'computer shy' applied economists. To begin with, the necessity to estimate the parameters of the stochastic frontier, to test the hypothesis that it is a significant improvement over the OLS model, is eliminated. A test of the null hypothesis that the third moment of the OLS residuals is greater than or equal to zero appears to be as good as any of the alternative tests. Secondly, even if the afore mentioned null hypothesis is rejected and the parameters of the stochastic frontier must be estimated, the COLS estimates, which can be obtained with a minimum of effort, appear to be of a similar quality to the maximum likelihood estimates. To encourage the use of COLS, expressions for the standard errors of the COLS estimators are presented in Appendix 1, and the SHAZAM code required to calculate COLS estimates, their standard errors and the third moment test, is listed in Appendix 2.

This analysis could be extended in any number of directions. Some topics warranting consideration include: investigation of the properties of pre-test estimators, extension of the results to panel data, and analysis of the influence of the choice of estimation method upon the performance of technical efficiency predictors.

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Appendix 1

Asymptotic Standard Errors of the COLS Estimators

The derivation of the standard errors follows the method used in the Appendix of OSW. Consider the composed error disturbance term $\varepsilon=V-U$, where $V\sim N(0,\sigma_V^2)$ and $U\sim|N(0,\sigma_U^2)|$. ALS show that the expectation of ε is

$$E(\varepsilon) = \mu = -\sqrt{2/\pi}.\sigma_U$$

and OSW present the first six central moments of ε in terms of σ_V^2 and σ_U^2. These results can also be expressed in terms of σ^2 and γ as

$$\mu = -\sqrt{2\gamma\sigma^2/\pi}$$

$$\mu_2 = \sigma^2\left[(1-\gamma)+\gamma\frac{(\pi-2)}{\pi}\right]$$

$$\mu_3 = \sigma^3\left[\sqrt{\frac{2}{\pi}}(1-\frac{4}{\pi})\gamma^{3/2}\right]$$

$$\mu_4 = \sigma^4\left[3(1-\gamma)^2+6\frac{(\pi-2)}{\pi}\gamma(1-\gamma)+(3-\frac{4}{\pi}-\frac{12}{\pi^2})\gamma^2\right]$$

$$\mu_5 = \sigma^5\gamma^{3/2}\sqrt{\frac{2}{\pi}}\left[10(1-\frac{4}{\pi})(1-\gamma)+(7-\frac{20}{\pi}-\frac{16}{\pi^2})\gamma\right]$$

$$\mu_6 = \sigma^6\left[15(1-\gamma)^3+45\frac{(\pi-2)}{\pi}(1-\gamma)^2\gamma+15(3-\frac{4}{\pi}-\frac{12}{\pi^2})(1-\gamma)\gamma^2+(15-\frac{6}{\pi}-\frac{100}{\pi^2}-\frac{40}{\pi\pi^3})\gamma^3\right].$$

These expressions will be used in the following asymptotic variances and covariances of the second and third sample central moments:

$$V(m_2) = \frac{1}{N}(\mu_4-\mu_2^2)$$

$$V(m_3) = \frac{1}{N}(\mu_6-\mu_3^2-6\mu_2\mu_4+9\mu_2^3)$$

$$Cov(m_2,m_3) = \frac{1}{N}(\mu_5-4\mu_2\mu_3).$$

Before these expressions can be of use, we must re-express the COLS estimators of γ and β_0 in Equations 4 and 5 (in the body of the paper) to be functions of the second and third moments. These become:

$$\hat{\gamma} = \left\{ m_2 \left[\sqrt{\frac{\pi}{2}} . \frac{\pi}{(\pi-4)} m_3 \right]^{-2/3} + \frac{2}{\pi} \right\}^{-1}$$

$$\hat{\beta}_0 = \hat{\beta}_0(\text{OLS}) + \sqrt{\frac{2}{\pi}} \left[\sqrt{\frac{\pi}{2}} . \frac{\pi}{(\pi-4)} m_3 \right]^{1/3}$$

and the estimator of σ^2 from Equation 3 remains:

$$\hat{\sigma}^2 = m_2 + \frac{2}{\pi} \left[\sqrt{\frac{\pi}{2}} . \frac{\pi}{(\pi-4)} m_3 \right]^{2/3} .$$

We now use Taylor series expansions, truncated after the first order terms, to obtain approximations of the asymptotic standard errors of the COLS estimators. To do this for $\hat{\beta}_0$ we need to specify the asymptotic covariance between $\hat{\beta}_0(\text{OLS})$ and the third sample moment. OSW note that it is equal to zero when the model contains an intercept and no other regressors. Assuming this result extends to the general case, we obtain:

$$V(\hat{\beta}_0) = \left[\frac{\partial \hat{\beta}_0}{\hat{\beta}_0(\text{OLS})} \right]^2 . V(\partial \hat{\beta}_0(\text{OLS})) + \left[\frac{\partial \hat{\beta}_0}{\partial m_3} \right]^2 . V(m_3)$$

$$V(\hat{\sigma}^2) = \left[\frac{\partial \hat{\sigma}^2}{\partial m_2} \right]^2 . V(m_2) + \left[\frac{\partial \hat{\sigma}^2}{\partial m_3} \right]^2 . V(m_3) + \left[\frac{\partial \hat{\sigma}^2}{\partial m_2} \right] \left[\frac{\partial \hat{\sigma}^2}{\partial m_3} \right] \text{Cov}(m_2, m_3)$$

$$V(\hat{\gamma}) = \left[\frac{\partial \hat{\gamma}}{\partial m_2} \right]^2 . V(m_2) + \left[\frac{\partial \hat{\gamma}}{\partial m_3} \right]^2 . V(m_3) + \left[\frac{\partial \hat{\gamma}}{\partial m_2} \right] \left[\frac{\partial \hat{\gamma}}{\partial m_3} \right] \text{Cov}(m_2, m_3),$$

where

$$\left[\frac{\partial\hat{\beta}_0}{\partial\hat{\beta}_0(\mathrm{OLS})}\right] = 1$$

$$\left[\frac{\partial\hat{\beta}_0}{\partial m_3}\right] = \frac{\pi}{(\pi-4)}\cdot\frac{1}{3}\left[\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}m_3\right]^{-2/3}$$

$$\left[\frac{\partial\hat{\sigma}^2}{\partial m_2}\right] = 1$$

$$\left[\frac{\partial\hat{\sigma}^2}{\partial m_3}\right] = \sqrt{\frac{2}{\pi}}\cdot\frac{\pi}{(\pi-4)}\cdot\frac{2}{3}\left[\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}m_3\right]^{-1/3}$$

$$\left[\frac{\partial\hat{\gamma}}{\partial m_2}\right] = -\left[\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}m_3\right]^{-2/3}\left\{m_3\left[\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}m_3\right]^{-2/3}+\frac{2}{\pi}\right\}^{-2}$$

$$\left[\frac{\partial\hat{\gamma}}{\partial m_3}\right] = m_2\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}\cdot\frac{2}{3}\left[\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}m_3\right]^{-5/3}\left\{m_3\left[\sqrt{\frac{\pi}{2}}\cdot\frac{\pi}{(\pi-4)}m_3\right]^{-2/3}+\frac{2}{\pi}\right\}^{-2}.$$

Appendix 2

SHAZAM code for COLS

This code assumes we have a data file (eg.dta) which contains three columns of data, each of length 60, on output and two inputs. The code obtains OLS estimates of a Cobb Douglas production function. It then calculates the third moment test statistic before calculating the COLS estimates and their standard errors.

```
sample 1 60
read(eg.dta) y x1 x2
genr ly=log(y)
genr lx1=log(x1)
genr lx2=log(x2)
ols ly lx1 lx2/resid=e coef=b stderr=se
gen1 b0ols=b:3
gen1 vb0ols=(se:3)**2
genr e2=e**2
stat e2/mean=m2
genr e3=e**3
stat e3/mean=m3
gen1 n=60
gen1 m3t=m3/sqrt(6*m2**3/n)
* third moment test:
print m3t
gen1 pi=3.142
gen1 s2=m2+2/pi*(sqrt(pi/2)*pi/(pi-4)*m3)**(2/3)
gen1 g=(sqrt(pi/2)*pi/(pi-4)*m3)**(2/3)/s2
gen1 b0=b0ols+sqrt(2*g*s2/pi)
* COLS estimates:
print s2 g b0
gen1 u2=s2*((1-g)+g*(pi-2)/pi)
gen1 u3=s2**(3/2)*(sqrt(2/pi)*(1-4/pi)*g**(3/2))
gen1 u4=s2**2*(3*(1-g)**2+6*(pi-2)/pi*g*(1-g)+(3-4/pi-12/pi**2)*g**2)
gen1 u5=s2**(5/2)*g**(3/2)*sqrt(2/pi)*(10*(1-4/pi)*(1-g)+(7-20/pi-16/pi**2)*g)
gen1 u6=s2**3*(15*(1-g)**3+45*(pi-2)/pi*(1-g)**2*g &
        +15*(3-4/pi-12/pi**2)*(1-g)*g**2+(15-6/pi-100/pi**2-40/pi**3)*g**3)
gen1 vm2=1/n*(u4-u2**2)
gen1 vm3=1/n*(u6-u3**2-6*u2*u4+9*u2**3)
gen1 cov=1/n*(u5-4*u2*u3)
gen1 tmp=sqrt(pi/2)*pi/(pi-4)*m3
gen1 db0dm3=pi/(pi-4)/3*tmp**(-2/3)
gen1 ds2dm3=sqrt(2/pi)*pi/(pi-4)*2/3*tmp**(-1/3)
```

```
gen1 dgdm2=-(tmp**(-2/3))*(tmp**(-2/3)+2/pi)**(-2)
gen1 dgdm3=m2*sqrt(pi/2)*pi/(pi-4)*2/3*tmp**(-5/3)*(m2*tmp**(-2/3)+2/pi)**(-2)
gen1 seb0=sqrt(vb0ols+db0dm3**2*vm3)
gen1 ses2=sqrt(vm2+ds2dm3**2*vm3+2*ds2dm3*cov)
gen1 seg=sqrt(dgdm2**2*vm2+dgdm3**2*vm3+2*dgdm2*dgdm3*cov)
* standard errors of COLS estimates:
print seb0 ses2 seg
stop
```

TABLE 1

MAXIMUM LIKELIHOOD ESTIMATES

		BIAS			VARIANCE			MSE		
γ	N	$\hat{\beta}_0$	$\hat{\sigma}^2$	$\hat{\gamma}$	$\hat{\beta}_0$	$\hat{\sigma}^2$	$\hat{\gamma}$	$\hat{\beta}_0$	$\hat{\sigma}^2$	$\hat{\gamma}$
0.00	50	0.4811	0.4355	0.3579	0.2399	0.3321	0.1344	0.4714	0.5217	0.2625
0.00	100	0.3885	0.3001	0.2774	0.1683	0.1763	0.0937	0.3192	0.2664	0.1706
0.00	400	0.2729	0.1580	0.1721	0.0851	0.0526	0.0449	0.1595	0.0775	0.0745
0.00	800	0.2348	0.1215	0.1412	0.0670	0.0321	0.0339	0.1221	0.0469	0.0538
0.05	50	0.2863	0.3815	0.2967	0.2284	0.3389	0.1303	0.3104	0.4845	0.2183
0.05	100	0.1847	0.2453	0.2117	0.1604	0.1597	0.0926	0.1945	0.2199	0.1374
0.05	400	0.0898	0.1229	0.1243	0.0838	0.0502	0.0460	0.0919	0.0653	0.0614
0.05	800	0.0553	0.0882	0.0938	0.0661	0.0314	0.0343	0.0691	0.0392	0.0431
0.10	50	0.2031	0.3347	0.2503	0.2100	0.2821	0.1258	0.2512	0.3942	0.1885
0.10	100	0.1224	0.2171	0.1748	0.1532	0.1626	0.0912	0.1682	0.2097	0.1217
0.10	400	0.0159	0.0883	0.0785	0.0816	0.0464	0.0458	0.0819	0.0542	0.0520
0.10	800	-0.0068	0.0612	0.0547	0.0646	0.0300	0.0353	0.0647	0.0338	0.0383
0.25	50	0.0611	0.2310	0.1192	0.2138	0.2789	0.1333	0.2175	0.3323	0.1475
0.25	100	-0.0035	0.1315	0.0607	0.1468	0.1461	0.1005	0.1468	0.1634	0.1041
0.25	400	-0.0905	0.0184	-0.0239	0.0830	0.0481	0.0568	0.0912	0.0484	0.0574
0.25	800	-0.0884	0.0000	-0.0329	0.0634	0.0317	0.0414	0.0712	0.0317	0.0425
0.50	50	-0.1184	0.0062	-0.0959	0.1593	0.1718	0.1319	0.1734	0.1719	0.1411
0.50	100	-0.1028	-0.0054	-0.0850	0.1127	0.1083	0.0970	0.1232	0.1083	0.1042
0.50	400	-0.0763	-0.0324	-0.0728	0.0523	0.0366	0.0473	0.0581	0.0376	0.0526
0.50	800	-0.0545	-0.0273	-0.0556	0.0314	0.0254	0.0318	0.0343	0.0261	0.0349
0.75	50	-0.1200	-0.0311	-0.1450	0.1310	0.1736	0.1282	0.1454	0.1746	0.1492
0.75	100	-0.1011	-0.0704	-0.1170	0.0737	0.0742	0.0797	0.0839	0.0791	0.0934
0.75	400	-0.0368	-0.0415	-0.0400	0.0107	0.0191	0.0121	0.0120	0.0208	0.0137
0.75	800	-0.0221	-0.0265	-0.0218	0.0046	0.0094	0.0047	0.0050	0.0101	0.0052
0.90	50	-0.0570	-0.0507	-0.0715	0.0480	0.1018	0.0492	0.0512	0.1044	0.0543
0.90	100	-0.0203	-0.0237	-0.0196	0.0130	0.0389	0.0089	0.0134	0.0394	0.0093
0.90	400	-0.0124	-0.0188	-0.0108	0.0032	0.0109	0.0015	0.0033	0.0112	0.0016
0.90	800	-0.0099	-0.0138	-0.0066	0.0014	0.0053	0.0006	0.0014	0.0055	0.0007
0.95	50	-0.0283	-0.0467	-0.0326	0.0221	0.0630	0.0162	0.0229	0.0652	0.0173
0.95	100	-0.0088	-0.0172	-0.0168	0.0103	0.0376	0.0062	0.0104	0.0379	0.0064
0.95	400	-0.0060	-0.0162	-0.0032	0.0016	0.0083	0.0004	0.0016	0.0086	0.0004
0.95	800	-0.0032	-0.0061	-0.0014	0.0008	0.0043	0.0001	0.0008	0.0044	0.0002
1.00	50	-0.0311	-0.0605	-0.0062	0.0067	0.0592	0.0008	0.0077	0.0629	0.0008
1.00	100	-0.0140	-0.0331	-0.0011	0.0045	0.0573	0.0000	0.0047	0.0584	0.0000
1.00	400	-0.0413	-0.0767	-0.0002	0.0059	0.0156	0.0000	0.0076	0.0215	0.0000
1.00	800	-0.0547	-0.0846	0.0000	0.0021	0.0037	0.0000	0.0051	0.0108	0.0000

TABLE 2

CORRECTED ORDINARY LEAST SQUARES

		BIAS			VARIANCE			MSE		
γ	N	$\hat{\beta}_0$	$\hat{\sigma}^2$	$\hat{\gamma}$	$\hat{\beta}_0$	$\hat{\sigma}^2$	$\hat{\gamma}$	$\hat{\beta}_0$	$\hat{\sigma}^2$	$\hat{\gamma}$
0.00	50	0.4379	0.3460	0.3289	0.1860	0.2121	0.1024	0.3778	0.3318	0.2106
0.00	100	0.3730	0.2728	0.2679	0.1538	0.1354	0.0826	0.2929	0.2099	0.1543
0.00	400	0.2777	0.1633	0.1803	0.0876	0.0496	0.0436	0.1647	0.0763	0.0761
0.00	800	0.2386	0.1267	0.1488	0.0703	0.0308	0.0330	0.1273	0.0469	0.0552
0.05	50	0.2447	0.2982	0.2717	0.1794	0.2055	0.1017	0.2393	0.2945	0.1755
0.05	100	0.1736	0.2266	0.2064	0.1499	0.1295	0.0839	0.1800	0.1809	0.1266
0.05	400	0.0956	0.1267	0.1319	0.0845	0.0465	0.0438	0.0937	0.0625	0.0612
0.05	800	0.0568	0.0923	0.1001	0.0694	0.0302	0.0335	0.0726	0.0387	0.0435
0.10	50	0.1669	0.2610	0.2278	0.1693	0.1781	0.1017	0.1972	0.2462	0.1535
0.10	100	0.1141	0.1967	0.1711	0.1369	0.1266	0.0807	0.1500	0.1653	0.1100
0.10	400	0.0172	0.0924	0.0847	0.0852	0.0451	0.0454	0.0855	0.0536	0.0526
0.10	800	-0.0068	0.0652	0.0611	0.0688	0.0289	0.0347	0.0689	0.0331	0.0384
0.25	50	0.0185	0.1530	0.0914	0.1596	0.1754	0.1041	0.1600	0.1988	0.1125
0.25	100	-0.0134	0.1096	0.0545	0.1325	0.1160	0.0880	0.1327	0.1280	0.0910
0.25	400	-0.0912	0.0180	-0.0213	0.0828	0.0447	0.0542	0.0912	0.0450	0.0547
0.25	800	-0.0849	0.0034	-0.0265	0.0644	0.0300	0.0396	0.0716	0.0300	0.0403
0.50	50	-0.1406	-0.0318	-0.1120	0.1365	0.1316	0.1132	0.1563	0.1326	0.1258
0.50	100	-0.1093	-0.0171	-0.0876	0.1072	0.0909	0.0908	0.1191	0.0912	0.0985
0.50	400	-0.0749	-0.0329	-0.0716	0.0503	0.0355	0.0447	0.0559	0.0366	0.0499
0.50	800	-0.0544	-0.0271	-0.0541	0.0315	0.0245	0.0306	0.0345	0.0252	0.0335
0.75	50	-0.1592	-0.0969	-0.1784	0.1135	0.1267	0.1113	0.1388	0.1361	0.1431
0.75	100	-0.1148	-0.0911	-0.1316	0.0694	0.0710	0.0732	0.0826	0.0793	0.0905
0.75	400	-0.0496	-0.0584	-0.0539	0.0105	0.0179	0.0120	0.0130	0.0213	0.0149
0.75	800	-0.0339	-0.0423	-0.0345	0.0045	0.0088	0.0048	0.0057	0.0106	0.0060
0.90	50	-0.0911	-0.0885	-0.1128	0.0506	0.1136	0.0474	0.0588	0.1214	0.0601
0.90	100	-0.0468	-0.0550	-0.0500	0.0195	0.0501	0.0132	0.0217	0.0531	0.0158
0.90	400	-0.0310	-0.0450	-0.0289	0.0046	0.0130	0.0031	0.0055	0.0150	0.0039
0.90	800	-0.0273	-0.0392	-0.0225	0.0020	0.0059	0.0014	0.0027	0.0074	0.0019
0.95	50	-0.0706	-0.0871	-0.0858	0.0324	0.0858	0.0220	0.0374	0.0933	0.0293
0.95	100	-0.0468	-0.0651	-0.0561	0.0162	0.0465	0.0096	0.0184	0.0507	0.0128
0.95	400	-0.0310	-0.0526	-0.0246	0.0034	0.0112	0.0015	0.0044	0.0140	0.0021
0.95	800	-0.0240	-0.0371	-0.0188	0.0018	0.0064	0.0008	0.0024	0.0077	0.0012
1.00	50	-0.0817	-0.1061	-0.0754	0.0194	0.0804	0.0072	0.0261	0.0916	0.0129
1.00	100	-0.0561	-0.0748	-0.0487	0.0102	0.0390	0.0033	0.0134	0.0446	0.0057
1.00	400	-0.0386	-0.0584	-0.0273	0.0029	0.0105	0.0008	0.0044	0.0139	0.0015
1.00	800	-0.0292	-0.0402	-0.0206	0.0013	0.0052	0.0003	0.0021	0.0068	0.0008

TABLE 3

HYPOTHESIS TESTS: PERCENTAGE OF REJECTIONS OF THE NULL

γ	N	Wald	Likelihood Ratio (LR)	One-sided LR	Bera Jarque	Third Moment
0.00	50	32.5	3.0	6.0	2.5	3.0
0.00	100	24.0	1.5	4.0	1.0	3.5
0.00	400	14.5	2.5	4.5	2.0	4.5
0.00	800	15.5	1.0	4.5	0.0	4.5
0.05	50	30.0	4.5	7.5	2.5	2.0
0.05	100	24.5	2.0	4.0	1.0	4.5
0.05	400	14.0	2.5	4.0	2.0	4.5
0.05	800	15.5	2.0	5.0	0.0	4.0
0.10	50	28.0	3.5	6.0	2.0	2.5
0.10	100	23.5	2.0	4.0	2.5	4.0
0.10	400	15.0	2.0	3.5	1.5	4.0
0.10	800	17.5	0.5	4.5	0.5	3.5
0.25	50	31.5	4.5	7.0	1.5	3.0
0.25	100	29.0	3.5	8.5	1.5	6.0
0.25	400	24.5	4.0	7.0	3.0	7.0
0.25	800	27.5	5.0	8.0	2.5	8.0
0.50	50	39.5	6.0	10.5	4.5	6.5
0.50	100	42.0	5.5	10.5	3.0	8.0
0.50	400	60.0	10.5	22.0	9.5	21.0
0.50	800	74.5	32.0	42.0	20.5	43.0
0.75	50	61.0	20.0	31.5	10.5	18.5
0.75	100	73.0	28.5	40.5	15.0	36.5
0.75	400	97.0	80.5	88.5	62.5	88.0
0.75	800	99.0	97.0	98.0	94.5	98.0
0.90	50	86.0	53.0	61.0	23.5	47.5
0.90	100	97.5	78.5	88.5	53.0	79.5
0.90	400	100.0	100.0	100.0	99.5	100.0
0.90	800	100.0	100.0	100.0	100.0	100.0
0.95	50	96.0	70.5	78.5	34.0	61.0
0.95	100	99.5	93.5	95.5	69.0	90.5
0.95	400	100.0	100.0	100.0	100.0	100.0
0.95	800	100.0	100.0	100.0	100.0	100.0
1.00	50	98.0	98.0	98.5	47.0	82.0
1.00	100	97.0	100.0	100.0	95.0	99.0
1.00	400	99.0	100.0	100.0	100.0	100.0
1.00	800	100.0	100.0	100.0	100.0	100.0

WORKING PAPERS IN ECONOMETRICS AND APPLIED STATISTICS

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