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A COMPARISON OF ASYMPTOTIC THEORY AND SIMULATION APPROACHES

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ABSTRACT

Impulse responses are standard tools in applied work for analysing the interrelationships between the variables of vector autoregressive (VAR) models. Asymptotic theory or simulation and bootstrapping methods are usually used for measuring the estimation variability of estimated impulse responses. In this study the small sample properties of these different approaches are compared in a Monte Carlo investigation. The results indicate that, in terms of their actual level, confidence intervals based on asymptotic theory are at least as good as confidence intervals obtained with simulation and bootstrapping methods, even in situations where the asymptotic theory is used incorrectly.

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1. Introduction

In a seminal paper Sims (1980) criticized conventional macroeconometric modelling procedures and proposed an alternative strategy based on vector autoregressive (VAR) models. Since that time the VAR approach has been widely used in applied work. Important information provided by a VAR model is the set of impulse response coefficients. Usually it is difficult, if not impossible, to directly interpret the coefficients of an estimated VAR model. Thus, impulse responses are often computed in order to study the interrelationships within the variables of a system.

The impulse responses represent the reactions of the system to exogenous shocks. They are functions of the VAR parameters and are estimated accordingly. Different approaches have been used to measure the sampling variability of the resulting estimators. One possibility is to employ standard asymptotic theory and use the asymptotic distribution to compute standard errors, t-ratios or confidence intervals for the impulse responses. Alternatively, bootstrap and simulation methods have been used for that purpose. Yet, little seems to be known about the relative merits of these different procedures. Some results on this question are provided in this paper. Confidence intervals for the impulse responses, based on asymptotic theory and on different simulation and bootstrap type procedures, are compared on the basis of their finite sample accuracy.

The plan of the paper is as follows. In the next section the general framework is laid out and the different methods for obtaining confidence intervals for the impulse responses are described. In Section 3 the details of the Monte Carlo experiment are given; the results are discussed in Section 4, followed by conclusions in Section 5.

2. The General Framework

2.1 VAR Processes and Impulse Responses

Suppose a set of K variables $y_t = (y_{1t}, \dots, y_{Kt})'$ is generated by a VAR(p) process of the form

$$y_t - \mu = A_1(y_{t-1} - \mu) + \dots + A_p(y_{t-p} - \mu) + u_t, \quad (2.1)$$

where $\mu = (\mu_1, \dots, \mu_K)'$ is the $(K \times 1)$ mean vector of the process, the A_i are $(K \times K)$ coefficient matrices and $u_t = (u_{1t}, \dots, u_{Kt})'$ is zero mean K -dimensional white noise with nonsingular covariance matrix $\Sigma_u = E(u_t u_t')$.

Alternative forms of impulse responses have been considered in VAR analyses. Some authors prefer the responses of the system to forecast errors whereas others consider the responses to orthogonalised or uncorrelated residuals. The former may be obtained recursively from

$$\Phi_i = [\phi_{kl,i}]_{k,l} = \Phi_i(A_1, \dots, A_p) = \sum_{j=1}^i \Phi_{i-j} A_j, \quad i = 1, 2, \dots, \quad (2.2)$$

where $\Phi_0 = I_K$, $A_j = 0$ for $j > p$ and $\phi_{kl,i}$ is the kl -th element of Φ_i , and represents the response of variable k to a one unit forecast error or residual in variable l , i periods ago, providing no other shocks contaminate the system (e.g., Lütkepohl (1990)). The notation $\Phi_i(A_1, \dots, A_p)$ is used to indicate that the impulse responses are functions of the VAR coefficients.

The responses to orthogonalised residuals may be obtained as

$$\Theta_i = [\theta_{kl,i}]_{k,l} = \Theta_i(A_1, \dots, A_p, \Sigma_u) = \Phi_i P, \quad i = 0, 1, \dots, \quad (2.3)$$

where P is a lower triangular matrix with positive diagonal elements satisfying $PP' = \Sigma_u$. In other words, P is determined by a Choleski decomposition of Σ_u . The kl -th element of Θ_i , $\theta_{kl,i}$, is interpreted as the response of variable k to an impulse in variable l one residual standard

deviation in magnitude, i periods ago. The ϕ and θ impulse responses are the quantities of interest in the following.

2.2 Estimation of the VAR Coefficients and Impulse Responses

Suppose a sample of size T , y_1, \dots, y_T , and p presample values are available for estimating the VAR(p) process (2.1). The sample mean $\bar{y} = \sum_{t=1}^T y_t / T$ is often used as an estimator for the mean vector μ and the coefficients $\underline{A} = [A_1, \dots, A_p]$ are usually estimated by multivariate least squares (LS) or by Yule-Walker estimation, and sometimes Bayesian restrictions are imposed. We use the LS estimator for \underline{A} based on mean adjusted data,

$$\hat{\underline{A}} = YX'(XX')^{-1}, \quad (2.4)$$

where

$$Y = [y_1 - \bar{y}, \dots, y_T - \bar{y}] \quad \text{and} \quad X = [X_0, \dots, X_{T-1}] \quad \text{with} \quad X_t = \begin{bmatrix} y_t - \bar{y} \\ \vdots \\ y_{t-p+1} - \bar{y} \end{bmatrix}$$

$(K \times T)$ $(Kp \times T)$ $(Kp \times 1)$

If the process is Gaussian with $u_t \sim N(0, \Sigma_u)$ this is the maximum likelihood (ML) estimator conditional on the presample values. In small samples it differs slightly from the Yule-Walker estimator. Our estimator for Σ_u is

$$\hat{\Sigma}_u = (Y - \hat{\underline{A}}X)(Y - \hat{\underline{A}}X)' / T. \quad (2.5)$$

Using these estimators in (2.2) and (2.3) gives estimators $\hat{\phi}_1$ and $\hat{\theta}_1$ of the impulse responses.

For a stationary Gaussian process the estimators are consistent and asymptotically normally distributed,

$$\sqrt{T} \begin{bmatrix} \text{vec}(\hat{\underline{A}} - \underline{A}) \\ \text{vech}(\hat{\Sigma}_u - \Sigma_u) \end{bmatrix} \xrightarrow{d} N \left(0, \begin{bmatrix} \Sigma_\alpha & 0 \\ 0 & \Sigma_\sigma \end{bmatrix} \right), \quad (2.6)$$

where vec is the column vectorizing operator, vech is its counterpart stacking only the elements on and below the diagonal,

$$\Sigma_{\alpha} = \text{plim } (XX'/T)^{-1} \otimes \Sigma_{\mathbf{u}} \quad (2.7)$$

and

$$\Sigma_{\sigma} = 2D_K^+(\Sigma_{\mathbf{u}} \otimes \Sigma_{\mathbf{u}})D_K^{+'}, \quad (2.8)$$

with $D_K^+ = (D_K'D_K)^{-1}D_K'$ being the Moore-Penrose generalised inverse of the $(K^2 \times K(K+1)/2)$ duplication matrix D_K (see, e.g., Magnus & Neudecker (1988, pp. 48-49) for the definition of this matrix). As usual, \otimes denotes the Kronecker product. Obvious consistent estimators of the asymptotic covariance matrices are

$$\hat{\Sigma}_{\alpha} = (XX'/T)^{-1} \otimes \hat{\Sigma}_{\mathbf{u}} \text{ and } \hat{\Sigma}_{\sigma} = 2D_K^+(\hat{\Sigma}_{\mathbf{u}} \otimes \hat{\Sigma}_{\mathbf{u}})D_K^{+'}. \quad (2.9)$$

From these asymptotic results it follows that $\hat{\Phi}_i$ and $\hat{\Theta}_i$ are also consistent estimators with asymptotic normal distributions,

$$\sqrt{T} \text{vec}(\hat{\Phi}_i - \Phi_i) \xrightarrow{d} N(0, \Sigma_i), \quad i = 1, 2, \dots, \quad (2.10)$$

where $\Sigma_i = G_i \Sigma_{\alpha} G_i'$, with $G_i = \partial \text{vec}(\Phi_i) / \partial \text{vec}(\underline{A})'$. Similarly,

$$\sqrt{T} \text{vec}(\hat{\Theta}_i - \Theta_i) \xrightarrow{d} N(0, \Omega_i), \quad i = 0, 1, \dots, \quad (2.11)$$

where $\Omega_i = C_i \Sigma_{\sigma} C_i' + F_i \Sigma_{\sigma} F_i'$ with $C_i = \partial \text{vec}(\Theta_i) / \partial \text{vec}(\underline{A})'$ and

$F_i = \partial \text{vec}(\Theta_i) / \partial \text{vech}(\Sigma_{\sigma})$. Closed form expressions for G_i , C_i , and F_i are given for instance, in Lütkepohl (1990).

Consistent estimators of the covariance matrices are obtained by replacing all unknown quantities with the estimators described in the foregoing. These estimators may be used in the usual way to obtain "t-ratios" and confidence intervals for the individual coefficients. It should be noted, however, that Σ_i and Ω_i may be singular with zero elements on the diagonal. For instance, if y_t is actually a VAR(0) white noise process and a VAR(1) process is fitted, $\Sigma_i = 0$ for $i = 2, 3, \dots$. In such a case the corresponding parameter estimators converge to their actual values more rapidly than at the usual \sqrt{T} -rate and the usual "t-ratios" will in general not have an asymptotic standard normal distribution. Under these circumstances the actual confidence level of confidence intervals from (2.10) and (2.11) may be different from the assumed level.

The asymptotic results given here hold for stationary processes. In their general form they also remain valid for cointegrated processes (see Park and Phillips (1989)). The covariance matrix Σ_α will be different from (2.7) in this case. However, the estimator given in (2.9) remains a consistent estimator for Σ_α . Hence, from a practical point of view we may proceed as in the stationary case.

An alternative asymptotic theory exists that can be used if the VAR order is unknown and potentially infinite. That theory proceeds on the assumption that the order of the process fitted to the data goes to infinity with the sample size (e.g., Lütkepohl, (1988)). As a result of the Monte Carlo setup used in the present study such an assumption is not reasonable here and is therefore not given further consideration.

2.3 Simulation Approaches

As alternatives to the use of asymptotic theory for the construction of impulse-response confidence intervals, simulation and bootstrapping procedures

are often used. The motivation for using such procedures is the belief that, in finite samples that are not large, they will provide a more accurate assessment of estimator reliability. Also, although the general asymptotic theory is valid for nonnormal processes, its performance may deteriorate if the process distribution is markedly different from the normal. Moreover, the asymptotic distribution of the orthogonal (θ) impulse responses depends on the process distribution because Σ_{σ} , the asymptotic covariance matrix of the elements of $\hat{\Sigma}_u$, depends on that distribution. If the process distribution is unknown and nonnormal and one proceeds under an incorrect assumption that the process is normal and thus uses the incorrect asymptotic distribution of the θ impulse-responses, we might expect bootstrap methods that are based on the empirical distribution of the residuals to have an advantage.

To use simulation or bootstrapping methods we proceed with the following steps.

Step 1: Given a sample of size T plus p presample values compute the LS/ML estimators \hat{A} and $\hat{\Sigma}_u$.

Step 2: Generate N sets of residuals $U(n) = [u_1(n), \dots, u_T(n)]$, $n = 1, \dots, N$, and, based on these residuals, generate new samples

$$y_t(n) = \bar{y} + \hat{A} \begin{bmatrix} y_{t-1}^{(n)} - \bar{y} \\ \vdots \\ y_{t-p}^{(n)} - \bar{y} \end{bmatrix} + u_t(n), \quad t = 1, \dots, T.$$

Step 3: For each generated sample determine $\bar{y}(n)$ and LS estimators $\hat{A}(n)$, $\hat{\Sigma}_u(n)$ and the corresponding impulse response estimates $\hat{\phi}_1(n)$ and $\hat{\theta}_1(n)$.

Step 4: From the resulting empirical distributions of the impulse responses

- a) determine the empirical standard deviations and set up confidence intervals with quantiles from a normal distribution table; or

- b) determine the empirical quantiles and use them to set up confidence intervals.

In Step 2 we use two different ways to generate residuals. The first possibility is to draw the $u_t(n)$ from a multivariate $N(0, \hat{\Sigma}_u)$ distribution whereas the second possibility is to draw randomly, with replacement, from the LS residuals $\hat{U} = [\hat{u}_1, \dots, \hat{u}_T] = Y - \hat{A}X$. In the following the latter approach will be referred to as bootstrapping and the former is called simulation with normal residuals.

In total we have described five different methods to set up confidence intervals for individual impulse response coefficients. We will number them as follows:

- 1 - based on asymptotic theory,
- 2 - based on standard errors obtained from simulation with normal residuals,
- 3 - based on empirical quantiles obtained from simulation with normal residuals,
- 4 - based on standard errors obtained from bootstrapping,
- 5 - based on empirical quantiles obtained from bootstrapping.

3. The Monte Carlo Setup

The finite sample accuracies of the five confidence interval methods were evaluated within the framework of a Monte Carlo study. In a study of this type the number of alternative setups is enormous. Choices have to be made concerning the dimension and order of the VAR process, the settings for μ , A_1 , A_2, \dots, A_p and Σ_u , the error distribution, whether the order of the VAR process is assumed known or estimated, the sample size T , the number of simulations (N) for the simulation techniques, and the number of replications. Furthermore, the computational task is immense in a Monte Carlo experiment

where each replication involves a large number of simulations. We therefore had to be selective with respect to the number of different processes considered. We focussed on bivariate VAR(1) processes with $\mu = 0$ and VAR coefficient matrices

$$A_1 = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix},$$

where a_{11} , a_{21} , and a_{22} are varying. If $-1 < a_{11}, a_{22} < 1$, the corresponding process is stationary. If $a_{11} = 1$, the two variables of the system are cointegrated, and if $a_{21} = a_{22} = 0$ some of the impulse responses will have zero variances so that the standard asymptotic theory does not apply.

The choice of the process mean $\mu = 0$ should not have an impact on the results since the impulse responses do not involve the process mean. Note that we still subtracted the sample mean from the data as explained in the previous section although the mean is actually zero.

Three different error distributions were used, namely multivariate normal, t- and χ^2 -distributions. For $u_t \sim N(0, \Sigma_u)$ a covariance matrix

$$\Sigma_u = \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix} \quad (3.1)$$

was chosen. It may be worth noting that multiplying this matrix by some constant does not affect the results. Therefore normalising the variances is justified. The choice of the off-diagonal elements, that is the choice of the contemporaneous correlation of the residuals may have some impact on the results.

The bivariate t distribution was one with 4 degrees of freedom and with inverse precision matrix Σ_u as specified in (3.1). Bivariate χ^2 errors with 3 degrees of freedom were generated through the sum of squares of three

independent bivariate normal random variables. The correlation for each bivariate normal random variable was set, and the bivariate $\chi_{(3)}^2$ variables were transformed, to yield "normalized" bivariate $\chi_{(3)}^2$ errors with zero mean vector and covariance matrix Σ_u as given in (3.1). In the simulation with normal residuals we used normal residuals even if the true error distribution was nonnormal. Also we have always assumed an asymptotic covariance matrix Σ_σ as in (2.8) for the elements of $\hat{\Sigma}_u$ although this is incorrect for nonnormal error distributions. In practice, where the true error distribution is unknown, it is common to assume a normal distribution.

In the cases with normal and $t_{(4)}$ errors, the initial vector y_0 in each replication was generated from, respectively, a normal or $t_{(4)}$ distribution that was consistent with the error distribution. That is, y_0 had zero mean and inverse precision matrix $\Sigma_y = E[y_t y_t']$ where $\text{vec}(\Sigma_y) = (I_4 - (A_1 \otimes A_1))^{-1} \text{vec}(\Sigma_u)$. When the errors follow a $\chi_{(3)}^2$ distribution the observations y_t will not also follow a $\chi_{(3)}^2$ distribution since the weighted average of $\chi_{(3)}^2$ errors, as given by the moving-average representation, is not a $\chi_{(3)}^2$ distribution. Thus, in this case, y_0 was generated from a $N(0, \Sigma_y)$ distribution. In all cases y_0 was used as the presample vector in LS estimation.

In the simulation method that assumed normal residuals we used

$$y_0^{(n)} \begin{cases} \sim N(0, \hat{\Sigma}_y) & \text{if } \hat{A}_1 \text{ had both characteristic roots less than 1} \\ & \text{and thus the estimated process was stationary} \\ = y_0 & \text{if } \hat{A}_1 \text{ had at least one characteristic root greater} \\ & \text{than or equal to 1} \end{cases}$$

for $n = 1, \dots, N$. In the bootstrap we always used the initial vector from the original sample, that is, $y_0^{(n)} = y_0$.

The impulse responses considered are $\phi_{kl, i}$, $i = 1, 2, 3$ and $\theta_{kl, j}$, $j = 0, 1, 2, 3$. Among other things we determined the number of inclusions of the

true impulse response coefficients in 90% and 95% confidence intervals estimated by the 5 methods listed in the previous section.

Two sample sizes were considered, $T = 50$ and $T = 100$. The results for $T = 50$ are based on a different set of random numbers than those for $T = 100$. In each replication the number of bootstrap and simulation runs is $N = 100$. Since preliminary experiments with $N = 200$ did not lead to much change in the results, the smaller N was settled upon. The number of replications for each set of parameter values is $R = 200$. Finally, the programs used to perform the computations were written in SHAZAM.

4. The Results

Qualitatively similar results were obtained for 90% and 95% confidence intervals. Therefore we will concentrate on 95% intervals as they seem to be the more common ones used in practice. We will discuss the dependence of the results on the parameter values, the error distribution and the sample size.

We will begin with a discussion of the results for the ϕ impulse responses. For these we have set $\sigma_{12} = 0.3$. Preliminary simulations with other Σ_u matrices did not yield results that prompted further investigation. A small correlation coefficient was felt to be a common occurrence in applied work with real data. In Figure 1 the proportions of inclusions of $\phi_{11,i}$ and $\phi_{21,i}$ in estimated 95% confidence intervals are plotted for normal errors, sample size $T = 50$, $a_{21} = a_{22} = 0.5$, and different values of a_{11} . A nonstationary, cointegrated process is included as a boundary case for $a_{11} = 1$. The asymptotic variances of all impulse response coefficients are nonzero so that the standard asymptotic theory remains valid. From the figure it can be seen that for lag 1 (where $\phi_{11,1} = a_{11}$ and $\phi_{21,1} = a_{21}$) all methods perform about equally well. An exception is the confidence intervals for

$\phi_{11,1}$ for values of a_{11} that approach 1; under these circumstances the two empirical quantile methods are considerably worse than the other methods and fall well below what could be attributed to sampling variability. In a Monte Carlo experiment with 200 replications the standard error for the proportion estimates is 0.015, so, roughly speaking, proportion estimates that lie between 0.92 and 0.98 can be considered reasonable (or attributable to sample variability). For higher lags the actual confidence level differs even more from the intended theoretical level of 95%, and for all methods and for a larger part of the parameter space. It is interesting to note that the direction of the deviation from the intended level is the same for all methods and there is no clearly superior or clearly inferior method over the whole parameter space, although for the cointegrated process, and for large positive a_{11} , the quantile methods 3 and 5 perform markedly poorer than the other methods.

In Figure 2 we have depicted the proportions of inclusions of zero in estimated confidence intervals for $\phi_{11,i}$ and $\phi_{21,i}$. Often one will be interested in whether there is an effect at all from an impulse in one variable. As a crude test one might check whether zero falls within the confidence intervals for the response coefficients. Thus, in Figure 2 the power of such a test for an individual coefficient is depicted. For lag 1 the power of all methods is quite similar. For lag 2 a slight superiority of the quantile methods becomes visible for a_{11} values close to zero. For lag 3 this superiority is quite strong, and extends to negative values of a_{11} when testing $\phi_{11,3}$.

Figure 3 corresponds exactly to Figure 1 except that the error distribution of the underlying processes is a bivariate t rather than a normal. For nonnormal errors one might expect an advantage from bootstrapping

methods, while the simulation methods are actually performed on false assumptions. This, however, is not reflected in the results which are largely similar to those from the normal error case. The same turned out to be true for the $\chi^2_{(3)}$ errors. We will therefore not report the results here.

Increasing the sample size from $T = 50$ to $T = 100$ did not change the situation drastically. In particular the general patterns in Figures 1 - 3 did not change much although the estimated confidence levels were overall closer to the intended level of 95% and the powers increased where they had not been 1 before. It is interesting, however, that the relative power advantage of the quantile methods 3 and 5 for values of a_{11} close to zero was maintained. To conserve space we do not give the results in detail here.

We have also considered processes with $A_1 = 0$ for which some asymptotic variances are zero. Of course, these processes are actually VAR(0) or white noise processes. In practice this information is often not available and we fitted VAR(1) processes to the generated data. In this situation it can be shown that the asymptotic variances of $\hat{\phi}_{kl,i}$ are zero for $i = 2, 3, \dots$. Thus, for these impulse responses the standard asymptotic approach may give misleading results because the estimators converge to the true values of zero more rapidly than at the usual \sqrt{T} -rate. In Table 1 we give some estimated confidence levels based on normal residuals. They clearly show that for lags 2 and 3 the intended level of 95% understates the nominal level markedly. Surprisingly, however, the simulation and bootstrapping methods are biased in the same direction as the asymptotic theory. In other words, they also provide confidence intervals with more than 95% probability content. The situation does not improve significantly for $T = 100$.

Let us now turn to the results for orthogonalised impulse responses. In Figure 4 the proportions of inclusions of $\theta_{11,i}$ and $\theta_{21,i}$ in estimated 95%

TABLE 1. Proportions of Inclusions of $\phi_{kl,i}$ in 95% Confidence Intervals
in 200 Replications for a Normal VAR(0) Process

i	method	T = 50		T = 100	
		$\phi_{11,i}$	$\phi_{21,i}$	$\phi_{11,i}$	$\phi_{21,i}$
1	1	.935	.945	.950	.960
	2	.940	.965	.945	.960
	3	.935	.950	.965	.960
	4	.920	.945	.945	.970
	5	.935	.955	.955	.960
2	1	1.000	1.000	1.000	1.000
	2	.995	1.000	1.000	1.000
	3	.990	1.000	.980	.995
	4	1.000	1.000	1.000	1.000
	5	.990	.995	.980	1.000
3	1	1.000	1.000	1.000	1.000
	2	1.000	1.000	1.000	1.000
	3	.995	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000
	5	.995	1.000	.995	1.000

confidence intervals are displayed for Gaussian (normal) VAR(1) processes with $\sigma_{13} = 0.3$, $a_{12} = a_{22} = 0.5$, varying a_{11} values and sample size $T = 50$. Thus, Figure 4 corresponds to Figure 1 which relates to the ϕ impulse responses. The overall conclusions emerging from Figure 4 are similar to those from Figure 1. Specifically, all methods tend to be biased in the same direction. That is, the proportions of inclusions of the true parameter value in 95% confidence intervals tend to be lower than 95% for all the methods or they tend to be larger than 95% for all the methods, depending on the value of a_{11} and the lag i of the impulse responses. None of the methods is superior over the whole range of a_{11} values, and the quantile methods tends to be inferior for large positive values of a_{11} and the cointegrated case.

Although we do not give the results here in detail in order to conserve space, we note that the quantile methods 3 and 5 had power advantages for higher lags ($i = 2, 3$) and small values of a_{11} , as for the ϕ impulse responses.

In Figure 5 similar results for varying values of the residual correlation σ_{12} are given for $T = 50$ and $a_{11} = a_{21} = a_{22} = 0.5$. Again a similar picture emerges as in Figure 4. With the exceptions of $\theta_{21,2}$ and $\theta_{21,3}$ the results are fairly insensitive to the residual correlation. None of the five methods does particularly well in matching the true and intended confidence intervals of 95% when there is a high positive residual correlation. In some cases the quantile methods are markedly inferior to the other methods.

As pointed out in Section 3, the asymptotic distribution of the θ impulse responses depends critically on the true distribution of the process. Therefore one would expect the quantile bootstrap method 5 to be superior for nonnormal error distributions since it is the only one that does not

incorporate any assumption regarding the process distribution. Some results for $\chi^2_{(3)}$ errors are depicted in Figure 6. Surprisingly the bootstrap methods are not generally superior to the asymptotic theory and the simulation methods that are based on normal residuals. The performance of all the methods is not satisfactory for this case and they are all biased in the same direction.

5. Conclusions

Since the Monte Carlo setup is necessarily limited some caution is required in drawing general conclusions from the results. There are, however, some observations that we can make. It is obvious that none of the methods is generally superior in terms of confidence level and power. Since all simulation methods are relatively expensive in terms of computer time it may be advisable to use the computationally efficient confidence bounds obtained on the basis of asymptotic theory, at least as a first check. In our Monte Carlo investigation their size was biased in the same direction as that of the other methods. In other words, when the asymptotic confidence intervals had a higher or lower level than the intended one the other methods had the same tendency. The power of the asymptotic methods may be lower, though, at higher lags and for small parameter values than that of the quantile methods. In other words, if the asymptotic confidence bounds indicate significant responses to an impulse in one variable, we can be reasonably sure that something is really going on in the system. If no significant impulse responses are found in this way a check with the quantile simulation methods may be advisable.

The poor performance of all methods for some parameter values is a concern especially as it does not go away quickly when the sample size increases. This lends support to those who doubt that precise quantitative

statements regarding the impulse responses can be made if unrestricted VAR models are being fitted.

Further research is required to confirm whether the foregoing results are of general validity in practical application of the VAR methodology. In particular it would be of interest to see whether the results depend on the dimension and order of the process. Also, in practice, the order of the data generation process is normally unknown. Therefore it is usually chosen according to some method or criterion. The impact of such strategies on the properties of the estimated impulse responses would also be of interest. Furthermore, investigating the effect of imposing parameter constraints either exact or of a Bayesian variety would be desirable. All these issues are left for future research.

References

- Lütkepohl, H. (1988), "Asymptotic Distribution of the Moving Average Coefficients of an Estimated Vector Autoregressive Process", Econometric Theory, 4, 77-85.
- Lütkepohl, H. (1990), "Asymptotic Distributions of Impulse Response Functions and Forecast Error Variance Decompositions of Vector Autoregressive Models", Review of Economics and Statistics, forthcoming.
- Magnus, J.R. and H. Neudecker (1988), Matrix Differential Calculus with Applications in Statistics and Econometrics, Chichester: John Wiley.
- Park, J.Y. and P.C.B. Phillips (1989), "Statistical Inference in Regressions with Integrated Processes: Part 2", Econometric Theory, 5, 95-131.
- Sims, C.A. (1980), "Macroeconomics and Reality", Econometrica, 48, 1-48.

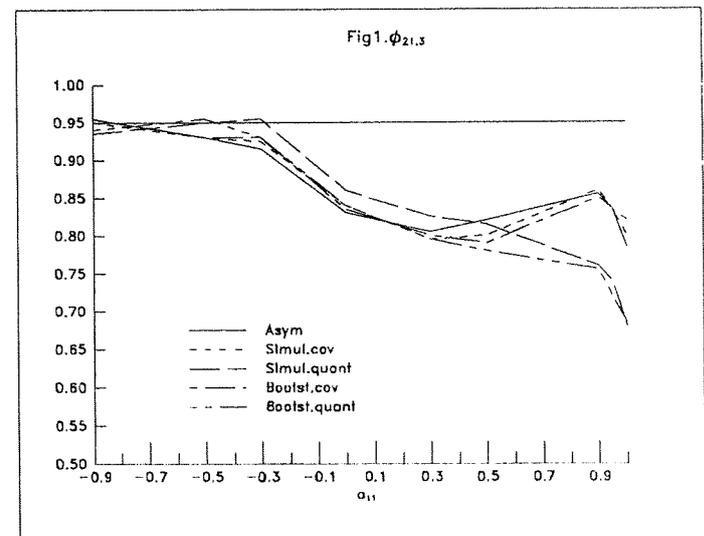
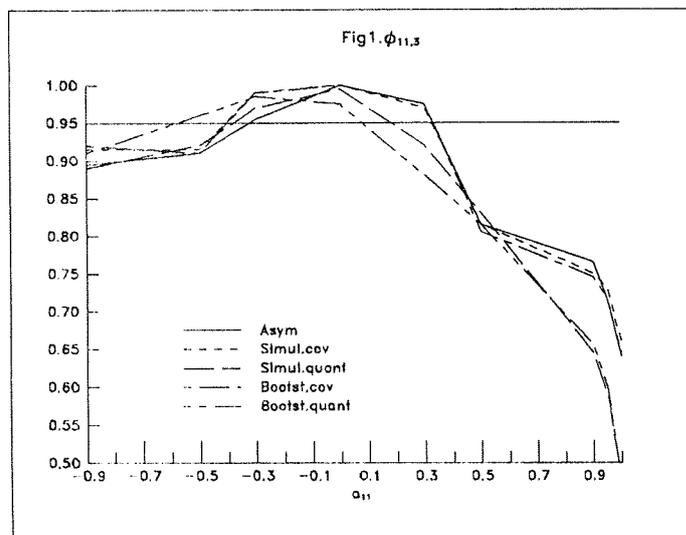
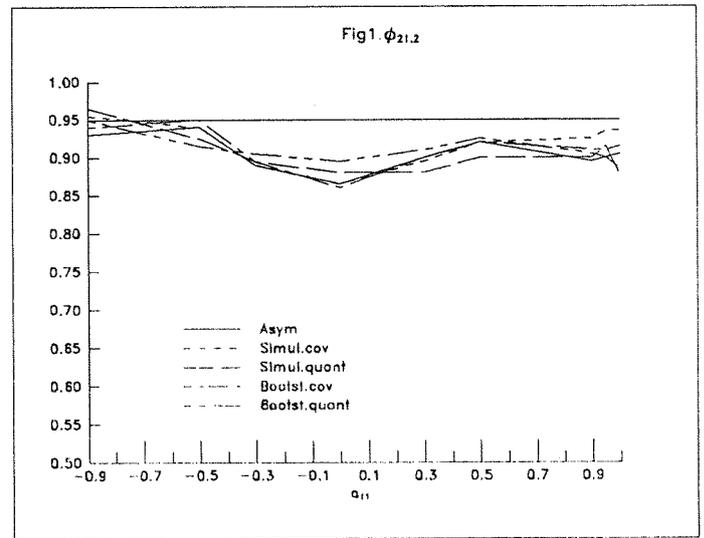
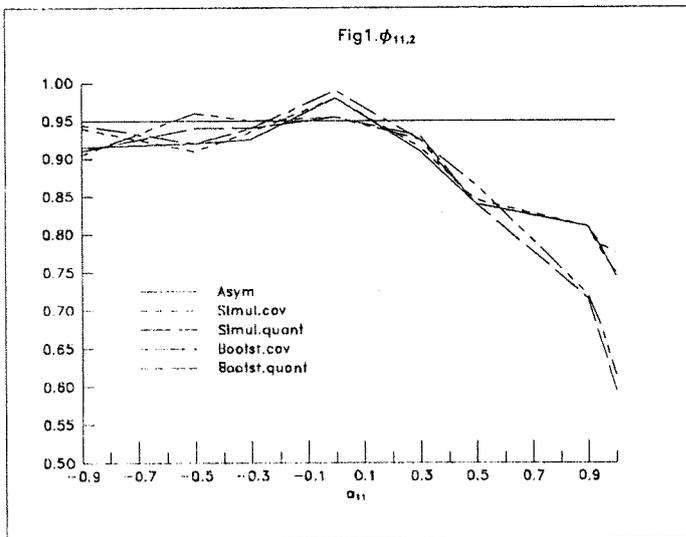
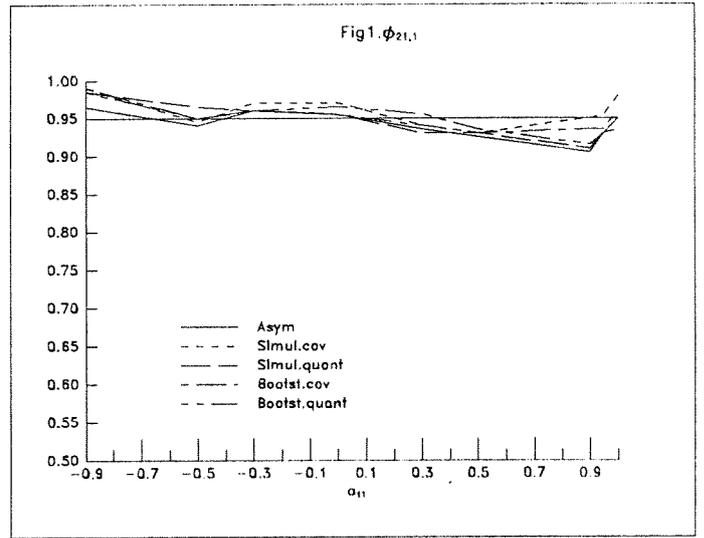
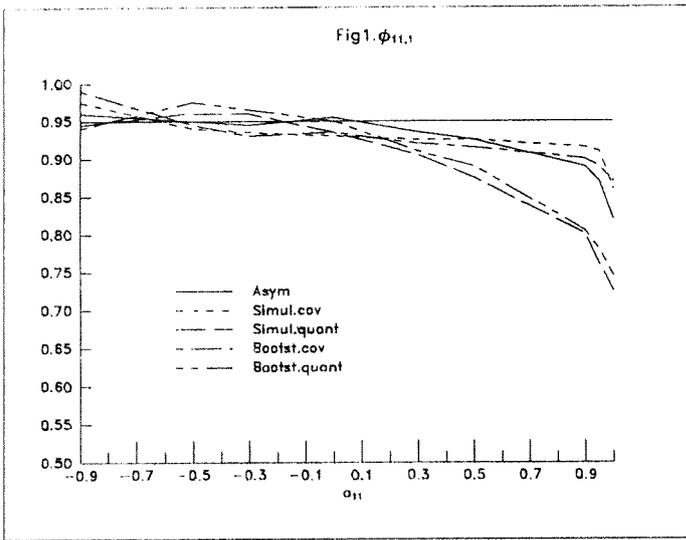


Figure 1. Confidence Interval Proportions for Impulse Response Coefficients: Normal Errors and $T = 50$

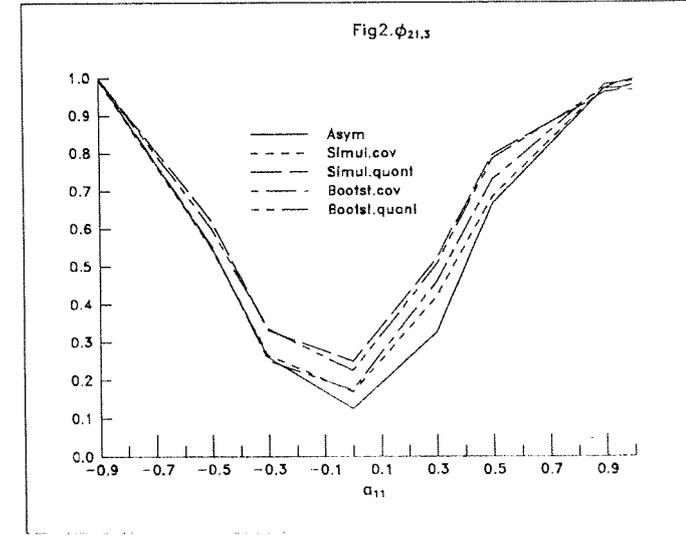
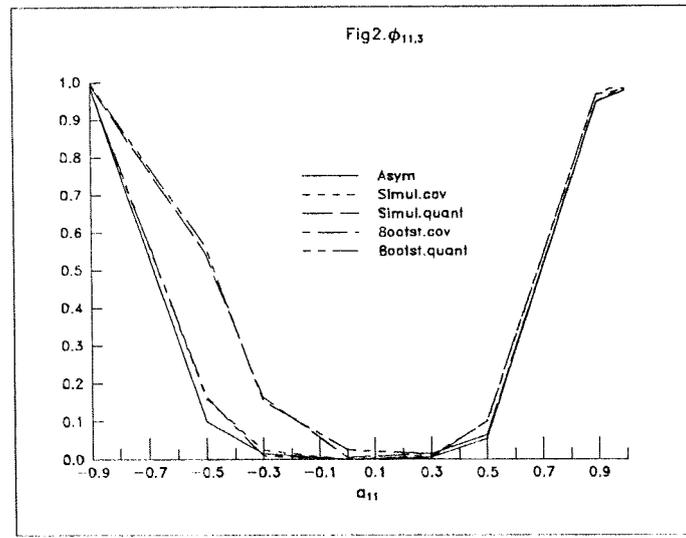
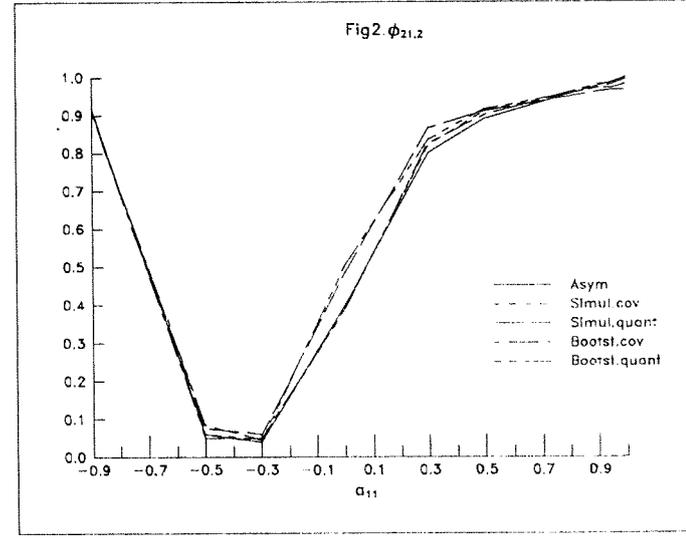
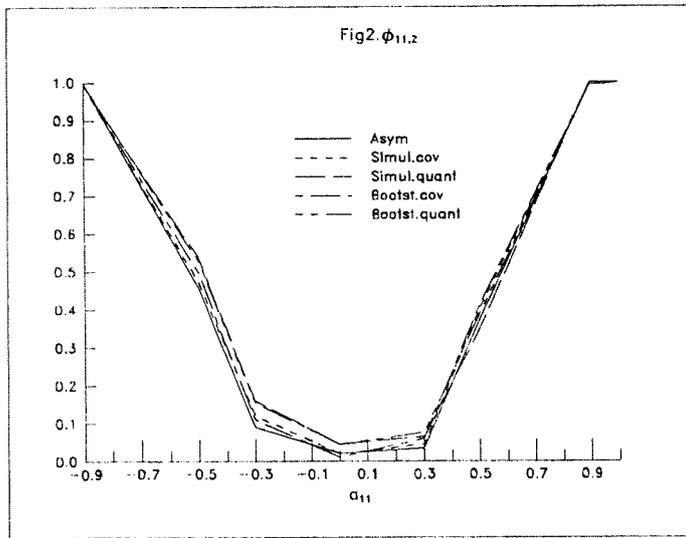
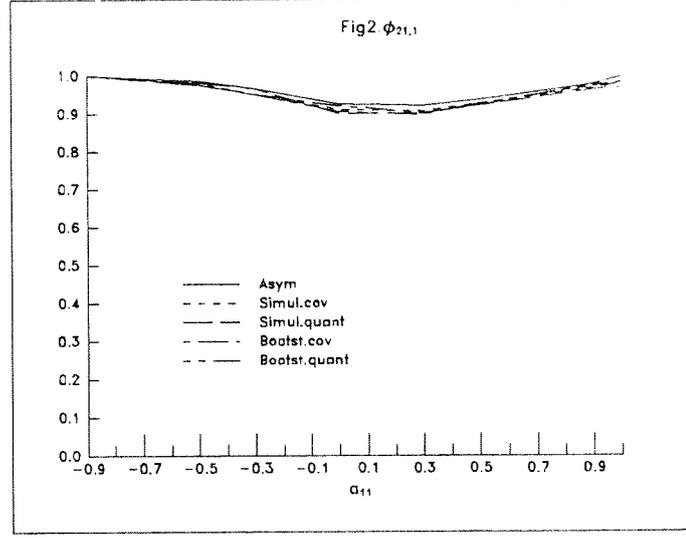
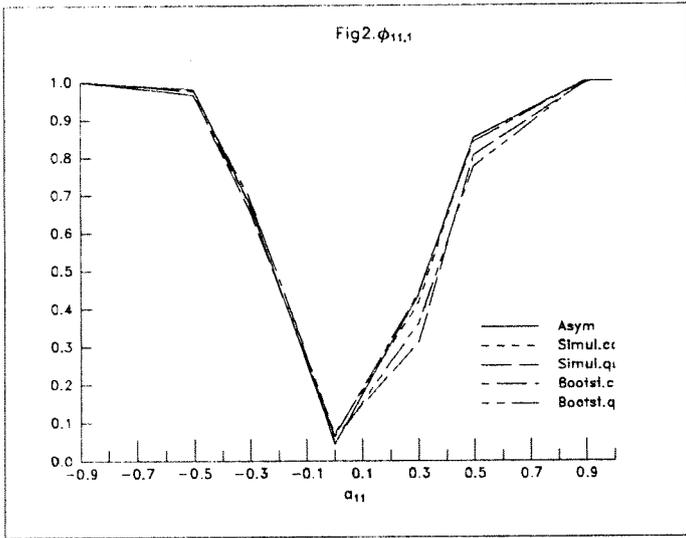


Figure 2. Power of Test for $H_0: \phi_{j1,k} = 0$: Normal Errors and $T = 50$

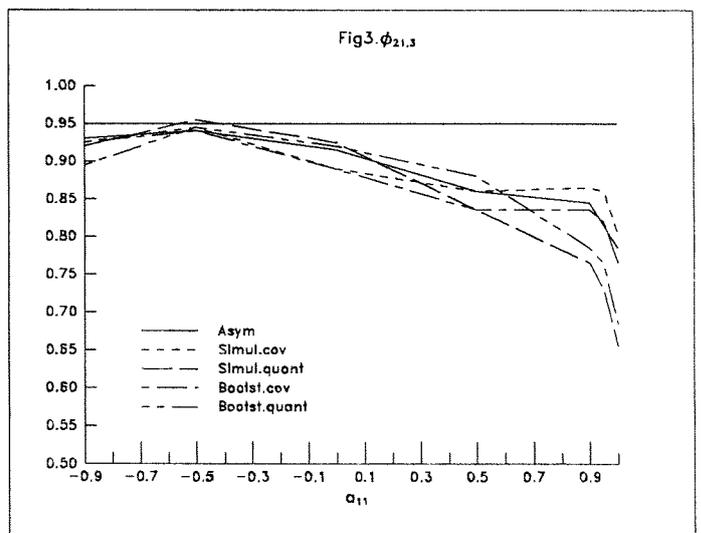
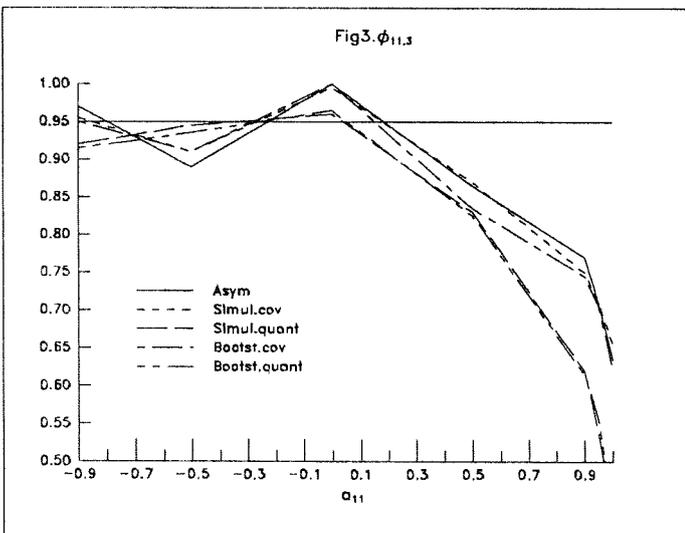
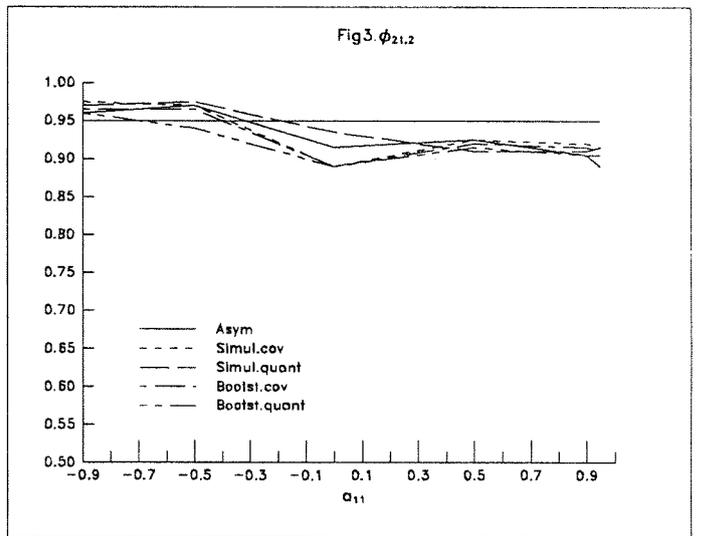
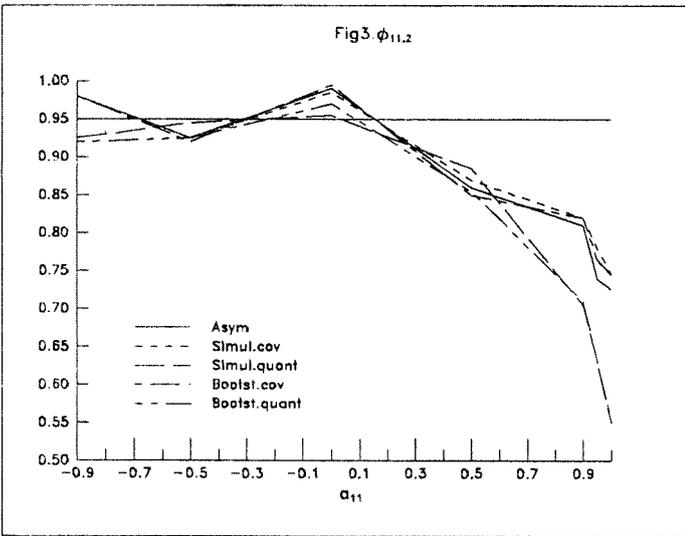
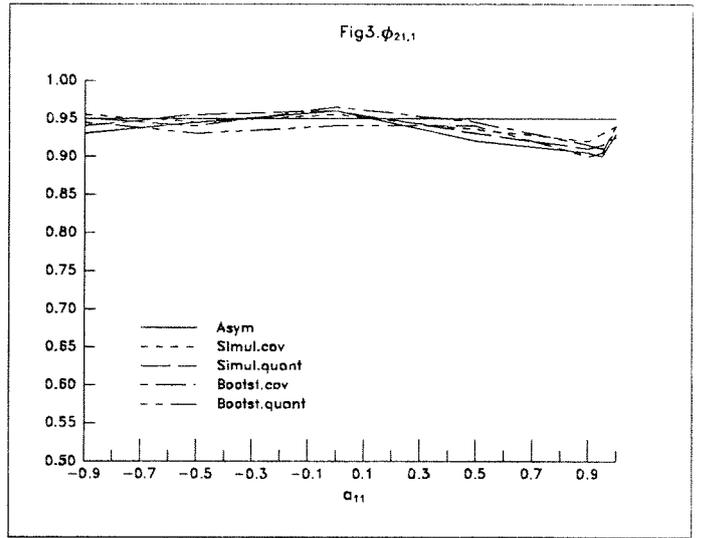
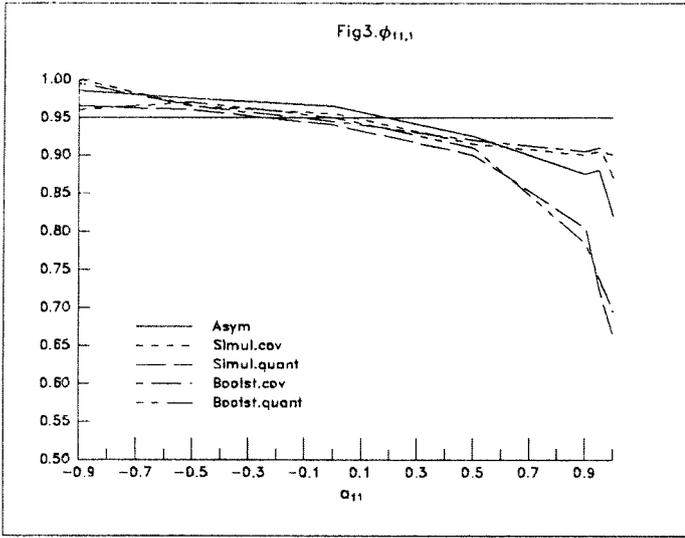


Figure 3. Confidence Interval Proportions for Impulse Response Coefficients: t Errors and $T = 50$

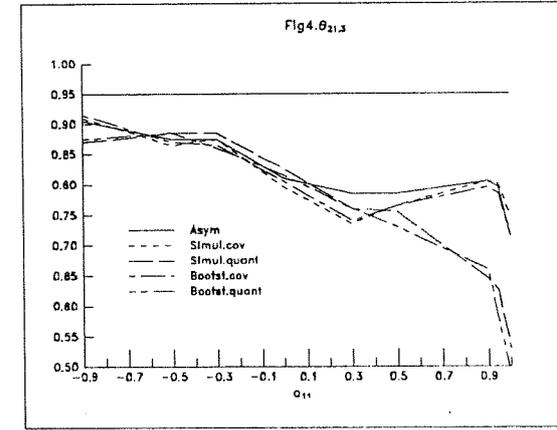
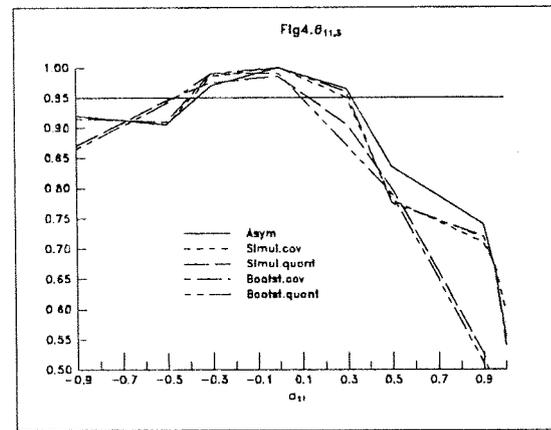
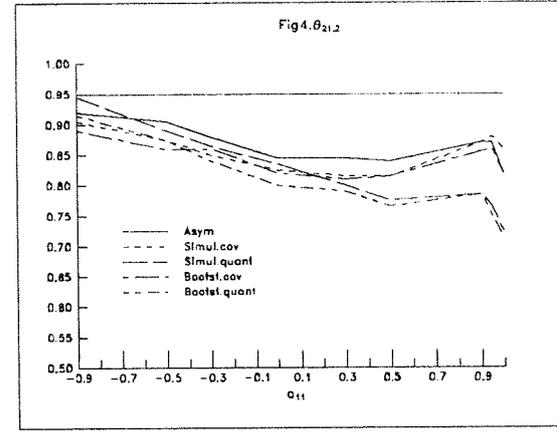
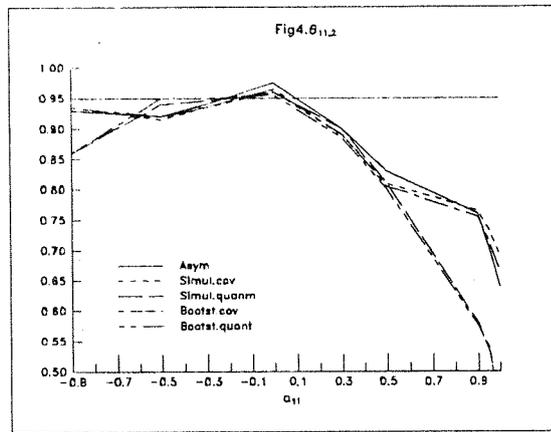
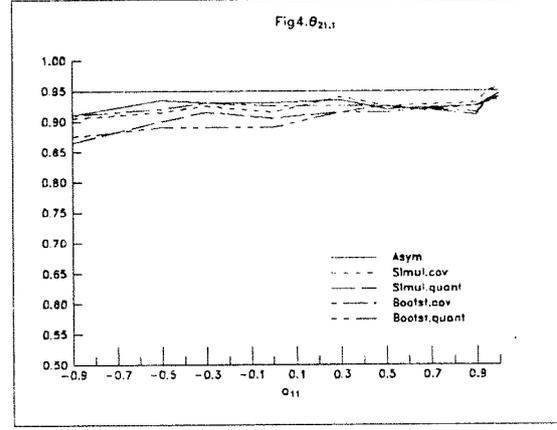
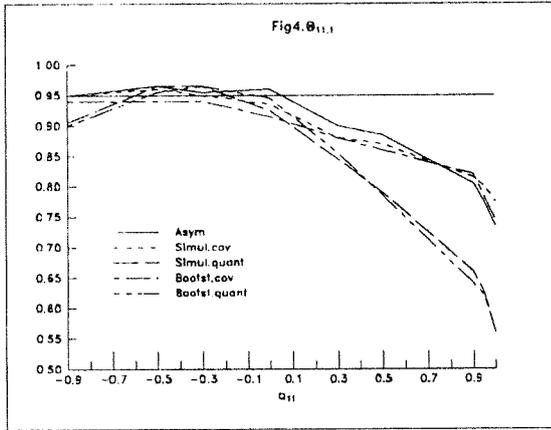
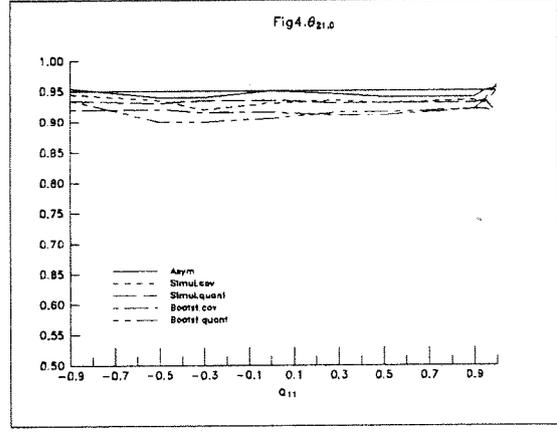
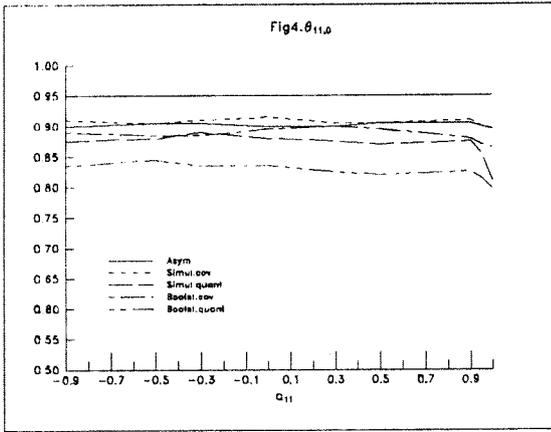


Figure 4. Confidence Interval Proportions for Orthogonalised Impulse Responses: Normal Errors and T = 50

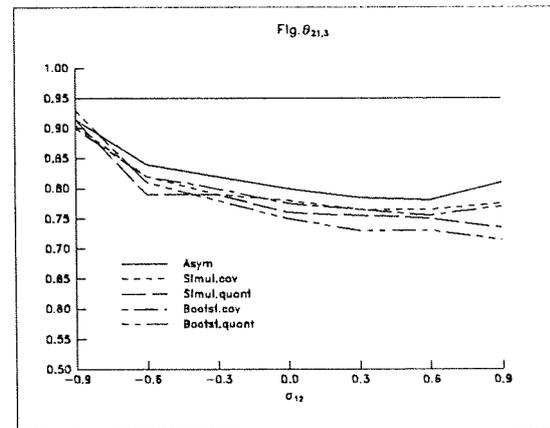
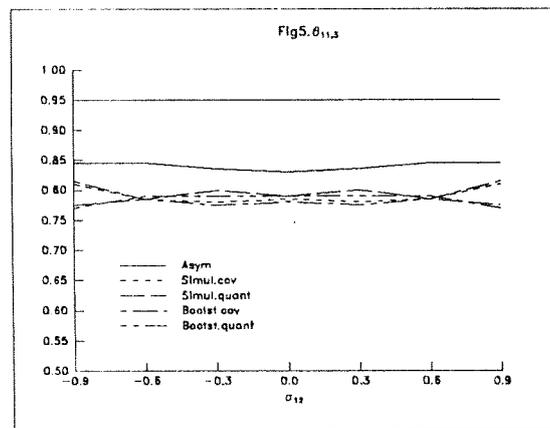
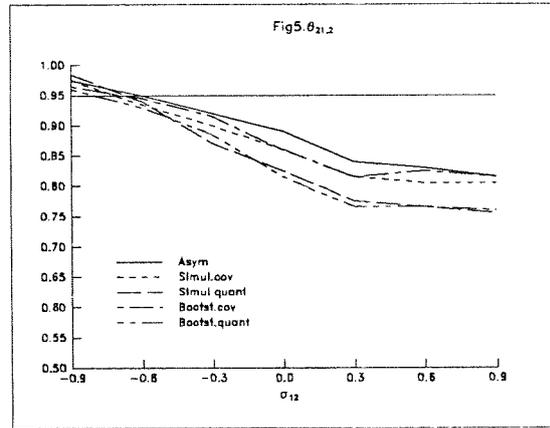
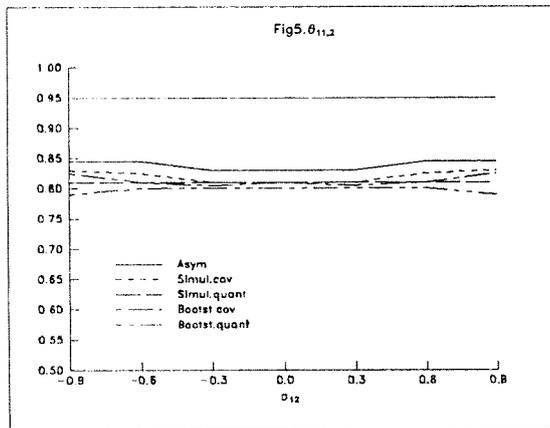
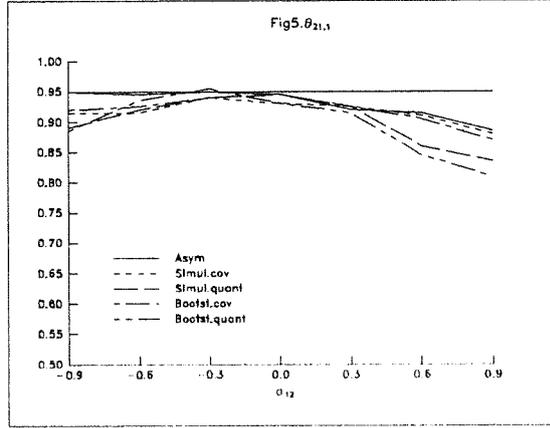
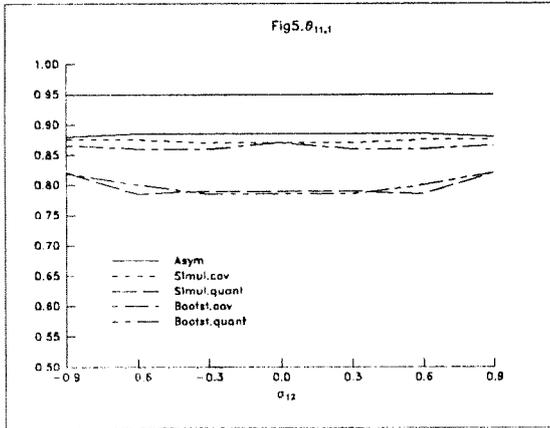
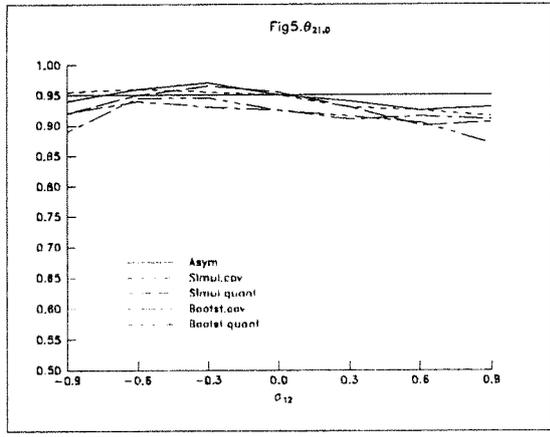
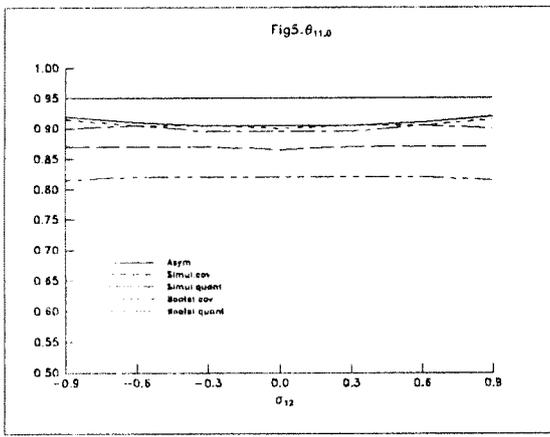


Figure 5. Confidence Interval Proportions for Orthogonalised Impulse Responses as a Function of Residual Correlation: Normal Errors and T = 50

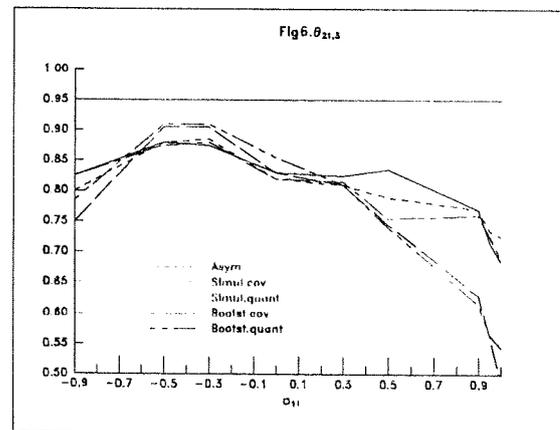
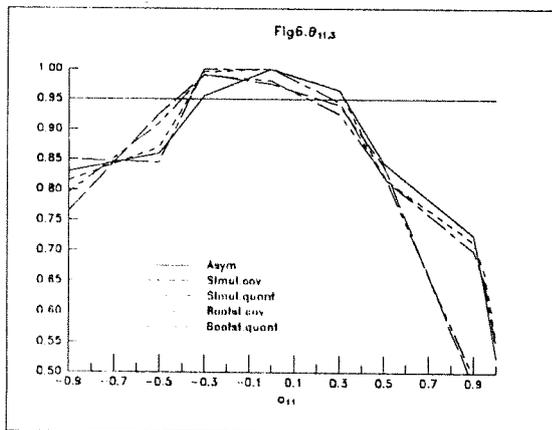
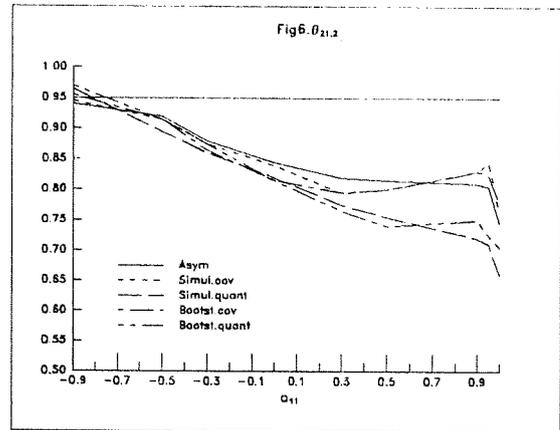
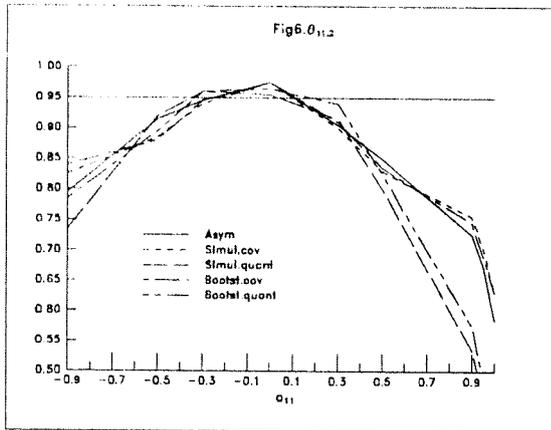
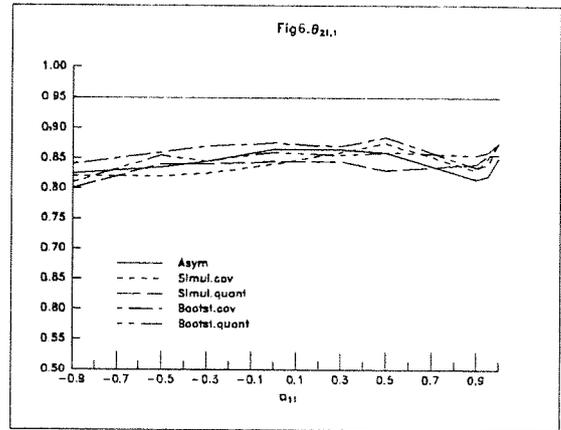
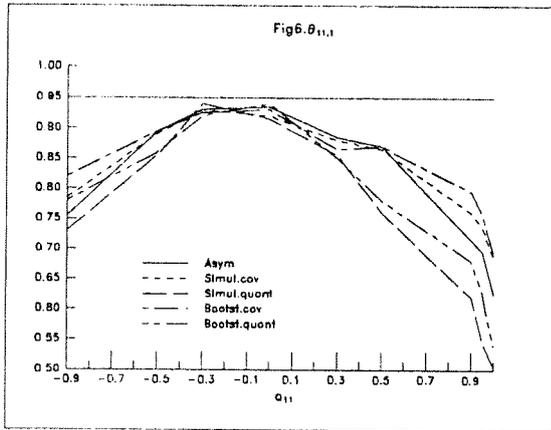
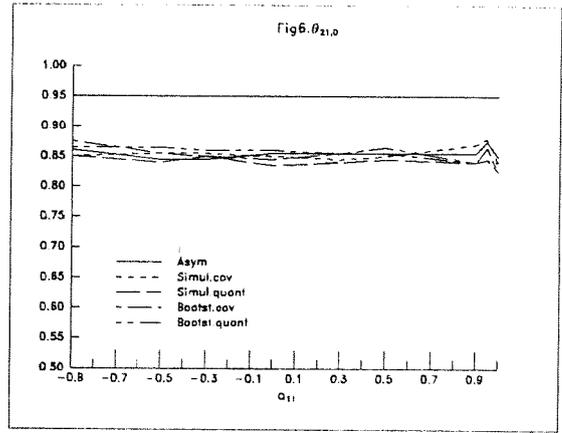
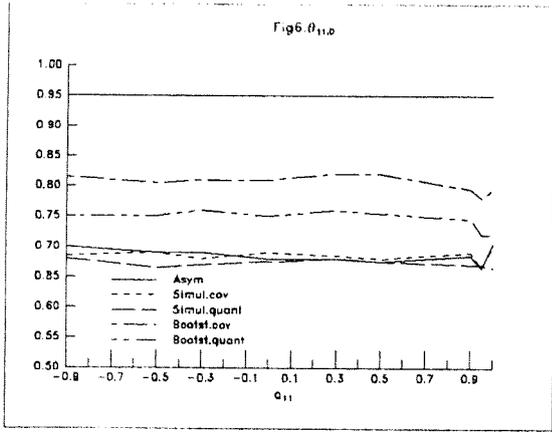


Figure 6. Confidence Interval Proportions for Orthogonalised Impulse Responses: χ^2 Errors and $T = 50$

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