A METHOD FOR THE COMPUTATION OF STANDARD ERRORS FOR GEARY-KHAMIS PARITIES AND INTERNATIONAL PRICES

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No. 47 - September 1990

ISSN 0 157-0188

ISBN 0 85834 892 6

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ABSTRACT

One of the areas receiving little attention in the past in index number theory is providing standard errors for the index number estimates. Recently, Clements and Izan (1987) and Selvanathan (1989,1990) used stochastic approach to index numbers to derive standard errors for the rate of inflation and Laspeyres and Paasche index numbers. In this paper we describe a method for computing standard errors associated with purchasing power parities computed using Geary-Khamis aggregation procedure in the International Comparisons Project of the United Nations. We assess the quality of the standard errors using Efron's (1979) bootstrap technique.

Key Words: Geary-Khamis, Purchasing Power Parity, International Price, Bootstrap.

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1. Introduction

The Geary-Khamis (G-K) method is the most widely used aggregation method for multilateral comparison of prices and real product. This method is presently used by the United Nations' International Comparison Project (ICP), the Organization for Economic Co-operation and Development (OECD), the Statistical Office of the European Economic Community (EEC) and the regional commission of the United Nations for inter-country comparisons of purchasing powers and real product, and by the Food and Agriculture Organization of the United Nations (FAO) in calculating regional and world indexes of food and agricultural production. The G-K method, due to Geary, R.C. (1958) and Khamis, S.H. (1969, 1970, 1972), uses the twin interdependent concepts of 'purchasing power parities (PPP)' of currencies and average 'international' prices of commodities. The G-K method derives values of the unknown parities and international prices from the solutions obtained from a system of linear homogeneous equations that define the international prices and parities as functions of the observed price and quantity data across countries. In essence, for any given price-quantity data set[1], the G-K method gives unique numerical values for the unknown parities and international prices[2]. Recently, Clements and Izan (1987) and Selvanathan (1989,1990) used the regression approach to obtain standard errors for the rate of inflation and Laspeyres and Paasche index numbers. So far no attempt has been made to derive any measures of reliability in terms of estimated standard errors, or some other suitable measures, of the numerical values resulting from the G-K method[3]. This paper provides a simple method for the computation of standard errors for the parities and international prices using the least-squares interpretation attached to the G-K equations discussed in Khamis (1984). A numerical illustration of the proposed method is provided in Section 3.

The organization of the paper is as follows. Section 2 briefly describes the Geary-Khamis method and provides the numerical values of parities and international prices based on aggregated data from the Phase IV of the International Comparisons Project. Section 3 discusses the least-squares approach and provides a simple procedure for the computation of the relevant standard errors and its application to Phase IV data. Section 4 suggests an alternative approach to obtain the standard errors derived in Section 3 and assess the quality of the results of that section using bootstrap technique. Finally in Section 5 we give our concluding comments.

2 Geary-Khamis Method

Let p_{ij} and q_{ij} denote the price and quantity[4] of commodity i for country j, i = 1,2, ..., N and j = 1,2, ..., M. Let P_i, PPP_j and R_j respectively denote the international price of i-th commodity, the purchasing power parity of j-th currency[5] and the implicit exchange rate for j-th currency. Obviously, for each j, $R_j = 1/PPP_j$. The Geary-Khamis method, first expounded in Geary (1958), defines the international prices and the purchasing power parities through the following system of (M+N) equations:

$$P_i = \frac{\sum_{j=1}^{M} PPP_j \, p_{ij} \, q_{ij}}{\sum_{j=1}^{M} q_{ij}} \qquad i = 1,2, \ldots, N \qquad (1)$$

$$PPP_j = \frac{\sum_{i=1}^{N} P_i \, q_{ij}}{\sum_{i=1}^{N} p_{ij} \, q_{ij}} \qquad j = 1,2, \ldots, M \qquad (2)$$

In general the system of equations (1) and (2), a set of (M + N) linear homogeneous equations in as many unknowns, has a unique positive solution for the P_i's and PPP_j's apart from an undetermined scalar multiplicitive factor [for details see Prasada Rao (1971) and Khamis (1972)]. The Geary-Khamis system in this form appears to be completely deterministic in that it results in numerical values of P_i and PPP_j that solve the linear equation system (1) and (2), given a price-quantity data set. For empirical application of the G-K method in the ICP, see Kravis et. al (1975, 1978 and 1982).

<u>A numerical illustration</u>

For the benefit of the readers who are not familiar with the method and the ICP, an illustration is provided using aggregated data from the Phase IV of the ICP (UN Publication, 1987). The list of countries includes the sixty countries that participated in the Phase IV exercise. The commodity list used here is restricted to eight highly aggregated commodity groups of the private consumption expenditure, viz. (i) food, beverages and tobacco, (ii) clothing and footwear, (iii) rent and fuel, (iv) house furnishing and operations (v) medical care (vi) transport and communication (vii) recreation and education and (viii) miscellaneous. Table 1 provides the purchasing power parities (PPP_j), implicit exchange rates (R_j) calculated as $1/PPP_j$ and the 1980 official exchange rates (E_j) for the sixty countries published in UN Publication (1987). Table 2 presents the international prices P_i's. The values of PPP_j's and P_i's are obtained by solving equations (1) and (2) using Phase IV data.

Results in Columns (3) and (4) of Table 1 and in Column (2) of Table 2 have been treated in the past as essentially deterministic. For empirical analyses based on ICP results, see Theil and Suhm (1981), Fiebig, Seale and Theil (1988) and Theil and Clements (1987). It is argued in the following sections that these parities are indeed stochastic and therefore it would be feasible to derive standard errors associated with these results.

3. Least-Squares Approach

The system of equations (1) and (2) underlying the Geary-Khamis method has been treated essentially as a deterministic system based on heuristic logic described in Geary (1958). However, a close scrutiny of the definitions shows that the international prices, P_i's, and the purchasing power parities PPP_j's, may be interpreted as weighted averages. This makes it possible to interpret P_i and PPP_j to be estimators of parameters from appropriately specified regression models. Such an interpretation may be found in Khamis (1984) and Prasada Rao (1972). This approach is examined further, in the following subsection, with the aim of deriving appropriate estimated standard errors associated with P_i and PPP_j.

Standard error for the Purchasing Power Parity PPP_j

Assuming the knowledge of the international prices P_i,[6] the unknown parities PPP_j may be characterized by the linear regression model, for each j (= 1,2, ..., M)

$$\frac{P_i}{p_{ij}} = PPP_j + u_{ij} \qquad i = 1,2, \ldots, N \qquad (3)$$

where p_{ij}/P_i represents the price relative of i-th commodity in country j relative to international price P_i. In fact this ratio represents a purchasing power parity based solely on commodity i and equation (3) postulates that the true, but unknown, PPP_j and P_i/p_{ij} deviate by a random disturbance term. Let u_{ij} be a random variable with zero mean and variance σ_{ij}^2.

Efficient estimation of the unknown PPP_j's would depend upon σ_{ij}^2. The following specification for the structure of σ_{ij}^2 leads to the G-K definition of PPP_j in (2). Suppose the disturbances u_{ij}'s are such that

Table 1:
Purchasing Power Parities, Implicit Exchange Rates and Official Exchange Rates (1980) for the Sixty Countries in Phase IV of ICP

Country	Currency Unit	Purchasing Power parity	Implicit Exchange rate	Official Exchange rate
j		PPP_j	$R_j = \frac{1}{PPP_j}$	E_j
(1)	(2)	(3)	(4)	(5)
1. USA	US Dollars	1.0000	1.0000	1.0000
2. Belgium	Francs	0.0269	37.1747	29.2430
3. Denmark	Kroner	0.1275	7.8431	5.6359
4. France	Francs	0.1856	5.3879	4.2260
5. Germany	D. Mark	0.4089	2.4456	1.8177
6. Greece	Drachmae	0.0286	34.9650	42.6170
7. Ireland	Ir Pounds	2.0705	0.4830	0.4859
8. Italy	Lire	0.0013	769.2308	856.5000
9. Luxembourg	Francs	0.0298	33.5571	29.2430
10. Netherlands	Guilders	0.4057	2.2188	1.9881
11. United Kingdom	Pounds	2.0430	0.4895	0.4303
12. Austria	Schillings	0.0651	15.3610	12.9380
13. Finland	Markkaa	0.2228	4.4883	3.7301
14. Hungary	Forint	0.0818	12.2249	32.7330
15. Norway	Kroner	0.1493	6.6979	4.9392
16. Poland	Zlotych	0.0565	17.6991	31.0510
17. Portugal	Escudos	0.0313	31.9489	50.0620
18. Spain	Pesetas	0.0158	63.2911	71.7700
19. Yugoslavia	Dinars	0.0543	18.4162	24.9110
20. Botswana	Pula	1.7554	0.5697	0.7769
21. Cameroon	Francs	0.0053	188.6792	211.3000
22. Ethiopia	Birr	1.1041	0.9057	2.0700
23. Cote d'Ivoire	Francs	0.0047	212.7660	211.3000
24. Kenya	Shillings	0.2318	4.3141	7.4202
25. Madagascar	Francs	0.0072	138.8890	211.3000
26. Malawi	Kwacha	2.5756	0.3883	0.8121
27. Mali	Francs	0.0037	270.2703	422.6000
28. Morocco	Dirhams	0.3582	2.7917	3.9367
29. Nigeria	Naira	1.5590	0.6414	0.5465
30. Senegal	Francs	0.0061	163.9344	211.3000
31. Tanzania	Shillings	0.1643	6.0864	8.1950
32. Tunisia	Dinars	3.8363	0.2607	0.4050
33. Zambia	Kwacha	1.3436	0.7443	0.7885
34. Zimbabwe	Dollars	2.1006	0.4761	0.6425
35. Israel	Shekels	0.2387	4.1894	5.1240
36. Hong Kong	HK Dollars	0.3123	3.2020	5.0000
37. India	Rupees	0.3017	3.3146	7.8630
38. Indonesia	Rupiahs	0.0036	277.7778	626.9900
39. Japan	Yen	0.0040	250.0000	226.7400
40. Korea	Won	0.0026	384.6154	607.4300
41. Pakistan	Rupees	0.3213	3.1124	9.9000
42. Philippines	Pesos	0.3389	2.9507	7.5114
43. Sri Lanka	Rupees	0.2966	3.3715	16.5340
44. Argentina	Pesos	0.0004	2500.0000	1837.2000
45. Bolivia	Pesos	0.0622	16.0772	24.5100
46. Brazil	Cruzeiros	0.0342	29.2398	52.7139
47. Chile	Pesos	0.0335	29.8508	39.0000
48. Colombia	Pesos	0.0492	20.3252	47.2800
49. Costa Rica	Colones	0.1818	5.5006	8.5700
50. Dominican Rep.	Dollars	1.7819	0.5612	1.0000
51. Ecuador	Sucres	0.0739	13.5318	25.0000
52. El Salvador	Colones	0.7846	1.2745	2.5000
53. Guatemala	Quetzales	2.3503	0.4255	1.0000
54. Honduras	Lempiras	0.9294	1.0760	2.0000
55. Panama	Balboas	1.5955	0.6268	1.0000
56. Paraguay	Guaranies	0.0129	77.5194	126.0000
57. Peru	Soles	0.0078	128.2051	288.6500
58. Uruguay	New Pesos	0.1361	7.3475	9.1600
59. Venezuela	Bolivares	0.3301	3.0294	4.2925
60. Canada	Dollars	0.9872	1.0130	1.1690

Table 2

Geary-Khamis International Prices for the 8 Commodities

Commodity i (1)	G-K International Price P_i (2)
1. Food, beverages and tobacco	.9951
2. Clothing and footwear	.9946
3. Rent and fuel	1.0252
4. House furnishings and operations	1.0014
5. Medical care	.8987
6. Transport and communication	1.0900
7. Recreation and education	1.0306
8. Miscellaneous	.9230

(i) $E\left(u_{ij}\right) = 0$ (ii) $V\left(u_{ij}\right) = \frac{\sigma_u^2}{p_{ij}\, q_{ij}}$ and

(iii) $E\left(u_{ij}\, u_{lk}\right) = 0 \quad \forall$ i, j, l, k and $i \neq l$, $j \neq k$.

Then the generalized least squares estimator of PPP_j from (3) is given by

$$\hat{PPP}_j = \frac{\sum_{i=1}^{N} P_i\, q_{ij}}{\sum_{i=1}^{N} p_{ij}\, q_{ij}} \tag{4}$$

which is equation (2). This is obtained by applying ordinary least squares method to the transformed model

$$\frac{P_i}{p_{ij}}\sqrt{p_{ij}\, q_{ij}} = PPP_j\sqrt{p_{ij}\, q_{ij}} + u_{ij}\sqrt{p_{ij}\, q_{ij}} \qquad i = 1, \ldots, N \tag{5}$$

The estimated standard error associated with $\hat{PPP}_j$, $SE\left(\hat{PPP}_j\right)$, is given by, for each j,

$$SE\left(\hat{PPP}_j\right) = \left[\frac{\hat{\sigma}_u^2}{\sum_{i=1}^{N} p_{ij}\, q_{ij}}\right]^{1/2} \tag{6}$$

The values of $\hat{PPP}_j$ and $SE\left(\hat{PPP}_j\right)$ can be obtained as standard output from most regression packages applied to equation (5).

A numerical illustration

Results, based on the data described before in Section 2, obtained from the estimation of equation (5) are presented in Table 3 below. The estimates of $\hat{PPP}_j$ in Column (2) of the table are reproduced from Column (3) of Table 1. Columns (2) and (3) respectively show the estimated $\hat{PPP}_j$ and the associated standard errors. Column (4) expresses the standard error as a percentage of $\hat{PPP}_j$. These figures are similar to the coefficient of variation used in empirical analysis.

If the implicit exchange rates, $\hat{R}_j$, are of interest then the relevant standard errors may be approximated as follows. Since $\hat{R}_j = 1/\hat{PPP}_j$, we can show that the approximate standard error of $\hat{R}_j$ will be given by

$$SE\left(\hat{R}_j\right) \approx \frac{SE\left(\hat{PPP}_j\right)}{\left(\hat{PPP}_j\right)^2}$$

The estimates of $\hat{R}_j$'s and their standard errors are presented in Columns (5) and (6) of Table 3. Columns (7) and (8) of the table present the bootstrap simulation results which we shall discuss in the next section.

As the results indicate, the $\hat{PPP}_j$ and $SE\left(\hat{PPP}_j\right)$ are derived conditional on using US dollars as the numeraire currency. It can be shown that use of another currency as numeraire would result in $\hat{PPP}_j$'s and $SE\left(\hat{PPP}_j\right)$'s to be a scalar transformation of the results in Table 3. Consequently the CV's in Column (4) remain unchanged.

Standard error for the international price P_i

Based on the values of PPP_j, the international prices P_i may be characterized by the following stochastic model for each i (= 1,2, ..., N)

$$PPP_j\, p_{ij} = P_i + v_{ij} \qquad j = 1,2, \ldots, M \qquad (7)$$

where $PPP_j\, p_{ij}$ is the converted national price and v_{ij} is a random variable with zero mean and variance σ_v^2. The intuitive meaning of equation (7) is that the "international" price differs from

Table 3
Data Based Estimates and Bootstrap Simulations for Pruchasing Power Parities

	Data Based					Mean Bootstrap Estimates and Standard Deviations (1000 simulations)	
Country	$\hat{PPP}_j$	$SE\left(\hat{PPP}_j\right) \times 100$	$\frac{SE\left(\hat{PPP}_j\right)}{\hat{PPP}_j} \times 100$	$\hat{R}_j$	$SE\left(\hat{R}_j\right)$	$\hat{PPP}^*_j$	$SD\left(\hat{PPP}^*_j\right) \times 100$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1. USA	1.0000	9.0051	9.00	1.0000	.0901	1.0055	8.6746
2. Belgium	.0269	.1446	5.38	37.2199	2.0032	.0266	.1437
3. Denmark	.1275	.4654	3.65	7.8439	.2863	.1263	.4565
4. France	.1856	.9056	4.88	5.3866	.2628	.1820	.8944
5. Germany	.4089	1.8135	4.44	2.4457	.1085	.4048	1.7606
6. Greece	.0286	.1459	5.10	34.9085	1.7783	.0281	.1459
7. Ireland	2.0705	16.5321	8.22	.4830	.0386	2.0574	16.4170
8. Italy	.0013	.0085	6.54	746.8374	47.5535	.0013	.0084
9. Luxembourg	.0298	.1930	6.18	33.5187	2.1679	.0292	.1908
10. Netherlands	.4057	2.7255	6.72	2.4649	.1656	.4018	2.7358
11. UK	2.0430	7.6447	3.74	.4895	.0183	2.0334	7.3562
12. Austria	.0651	.2707	7.10	15.3585	.6385	.0645	.2840
13. Finland	.2228	1.8726	8.40	4.4880	.3772	.2293	1.8179
14. Hungary	.0818	1.4822	18.12	12.2291	2.2167	.0882	1.4813
15. Norway	.1493	.8023	5.38	6.6973	.3599	.1479	.8461
16. Poland	.0565	.8736	15.46	17.6918	2.7344	.0610	.8900
17. Portugal	.0313	.1593	5.09	31.9943	1.6308	.0312	.1608
18. Spain	.0158	.0768	4.86	63.2170	3.0677	.0154	.0734
19. Yugoslavia	.0543	.6867	12.65	18.4288	2.3321	.0564	.6829
20. Botswana	1.7554	17.2042	10.20	.5697	.0558	1.7642	16.925
21. Cameroon	.0053	.0702	13.25	190.4757	25.4674	.0055	.0681
22. Ethiopia	1.1041	28.0067	25.31	.9057	.2297	1.1817	27.8400
23. Cote d'Ivoire	.0047	.0433	9.21	214.3974	19.9016	.0048	.0444
24. Kenya	.2318	3.4046	14.69	4.3137	.6335	.2385	3.5287
25. Madagascar	.0072	.0842	11.69	139.1499	16.3054	.0075	.0818
26. Malawi	2.5756	35.2584	13.79	.3883	.0531	2.5313	35.0910
27. Mali	.0037	.0559	15.11	273.2522	41.7112	.0039	.0530
28. Morocco	.3582	3.4438	9.61	2.7917	.2684	.3697	3.3704
29. Nigeria	1.5590	22.8511	14.66	.6414	.0940	1.6944	22.1560
30. Senegal	.0061	.0831	13.62	164.8903	22.5960	.0062	.0844
31. Tanzania	.1643	4.0331	24.55	6.0866	1.4941	.1913	3.8818
32. Tunisia	3.8363	38.9565	10.15	.2607	.0264	3.9744	38.7870
33. Zambia	1.3436	9.6795	7.20	.7443	.0536	1.3406	9.5920
34. Zimbabwe	2.1006	18.8329	8.97	.4761	.0423	2.0625	18.7120
35. Israel	.2387	1.1295	4.73	4.1893	.1982	.2335	1.1108
36. Hong Kong	.3123	4.1524	13.30	3.2022	.4258	.3203	4.1329
37. India	.3017	5.9846	19.84	3.3150	.6577	.3318	5.8029
38. Indonesia	.0036	.0462	12.83	275.8786	35.1671	.0037	.0454
39. Japan	.0040	.0213	5.33	250.1424	13.3106	.0041	.0215
40. Korea	.0026	.0386	14.85	379.2683	55.5295	.0028	.0387
41. Pakistan	.3213	3.7785	11.76	3.1119	.3659	.3506	3.5520
42. Philippines	.3389	4.0364	11.91	2.9511	.3515	.3694	3.8024
43. Sri Lanka	.2966	7.2191	24.34	3.3719	.8208	.3469	6.8261
44. Argentina	.0004	.0028	7.00	2442.2008	165.4578	.0004	.0027
45. Bolivia	.0622	.8452	13.59	16.0679	2.1820	.0622	.8399
46. Brazil	.0342	.3197	9.35	29.2547	2.7362	.0328	.3170
47. Chile	.0335	.2982	8.90	29.8770	2.6614	.0336	.2920
48. Colombia	.0492	.4490	9.13	20.3354	1.8569	.0520	.4365
49. Costa Rica	.1818	1.4647	8.06	5.5018	.4434	.1912	1.4541
50. Dominican Rep	1.7819	24.9561	14.00	.5612	.0786	1.9491	24.5970
51. Ecuador	.0739	.7558	10.23	13.5329	1.3841	.0734	.7534
52. El Salvador	.7846	8.3012	10.58	1.2746	.1349	.8368	8.2208
53. Guatemala	2.3503	31.8120	13.54	.4255	.0576	2.5536	30.2280
54. Honduras	.9294	8.1561	8.78	1.0760	.0944	.9932	7.7103
55. Panama	1.5955	10.6639	6.68	.6268	.0419	1.6159	10.7520
56. Paraguay	.0129	.2057	15.95	77.7948	12.4486	.0137	.1999
57. Peru	.0078	.0705	9.04	128.6044	11.6529	.0080	.0694
58. Uruguay	.1361	1.7429	12.81	7.3521	.9421	.1352	1.7200
59. Venezuela	.3301	4.8159	14.59	3.0296	.4420	.3414	4.5928
60. Canada	.9872	12.3602	12.52	1.0130	.1268	.9967	12.1060

the observed national price, suitably converted to facilitate comparison, by a random variable with zero expectation.

Efficient estimation of the unknown international price P_i depends mainly on the variances and covariances of the disturbances v_{ij}. Among many possible specifications, consider the following structure where

(i) $E\left(v_{ij}\right) = 0$ (ii) $V\left(v_{ij}\right) = \dfrac{\sigma_v^2}{q_{ij}}$ and

(iii) $E\left(v_{ij}\, v_{lk}\right) = 0 \quad \forall\, i, j, l, k$ and $i \neq l, j \neq k$.

Given this specification, the BLUE of P_i for each i is given by

$$\hat{P}_i = \frac{\sum_{j=1}^{M} PPP_j\, p_{ij}\, q_{ij}}{\sum_{j=1}^{M} q_{ij}} \qquad (8)$$

which coincides with the G-K formula for computing international prices in equation (1). The standard error of $\hat{P}_i$ is given by

$$SE\left(\hat{P}_i\right) = \left(\frac{\hat{\sigma}_v^2}{\sum_{j=1}^{M} q_{ij}}\right)^{1/2} \qquad (9)$$

where $\hat{\sigma}_v^2$ is an estimate of the unknown σ_v^2.

A numerical illustration

Results based on the data described in Section 2 for P_i are presented in Table 4 for the eight broadly defined categories of private consumption expenditure. The estimates of $\hat{P}_i$ in Column (2) of the table are reproduced from Column (2) of Table 2. Column (3) of the table provides the standard errors evaluated using equation (9). Column (4) expresses the standard error as a percentage of the estimate $\hat{P}_i$. We shall discuss the last two Columns of the table in the next section.

These results indicate that $\hat{P}_i$ have very small standard errors. Further coefficient of variation associated with different commodity groups are very similar in magnitude. This

Table 4

Data Based Estimates and Bootstrap Simulations for International Prices

Commodity i	Data Based			Mean Bootstrap Estimates and Standard Deviations (1000 Simulations)	
	$\hat{P}_i$	$SE\left(\hat{P}_i\right)$	$\frac{SE\left(\hat{P}_i\right)}{\hat{P}_i} \times 100$	$\hat{P}_i^*$	$SD\left(\hat{P}_i^*\right)$
(1)	(2)	(3)	(4)	(5)	(6)
1. Food, beverages and tobacco	.9951	.0218	2.19	1.0236	.0214
2. Clothing and footwear	.9946	.0326	3.28	1.0262	.0324
3. Rent and fuel	1.0252	.0399	3.89	1.0736	.0400
4. House furnishings and operations	1.0014	.0296	2.96	.9954	.0304
5. Medical care	.8987	.0374	4.16	.9178	.0378
6. Transport and communication	1.0900	.0297	2.72	1.1240	.0302
7. Recreation and education	1.0306	.0393	3.81	.9663	.0391
8. Miscellaneous	.9230	.0212	2.30	.9344	.0218

contrasts with the results in Table 3 where the coefficient of variations are much higher for PPP_j's corresponding to the developing countries.

4. Alternative Standard Errors

In the last section we estimated regression models (3) and (7) by GLS and obtained standard errors for $\hat{PPP}_j$'s and $\hat{P}_i$'s. In this section we use distribution-free bootstrap simulations (see, e.g. Efron [1979], Freedman and Peters [1984], Selvanathan [1989]) to obtain alternative standard errors for $\hat{PPP}_j$'s and $\hat{P}_i$'s. By doing this we can also assess the quality of our data-based estimates and their standard errors.

In a nut-shell bootstrap technique works as follows: Consider a simple regression model of the form $y_i = \beta x_i + \varepsilon_i$, $i = 1, ..., n$. Bootstrapping this model involves the following three steps:

Step 1: Estimate the model and obtain data based estimate for β, $\hat{\beta}$ (say) and evaluate the residuals $\hat{\varepsilon}_i = y_i - \hat{\beta} x_i$. $i = 1, ..., n$.

Step 2: Assign mass 1/n to each residual $\hat{\varepsilon}_i$ (i = 1,2, ..., n) and draw n uniform random numbers with replacement in the range 1 to n. Let the drawn random numbers be $\{k_1, k_2, ..., k_n\}$.

Step 3: Define bootstrap errors $\varepsilon_i^* = \hat{\varepsilon}_{k_i}$, i = 1, ..., n and generate data for the dependent variable as $y_i^* = \hat{\beta}x_i + \varepsilon_i^*$, i = 1, ..., n

Using the generated data y_i^*, i = 1, ..., n together with the observed values of the independent variable x, we estimate the model to obtain a bootstrap estimate $\hat{\beta}^*$ for β. We repeat this procedure 1000 times to obtain 1000 bootstrap estimates for β. We then evaluate the mean and the standard deviation (SD) of the sampling distribution of the 1000 bootstrap estimates. As this SD is the bootstrap estimate of variability in the parameter estimate this SD can be considered as an alternative standard error for β.

We now bootstrap equations (3) and (7) and present the simulation results in Tables 3 and 4. Columns (2) and (3) of the tables present the data-based estimates and the standard errors of $\hat{PPP}_j$'s and $\hat{P}_i$'s, respectively. The corresponding bootstrap simulation results are presented in Columns (7) and (8) of Table 3 and Columns (5) and (6) of Table 4 respectively. A comparison of Column (2) with (7) in Table 3 and Column (2) with Column (5) in Table 4 shows very little difference between the data-based estimates and the mean bootstrap estimates for both $\hat{PPP}_j$'s and $\hat{P}_i$'s. However, due to smaller standard errors of the estimates, in many cases, eventhough the relative bias is very small, it appears the bias is significant. We also notice that the alternative bootstrap standard errors for $\hat{PPP}_j$'s given in Column (8) of Table 3 and for $\hat{P}_i$'s given in Column (6) of Table 4 are very close to the corresponding actual standard errors presented in Column (3) of both tables. Thus the overall conclusion of this section is that (i) we can use the bootstrap standard errors given in Column (8) of Table 3 and Column (6) of Table 4 as alternatives to the actual standard errors of the estimates given in Column (3) of the two tables and (ii) the bootstrap simulation results are re-assuring the quality of our data-based results.

5. Conclusions

This paper describes a method of computing standard errors associated with the purchasing power parities computed using the Geary-Khamis aggregation procedure in the International Comparisons Project (ICP) of the United Nations. The method discussed here is based on an interpretation of these parities using regression approach. The empirical results obtained suggest that the associated standard errors are sizeable in a number of instances. In view of the importance attached to these parities, the results from this paper strongly suggest routine computation and publication of the standard errors associated with the parities. The bootstrap simulation results validate the standard errors obtained using the procedure.

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FOOTNOTES

1 The data-set should satisfy some very mild regularity conditions discussed in Prasada Rao, D.S. (1971) and Khamis, S.H. (1972)

2 The solution is unique up to a factor of proportionality i.e., if one of the unknowns is fixed at an arbitrary level, the rest of the unknowns have a unique solution. Therefore, the ratios of the unknown parities and international prices are uniquely determined.

3 However sensitivity of the G-K parities to errors in the price and expenditure data is examined in Orlando (1990), but it relates to a different issue.

4 Generally these prices are expressed in national currencies. Thus the national expenditures, $p_{ij}\ q_{ij}$, on different commodities are not additive across countries.

5 Purchasing power parity of j-th currency represents the number of numeraire currency units equivalent to one unit of j-th currency. For example, if the numeraire currency is the US dollar and the j-th currency is the Japanese Yen, then 1¥ = US$.00625 represents the purchasing power parity of the Yen and US dollar.

6 Without loss of generality we may assume the values of P_i to be known as the solutions to the GK system are unique up to a factor of proportionality.

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