MAXIMUM LIKELIHOOD ESTIMATION OF
HOUSEHOLD EQUIVALENCE SCALES FROM AN
EXTENDED LINEAR EXPENDITURE SYSTEM:
APPLICATION TO THE 1988 AUSTRALIAN
HOUSEHOLD EXPENDITURE SURVEY

William E. Griffiths and Ma. Rebecca Valenzuela

No. 80 - November 1995

ISSN 0 157 0188
ISBN 1 86389 283 4
To overcome an identification problem which arises when estimating household equivalence scales from a linear expenditure system, Kakwani (1980) introduced additional information in the form of a micro-aggregate consumption function and suggested a two-step estimation procedure that allows for heteroskedasticity across households with differing household composition. The resulting "extended linear expenditure system" was utilized by Binh and Whiteford (1990) who apply Kakwani's estimation procedure to Australian data from the 1984 Household Expenditure Survey. In this paper, we derive and indicate how to compute a maximum likelihood estimator for the extended linear expenditure system. We allow for non-zero error correlation between observations that correspond to different commodities for a given household. Households with different compositions are allowed to have different error covariance matrices. Using the 1988 Australian Household Expenditure Survey, maximum likelihood estimates are obtained for 11 commodity groups and 8 different household types.
I. Introduction

The equivalence-scale concept assumes that household utility from consumption of a given commodity depends on the size and composition of the household. To make the utility generated from consumption of a particular commodity comparable for different household compositions, commodity-specific scales are used to "deflate" consumption of each commodity. A general or income equivalence scale for a given household composition is the amount by which the income of that household must be multiplied to equate its indirect utility function with that of a household with a "reference" composition. Equivalence scales are an important ingredient for assessing living standards and associated aspects such as income inequality, poverty, welfare payments and taxation. Binh and Whiteford (1990) review various definitions of equivalence scales, some of the conceptual issues, and estimates from Australian data. They compare the various estimates and provide a number of new alternative estimates based on the 1984 Household Expenditure Survey. Other recent reviews of the equivalence-scale literature are those of Browning (1992), Coulter, Cowell and Jenkins (1992) and Blundell, Breston and Walker (1994). Related work using Australian and New Zealand data is that of Chatterjee, Michelini and Ray (1994).

In this paper we focus on the extended linear expenditure system that was used by Binh and Whiteford (1990) to provide one of their sets of equivalence-scale estimates. As an alternative to the estimation procedure that was suggested by Kakwani (1980), and utilized by Binh and Whiteford (1990), we develop a maximum likelihood estimator that overcomes possible 2-step estimation problems and which allows for a more general assumption about the equation "errors", that of non-zero correlation between the errors for different commodities in the same household. This assumption is more in line with that usually made for Engel functions and other systems of demand equations. We use Kakwani's estimator and the maximum likelihood estimator to estimate the model and associated equivalence scales from data from the 1988 Household Expenditure Survey, and compare the results with those estimated by Binh and Whiteford (1990) from the 1984 Household Expenditure Survey.
II. The Model and Maximum Likelihood Estimation

Our starting point for outlining the extended linear expenditure system (ELES) is the Klein-Rubin utility function

\[ u_h = \sum_{i=1}^{n} b_i \ln \left( \frac{q_{ih}}{s_{ih}} - c_i \right) \]  

(1)

where

- \( q_{ih} \) is the quantity of the i-th commodity consumed by the h-type household;
- \( s_{ih} \) is the i-th commodity-specific equivalence scale for the h-type household;
- \( b_i \) is the marginal budget share for the i-th commodity satisfying the constraints \( 0 < b_i < 1, \sum_{i=1}^{n} b_i = 1 \);
- \( c_i \) is a parameter which, if interpreted as the subsistence quantity of the i-th commodity, satisfies the constraint \( c_i > 0 \); Pollack and Wales (1978) prefer not to give \( c_i \) a strict subsistence interpretation, letting negative values be a possibility.

Household types (h = 1,2,...,H) are defined in terms of the number of adults and children in the household.

Maximizing (1) subject to the budget constraint

\[ \sum_{i=1}^{n} p_i q_{ih} = v_h \]  

(2)

where

- \( p_i \) is the price of the i-th commodity and
- \( v_h \) is total expenditure for the h-type household,

leads to the linear expenditure system

\[ v_{ih} = a_{ih} + b_i (v_h - a_h) \]  

(3)

where
\[ v_{ih} = p_i q_{ih} \] is expenditure on the i-th commodity by the h-type household,

\[ a_{ih} = p_i c_i s_{ih} \] is subsistence expenditure for the i-th commodity and h-type household, and

\[ a_h = \sum_{i=1}^{n} a_{ih} \] is total subsistence expenditure for a h-type household.

The immediate problem is to estimate the \( a_{ih} \) and the \( b_i \), with these estimates later being used to estimate the scales \( s_{ih} \). However, without further information not all of the \( a_{ih} \) are identified.

One solution to the identification problem (see Griffiths and Chotikapanich (1995) for a "Bayesian solution") is to use an ELES which includes a micro-consumption function given by

\[ v_h = (1 - b)a_h + b x_h \] (4)

where

- \( x_h \) is net income for the h-type household, and
- \( b \) is a common marginal propensity to consume.

To estimate the parameters in (3) and (4), Kakwani (1980) appends errors to these equations, and assumes the error variances can be different for each household type and for each commodity. He suggests first estimating \( a_h \) and \( b \) from equation (4), and then replacing \( a_h \) in each of the commodity equations (3) with its estimate from equation (4). To estimate the \( a_{ih} \) and \( b_i \) in equation (3), weighted least squares which allows for the heteroskedasticity across different household types is applied in turn to each of these equations; using an external estimate of \( a_h \) identifies the remaining parameters. The estimation procedure that we develop attempts to improve on Kakwani's procedure in two ways. First, because Kakwani estimates each of the commodity equations separately, he is ignoring any correlation that might exist between the errors that correspond to different commodity equations for a given household. Second, the "2-step" nature of the procedure ignores the effect of using estimates from one equation on the properties of estimates from a second equation. An
estimator which allows for error correlation across different commodity equations and which estimates all parameters simultaneously would seem more desirable.

To investigate how all the parameters might be jointly estimated, we substitute (4) into (3) to obtain

\[ v_{ih} = a_{ih} + b_i[(a - b)a_h + b_xh - a_h] \]

\[ = a_{ih} + b_i(b_xh - a_h) \]

\[ = a_{ih} - b_i b a_h + b_i b x_h \]

\[ = \pi_{ih} + \eta_i x_h \] (5)

where

\[ \pi_{ih} = a_{ih} - b_i b a_h \] (6)

\[ \eta_i = b_i b \] (7)

Let us consider estimation of the \( \pi_{ij} \) and the \( \eta_i \) and then examine how estimates of the original parameters, \( a_{ih}, b_i, b \) and \( a_h \), can be retrieved from these estimates. To introduce some statistical assumptions, we modify equation (5) to

\[ v_{ih} = \pi_{ih} z_h + \eta_i x_h + e_{ih} \] (8)

where \( v_{ih} \) is an \((M_h x 1)\) vector of observations on expenditure for the i-th commodity and h-type household;

\( z_h \) is an \((M_h x 1)\) vector of ones;

\( x_h \) is an \((M_h x 1)\) vector of observations on income for households of type h;

\( e_{ih} \) is an \((M_h x 1)\) vector of errors.

Let \( E_h' = (e_{ih}', e_{ih}', ..., e_{ih}') \) be an \((nM_h x 1)\) vector containing all errors for all the commodity equations for the h-type household. We assume that

\[ E_h \sim N(0, \Omega_h \otimes I_{M_h}) \] (9)

Thus, the error covariance matrix \( \Omega_h \) is allowed to be different for different household types. Because \( \Omega_h \) is not diagonal, correlation between errors from equations for different commodities, and the same household, is permitted. Zero error correlation is
assumed across different households. (The sample is assumed to be random.) Thus, in addition to (9), \( E(\varepsilon_h^e\varepsilon_k^e) = 0 \) for \( h \neq k \).

A maximum likelihood estimator for the \( \pi_{ih} \), the \( \eta_i \) and the \( \Omega_h \) is described in the Appendix. Specifically, maximum likelihood estimates are those values that satisfy equations (A16), (A22) and (A32). A convenient iterative procedure for computing these estimates is as follows:

1. Express \( v_{ih} \) and \( x_h \) in terms of deviations from their household type means. That is, compute \( v_{ih}^* = v_{ih} - \bar{v}_{ih} z_h \) and \( x_h^* = x_h - \bar{x}_h z_h \), where

   \[
   \bar{v}_{ih} = M_h^{-1} z_h v_{ih} \quad \text{and} \quad \bar{x}_h = M_h^{-1} z_h x_h.
   \]

2. Find the least squares estimates \( \hat{\eta}^h_i = (x_h^* x_h^*)^{-1} x_h^* v_{ih} \).

3. Find an initial estimate of \( \Omega_h \) as

   \[
   \left[ \hat{\Omega}_h \right]_{ij} = (v_{ih}^* - x_h^* \hat{\eta}^h_i) \left( v_{jh}^* - x_h^* \hat{\eta}^h_j \right) / M_h.
   \]

   (Note that steps (2) and (3) can be computed at the same time with a seemingly unrelated regression of each of \( v_{1h}, v_{2h}, \ldots, v_{nh} \) on \( x_h^* \), with no constant.)

4. Compute a pooled estimate for \( \eta \) as

   \[
   \hat{\eta} = \left[ \sum_{h=1}^{H} (x_h^* x_h^*) \hat{\Omega}_h^{-1} \right]^{-1} \sum_{h=1}^{H} (x_h^* x_h^*) \hat{\Omega}_h^{-1} \hat{\eta}^h
   \]

   where \( \hat{\eta}^h = (\hat{\eta}^h_1, \hat{\eta}^h_2, \ldots, \hat{\eta}^h_n) \).

5. Use the estimates from (4) to compute

   \[
   \left[ \hat{\Omega}_h \right]_{ij} = (v_{ih}^* - x_h^* \hat{\eta}_i) \left( v_{jh}^* - x_h^* \hat{\eta}_j \right) / M_h.
   \]

6. Repeat steps 4 and 5 until convergence.

7. Compute estimates of the \( \pi_{ih} \) from

   \[
   \hat{\pi}_{ih} = \bar{v}_{ih} - \bar{x}_h \hat{\eta}_i
   \]
In our experience the convergence of (11) and (12) took less than 6 iterations.

The asymptotic standard errors of the $\hat{\eta}_i$ are given by (see equations (A39) and (A44)) the square roots of the diagonal elements of

$$V(\hat{\eta}) = D^{-1} = \left(\sum_{h=1}^{H} \left( X_h' X_h - M_h \bar{x}_h^2 \right) \Omega_h^{-1} \right)^{-1}$$  \hspace{1cm} (14)

Those for the $\hat{\pi}_h$ are given by the square roots of the diagonal elements of (see equation (A43))

$$V(\hat{\Pi}_h) = M_h^{-1} \Omega_h + \bar{x}_h^2 D^{-1}$$  \hspace{1cm} (15)

Given estimates for $\pi_{ij}$ and $\eta_i$, estimates for $b$, $a_h$ and $a_{ih}$ can be obtained from

$$b = \sum_{i=1}^{n} \eta_i$$  \hspace{1cm} (16)

$$a_h = \frac{\sum_{i=1}^{n} \pi_{ih}}{1 - b}$$  \hspace{1cm} (17)

$$b_i = \frac{\eta_i}{b}$$  \hspace{1cm} (18)

$$a_{ih} = \pi_{ih} + b_i b a_h$$  \hspace{1cm} (19)

The asymptotic variances and covariance matrices for the estimates obtained from equations (16) - (19) are derived in the Appendix as follows:

$$V(\hat{b}) = z' D^{-1} z$$  \hspace{1cm} (20)

where $z$ is an n-dimensional vector of ones;

$$V(\hat{\beta}) = \frac{1}{(1 - b)^2} C D^{-1} C'$$  \hspace{1cm} (21)
where \( \hat{\beta}' = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n) \) and \( C = I_n + \frac{1}{1-b} \eta \eta' \);

\[
V(\hat{\alpha}_h) = C \left[ M_h^{-1} \Omega_h + (\bar{x}_h - a_h)^2 D^{-1} \right] C'
\] (22)

where \( \hat{\alpha}_h = (\hat{a}_{1h}, \hat{a}_{2h}, \ldots, \hat{a}_{nh}) \);

\[
V(\hat{\alpha}_h) = z' \left[ V(\hat{\alpha}_h) \right] z
\]

(23)

Noting that \( a_{ih} = p_i c_i s_{ih} \), we see that the commodity-specific equivalence scales can be estimated from

\[
\hat{s}_{ih} = \frac{\hat{a}_{ih}}{\hat{a}_{ir}}
\]

(24)

where \( \hat{a}_{ir} \) refers to the subsistence expenditure on the i-th commodity by the reference household. Note that, for the reference household, we set \( s_{ir} = 1 \). In the Appendix the asymptotic variance of \( \hat{s}_{ih} \) is derived as

\[
V(\hat{s}_{ih}) = \frac{1}{\hat{a}_{ir}^2} V(\hat{a}_{ih}) + \frac{\hat{a}_{ih}^2}{\hat{a}_{ir}^4} V(\hat{a}_{ir}) - \frac{2a_{ih}}{a_{ir}^2} \text{cov}(\hat{a}_{ih}, \hat{a}_{ir})
\]

(25)

where the \( \text{cov}(\hat{a}_{ih}, \hat{a}_{ir}) \) are given by the diagonal elements of

\[
(\bar{x}_h - a_h)(\bar{x}_r - a_r)C^{-1}C'.
\]

The final quantities of interest are the general scales; the general scale of the h-type household is the ratio of income of that household to the income of the reference household that is required to equate the indirect utility functions of the two households. From Binh and Whiteford (1990), it is given by

\[
s_h = \frac{a_h}{x_r} + \prod_{i=1}^{n} (s_{ih})^{-b_i} \left[ 1 - \frac{a_r}{x_r} \right]
\]

(26)
III. Data and Results

The data used in this study are derived from the 1988-89 Household Expenditure Survey (HES) conducted by the Australian Bureau of Statistics between the period July 1988 to July 1989. It is the fourth of a series of surveys designed to obtain details of expenditures, income and a wide range of demographic characteristics of Australian private households on a nation-wide basis. The public-use tape released in November 1990 contains a total sample of 7225 households representing over 5.4 million households from all over the country.

The HES is concerned with the expenditure patterns of private households. It is restricted to goods and services that are for private consumption. For estimation of the scales, we used the same commodity and household classifications as those employed by Binh and Whiteford (1990) in their study that used the earlier 1984 Household Expenditure Survey.

The n=11 expenditure categories are:

1. **Housing** includes expenses incurred for the payment of rent, mortgage, property rates, house and contents insurance as well as housing repairs and maintenance;

2. **Fuel and Power** includes all expenses towards electricity, gas, and other fuels;

3. **Food** includes all expenses towards bakery products, flour and other cereals, meat and fish, dairy products, fruits and vegetables, miscellaneous food (jams, jellies, coffee, tea), non-alcoholic beverages, meals out and take-away food;

4. **Alcohol and Tobacco** refers to all expenses towards the purchase of cigarettes and all types of alcoholic beverages;

5. **Clothing & Footwear** includes all expenses towards the purchase of clothing and footwear for men, women and children, clothing accessories (e.g. ties, gloves, handkerchiefs) as well as clothing and footwear services (e.g. drycleaning and shoe repairs);

6. **Household Furnishings & Equipment** includes all expenses towards furniture and floor coverings, blankets and rugs, household linen and furnishings, household appliances, glassware, tableware, household utensils and cleaning
agents. This category also includes expenditure incurred for the operation of the household such as gardening services, housekeeping, childcare, and the repair and maintenance of household durables;

7. **Medical & Health Care** covers items such as accident and health insurance premiums, practitioner’s fees, prescriptions, medicines, pharmaceutical products, hospital and other health charges;

8. **Transport** refers to all expenses made for the purchase of motor vehicles, petrol and fuels, vehicle registration and insurance, vehicle servicing and repairs, driver’s licenses, driving lessons, subscriptions to motor organisations, vehicle hire, as well as public transport fees;

9. **Recreation & Entertainment** includes expenses incurred for the purchase of television and other audio-visual equipment, books, newspapers, and other printed material, recreational equipment (cameras, musical instruments, toys), gambling, entertainment and recreational services. Holiday expenses as well as those incurred for animal pets are also included in this category;

10. **Personal Care** pertains to expenses towards toiletries, cosmetics, hair dressing and beauty services;

11. **Others** includes expenses for miscellaneous goods (watches, jewellery, stationary), interest payments on selected credit services, education fees, and other miscellaneous services. This category also includes income tax payments, other capital housing costs (extensions, renovations, landscaping) and superannuation and life insurance;

The actual sample used is restricted to households of related persons with one or two adults and at most three children, resulting in $H = 8$ different household types. Adults are all those aged 17 or older and children refer to all those aged 16 or below. From the original sample set of 7225 households, 290 households with negative expenditure items were discarded as their occurrence was not consistent with the models as set up in the earlier discussions. They were also deemed unexplainable in practical terms. [72 percent of these negative expenditures were on transport while 27 percent were on recreation and entertainment.] Households with zero expenditures
were retained. The number and other characteristics of household types used in this study are given in Table 1.

Table 2 presents the parameter estimates of the extended linear expenditure system following the steps outlined for the maximum likelihood estimation procedure in part II. The iterative process (Steps 4 and 5) converged on the 5th iteration and yielded the \( \eta_i \) estimates found in the 2nd column. The other columns present the estimates of \( \pi_{ab} \) corresponding to each commodity group and household type. The table also provides estimates of the asymptotic standard errors of these parameters from equations (14) and (15). These estimates of \( \eta_i \) and \( \pi_{ab} \) do not carry a direct economic interpretation but are important to the procedure as they lead to the estimation of the marginal propensities and subsistence expenditures which are presented in Table 3. The standard errors are all relatively small, except perhaps for household type (1,3) where the number of households of that type is small.

In Table 3 the second column provides the marginal budget shares \( b_i \) and the third through the eighth columns give the estimates of subsistence expenditures \( a_{ih} \) for each expenditure category. For all types of households, expenditures on food, housing, transport and household furnishings and equipment make up between 62 to 67 per cent of subsistence expenditures. In general, the \( a_{ih} \) estimates are observed to increase with the size of the household. The inferences that can be drawn from Table 3 are similar to those that can be drawn from the commodity-specific scales given in Table 4, so we turn now to that table. A two-adult household with no children is chosen to be the reference household for which \( s_{ih} \) is set to 1. The per household equivalence scale for each commodity thus shows the relative requirements of other household types for them to be on the same level of utility as this reference household. In the ELES the commodity-specific scales are derived as ratios of subsistence expenditures (relative to the reference household). As expected, for most commodities there is an increase in the per household equivalent (phe) expenditure as the household size increases; these increases generally occur at a decreasing rate indicating economies of scale for additional children. There are some exceptions to these observations. The phe expenditure for Alcohol and Tobacco declines as the number of children in the household increases. Also, the phe expenditures for Medical and Health Care and
Others commodity groups exhibit no defined trend for 1 adult households. A more thorough investigation of expenditure patterns of households may be required for us to provide definitive explanations for such deviations but they may be because the presence of children in the household tends to influence expenses away from 'adult goods' under which alcohol, tobacco and many other miscellaneous goods are classified. After the first child there exist strong economies of scale for additional children, particularly for expenditures towards housing, food and household furnishings.

It is of interest to note whether there has been a change in the scales over time. thus, in Table 5, we compare scale estimates calculated from Binh and Whiteford's (1990) results, that used the 1984 data, with our results that used the 1988 data. Also, since it could be argued that a difference in results may be attributable to the new maximum likelihood estimation procedure, rather than a change in consumption patterns, we also include estimates from the 1988 data, obtained using Kakwani's estimation procedure. The two sets of 1988 scales are very similar with no one method exhibiting consistently higher or lower values. The estimated standard errors for both sets (available from the authors) show more divergence, but again do not display any consistent over or underestimation.

There are noticeable changes between the 1984 and 1988 scales. The phe expenditures appear to have increased significantly in 1988 for one-adult households with no children, but decreased for households with children. A significant difference observed in the table is that in 1984, relative costs for Alcohol and Tobacco increased with the increase in household size. This trend is reversed in 1988. There are fewer economies of scale in housing in the later data set, but greater economies of scale in food. The largest differences in the phe expenditures occurred in the one-adult, three-children household group; since the number of households in this group is relatively small, and the standard errors of the estimated scales are relatively high, these differences may reflect sampling error.

The general scales computed from equation (26) are presented in Table 6. Because these scales depend on income they are computed for three income levels, the
same levels as utilized by Binh and Whiteford (1990). Also, as in Table 5, we have presented three estimates of each scale - the Binh and Whiteford 1984 estimates, the 1988 estimates using Kakwani's estimation procedure and the 1988 estimates using maximum likelihood estimation. There is virtually no difference between the two sets of 1988 estimates. Also, there is virtually no sensitivity to the level of income. Comparing the 1984 estimates with the 1988 estimates, we find the results for the 2-adult families are quite similar, although Binh and Whiteford's conclusion that "there is strong evidence of economies of scale in the second child but adding the third child increases these households' relative needs considerably" no longer holds. For the 1988 data adding the third child was only slightly more expensive than adding the second child. For one-adult families the noticeable differences are an increase in the relative cost when there are no children, and a decrease in the relative cost when children are present.

IV. Conclusions

We have introduced a maximum likelihood estimation procedure for an extended linear expenditure system that has different intercepts and different error covariance matrices for groups of households with different compositions. Estimates of household equivalence scales, both general and commodity specific, can be obtained from this system. The procedure was applied to data from the 1988 Australian Household Expenditure Survey. For this particular sample the maximum likelihood estimates were similar to those from a 2-step estimation procedure proposed by Kakwani (1980). However, with respect to methodology, the maximum likelihood procedure is an improvement over that of Kakwani, and we cannot conclude that the two techniques will yield similar estimates in other samples. Furthermore, the more general error covariance assumptions make it likely that the standard errors will more accurately assess the reliability of estimation. A comparison of the estimates with those obtained using 1984 data uncovered some changes in the equivalence scales; in particular the general scales for one-adult households and households with two adults and two children were noticeably different.
Appendix: Maximum Likelihood Estimation And Related Covariance Matrices

The system of equations we wish to estimate is given by

\[
\begin{bmatrix}
V_{1h} \\
V_{2h} \\
\vdots \\
V_{nh}
\end{bmatrix} =
\begin{bmatrix}
\pi_{1h} \\
\pi_{2h} \\
\vdots \\
\pi_{nh}
\end{bmatrix} +
\begin{bmatrix}
x_{h} \\
x_{h} \\
\vdots \\
x_{h}
\end{bmatrix} \times
\begin{bmatrix}
\eta_{1} \\
\eta_{2} \\
\vdots \\
\eta_{n}
\end{bmatrix} +
\begin{bmatrix}
e_{1h} \\
e_{2h} \\
\vdots \\
e_{nh}
\end{bmatrix} (A1)
\]

or

\[ V_h = Z_h \Pi_h + X_h \eta + E_h \] (A2)

where \( h = 1, 2, \ldots, H \) refers to households with household composition type \( h \); \( n \) refers to the number of commodity groups;

- \( v_{ih} \) is an \((M_h \times 1)\) vector of observations on expenditure for the \( i \)-th commodity and \( h \)-type household;
- \( z_h \) is an \((M_h \times 1)\) vector of ones;
- \( x_h \) is an \((M_h \times 1)\) vector of observations on income for households of type \( h \);
- \( e_{ih} \) is an \((M_h \times 1)\) vector of errors;
- \( V_h \) is of dimension \((nM_h \times 1)\);
- \( Z_h = I_n \otimes z_h \) is a \((nM_h \times n)\) matrix of dummy variables;
- \( X_h = I_n \otimes x_h \);
- \( \Pi_h \) and \( \eta \) are \((n \times 1)\) vectors of unknown parameters;
- \( E_h \) is an \((nM_h \times 1)\) vector of errors which we assume are distributed as

\[ E_h \sim N[0, \Omega_h \otimes I_{M_h}] \] (A3)

Given estimates of \( \Pi_1, \Pi_2, \ldots, \Pi_H \) and \( \eta \), estimates of the original parameters can be obtained using the expressions

\[ b = \sum_{i=1}^{n} \eta_i \] (A4)
Moreover, the equivalence scale for the \( i \)-th commodity and \( h \)-type household is given by

\[
a_{ih} = \frac{\sum_{j=1}^{n} \pi_{ih}}{1 - \sum_{j=1}^{n} \eta_{j}}
\]  
(A5)

\[
b_{i} = \frac{\eta_{i}}{\sum_{j=1}^{n} \eta_{j}}
\]  
(A6)

\[
a_{bh} = \pi_{ib} + \frac{\eta_{i} \sum_{j=1}^{n} \pi_{jh}}{1 - \sum_{j=1}^{n} \eta_{j}}
\]  
(A7)

Moreover, the equivalence scale for the \( i \)-th commodity and \( h \)-type household is given by

\[
s_{ih} = \frac{a_{ih}}{a_{ih}}
\]  
(A8)

where \( r \) refers to the reference household. Our task is to derive expressions for the maximum likelihood estimates of \( \Pi_{h}, \Omega_{h} \) \( (h = 1, 2, ..., H) \) and \( \eta \), asymptotic covariance matrices for these estimates, and asymptotic covariance matrices for the consequent maximum likelihood estimates for the parameters in (A4) - (A8).

Noting that \( V_{1}, V_{2}, ..., V_{H} \) are independent we can write the log-likelihood function for all parameters, given data on all household types, as

\[
\log L = -\frac{nM}{2} \log(2\pi) - \frac{1}{2} \sum_{h=1}^{H} M_{h} \log|\Omega_{h}| \\
- \frac{1}{2} \sum_{h=1}^{H} (V_{h} - Z_{h} \Pi_{h} - X_{h} \eta)' \left( \Omega_{h}^{-1} \otimes I_{M_{h}} \right) (V_{h} - Z_{h} \Pi_{h} - X_{h} \eta)
\]  
(A9)

where \( M = \sum_{h=1}^{H} M_{h} \). To maximize this function we first investigate whether it is possible to concentrate out the \( \Pi_{h} \). Working in this direction, the last term can be written (without the summation) as
Now,

\[ \log L_{c3} \]

\[ = (V_h - X_h \eta) (\Omega^{-1}_h \otimes I_{M_h}) (V_h - Z_h \Pi_h - X_h \eta) \]

\[ = (V_h - X_h \eta) (\Omega^{-1}_h \otimes I_{M_h}) (V_h - X_h \eta) + \Pi_h Z_h' (\Omega^{-1}_h \otimes I_{M_h}) Z_h \Pi_h - 2 \Pi_h Z_h' (\Omega^{-1}_h \otimes I_{M_h}) (V_h - X_h \eta) \]

(A10)

Setting this derivative equal to zero and solving for the maximizing value \( \hat{\Pi}_h \) gives

\[ \hat{\Pi}_h = \left[ Z_h' (\Omega^{-1}_h \otimes I_{M_h}) Z_h \right]^{-1} Z_h' (\Omega^{-1}_h \otimes I_{M_h}) (V_h - X_h \eta) \]

(A12)

Now,

\[ Z_h' (\Omega^{-1}_h \otimes I_{M_h}) Z_h = (I_n \otimes z_h') (\Omega^{-1}_h \otimes I_{M_h}) (I_n \otimes z_h) \]

\[ = \Omega^{-1}_h \otimes z_h' z_h \]

\[ = \Omega^{-1}_h \otimes M_h = M_h \Omega^{-1}_h \]

(A13)

Also,

\[ Z_h' (\Omega^{-1}_h \otimes I_{M_h}) = \Omega^{-1}_h \otimes z_h' \]

(A14)

Using (A13) and (A14) in (A12) gives

\[ \hat{\Pi}_h = \left[ \Omega^{-1}_h \otimes M_h \right]^{-1} (\Omega^{-1}_h \otimes z_h') (V_h - X_h \eta) \]

\[ = \left( \Omega_h \otimes M_h^{-1} \right) (\Omega^{-1}_h \otimes z_h') (V_h - X_h \eta) \]

\[ = \left( I_n \otimes \frac{1}{M_h} z_h' \right) (V_h - X_h \eta) \]

(A15)

Considering the i-th row in equation (A15), we obtain
where $\bar{v}_{ih} = \frac{1}{M_h} z_h' v_{ih}$ is the average expenditure on commodity $i$ for all households of type $h$, and

$$\bar{x}_h = \frac{1}{M} z_h' x_h$$

is the average income of all $h$-type households.

The result in (A16) is an important one. It means that the $\hat{\pi}_{ih}$ do not depend on $\Omega_h$ and can be computed at the end of the maximum likelihood algorithm, after we have estimated the $\Omega_h$ and $\eta_i$.

Let $\bar{V} = (\bar{V}_1, \bar{V}_2, \ldots, \bar{V}_M)$ and $\bar{x}_h = I_n \otimes \bar{x}_h$. Then,

$$\Delta = \bar{V} - \bar{x}_h \eta$$

Substituting (A17) into (A10) yields

$$Q = \left((V_h - Z_h \bar{V}_h) - (X_h - Z_h \bar{x}_h) \eta\right)' \left(\Omega_h^{-1} \otimes I_{M_h}\right) \left((V_h - Z_h \bar{V}_h) - (X_h - Z_h \bar{x}_h) \eta\right)$$

where $V_h^* = V_h - Z_h \bar{V}_h$ is a vector of expenditures expressed in terms of deviations from the mean expenditures for each commodity and household type, and $X_h^* = X_h - Z_h \bar{x}_h$ is a vector of incomes expressed in terms of deviations from the mean incomes for each household type. The concentrated log-likelihood function can now be written as

$$\log L^* = -\frac{nM}{2} \log(2\pi) - \frac{1}{2} \sum_{h=1}^H M_h \log |\Omega_h|$$

$$-\frac{1}{2} \sum_{h=1}^H (V_h^* - X_h^*)' (\Omega_h^{-1} \otimes I_{M_h}) (V_h^* - X_h^*) \eta$$

$$= -\frac{nM}{2} \log(2\pi) + \frac{1}{2} \sum_{h=1}^H M_h \log |\Omega_h^{-1}| - \frac{1}{2} \sum_{h=1}^H \text{tr}[W_h \Omega_h^{-1}]$$
where $W_h$ is an $(n \times n)$ matrix with $(i,j)$th element given by

$$[W_h]_{ij} = \left( v_{ih}^* - x_{ih}^* \eta_i \right)' \left( v_{jh}^* - x_{jh}^* \eta_j \right)$$  \hspace{1cm} (A20)

See Judge et al. (1988, p.553) for details of the two alternative specifications in (A19a) and (A19b). Judge et al. also give details on how to differentiate (A19b) with respect to $\Omega_h^{-1}$. This differentiation yields

$$\frac{\partial \log L^*}{\partial \Omega_h^{-1}} = \frac{M_h}{2} \Omega_h - \frac{1}{2} W_h$$  \hspace{1cm} (A21)

Setting this derivative to zero yields the maximum likelihood estimator for $\Omega_h$, given $\eta$. Specifically,

$$\hat{\Omega}_h = \frac{1}{M_h} W_h$$  \hspace{1cm} (A22)

To find an expression for the maximum likelihood estimator for $\eta$, we return to the last term in (A19a) and rewrite it as

$$\sum_{h=1}^H Q_h^* = \sum_{h=1}^H \left( v_{ih}^* - x_{ih}^* \right)' \left( \Omega_h^{-1} \otimes I_{M_h} \right) \left( v_{ih}^* - x_{ih}^* \right)$$

$$= \sum_{h=1}^H \left[ v_{ih}^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) v_{ih}^* + \eta^t X_h^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) X_h^* \eta - 2 \eta^t X_h^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) v_{ih}^* \right]$$  \hspace{1cm} (A23)

Now,

$$\frac{\partial \log L^*}{\partial \eta} = -\frac{1}{2} \sum_{h=1}^H \frac{\partial Q_h^*}{\partial \eta}$$

$$= -\frac{1}{2} \sum_{h=1}^H \left[ 2 X_h^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) X_h^* \eta - 2 X_h^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) v_h^* \right]$$  \hspace{1cm} (A24)

Setting this quantity equal to zero yields

$$\left[ \sum_{h=1}^H X_h^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) X_h^* \right] \hat{\eta} = \sum_{h=1}^H X_h^* \left( \Omega_h^{-1} \otimes I_{M_h} \right) v_h^*$$  \hspace{1cm} (A25)

Now,
\[ X_h^*(\Omega_h^{-1} \otimes I_{M_h})X_h^* = (I_n \otimes x_h^*)(\Omega_h^{-1} \otimes I_{M_h})(I_n \otimes x_h^*) \]
\[ = \Omega_h^{-1} \otimes x_h^*x_h^* \]
\[ = x_h^*x_h^*\Omega_h^{-1} \quad (A26) \]

Also,
\[ X_h^*(\Omega_h^{-1} \otimes I_{M_h})V_h^* = (I_n \otimes x_h^*)(\Omega_h^{-1} \otimes I_{M_h})V_h^* \]
\[ = (\Omega_h^{-1} \otimes x_h^*)V_h^* \quad (A27) \]

In light of the 2nd line in equation (A26), let us write equation (A27) as
\[ X_h^*(\Omega_h^{-1} \otimes I_{M_h})V_h^* = [\Omega_h^{-1} \otimes x_h^*x_h^* \Omega_h^{-1} \otimes x_h^*x_h^*]^{-1}[\Omega_h^{-1} \otimes x_h^*x_h^*]V_h^* \]
\[ = (x_h^*x_h^*\Omega_h^{-1})(\Omega_h \otimes (x_h^*x_h^*)^{-1})(\Omega_h^{-1} \otimes x_h^*)V_h^* \]
\[ = x_h^*x_h^*\Omega_h^{-1}[I_n \otimes (x_h^*x_h^*)^{-1}]V_h^* \]
\[ = x_h^*x_h^*\Omega_h^{-1}\hat{\eta}^b \quad (A28) \]

where
\[ \hat{\eta}^b = [I_n \otimes (x_h^*x_h^*)^{-1}]V_h^* \quad (A29) \]

is the OLS estimator for \( \eta \) from observations corresponding only to the \( h \)-type household. The \( i \)-th element in \( \hat{\eta}^b \) is given by
\[ \hat{\eta}_{hi} = (x_h^*x_h^*)^{-1}x_h^*V_{hi}^* \quad (A30) \]

Substituting (A26) and (A28) into (A25) yields
\[ \sum_{h=1}^{H} (x_h^*x_h^*)\Omega_h^{-1}\hat{\eta} = \sum_{h=1}^{H} (x_h^*x_h^*)\Omega_h^{-1}\hat{\eta}^b \quad (A31) \]

or
\[ \hat{\eta} = \left[ \sum_{h=1}^{H} (x_h^*x_h^*)\Omega_h^{-1} \right]^{-1} \sum_{h=1}^{H} (x_h^*x_h^*)\Omega_h^{-1}\hat{\eta}^b \quad (A32) \]
The maximum likelihood estimator for $\eta$ is given by a matrix-weighted average of the h-type household OLS estimators $\hat{\eta}_h$ with weights given by $\left(x_h^* x_h^*\right)\Omega_h^{-1}$. Maximum likelihood estimators for all the $\pi_{ih}$, $\eta$ and $\Omega_h$ are given by the simultaneous solution of equations (A16), (A22) and (A32).

**Asymptotic Covariance Matrix**

To specify the asymptotic covariance matrix for the maximum likelihood estimator we need the second derivatives of the log-likelihood function. From (A11), (A13) and (A14):

$$\frac{\partial^2 \log L}{\partial \Pi_h \partial \Pi_k'} = -Z_h' \left(\Omega_h^{-1} \otimes I_{M_h}\right)Z_h = -M_h \Omega_h^{-1}$$  \hspace{1cm} (A33)

$$\frac{\partial^2 \log L}{\partial \Pi_h \partial \eta_i'} = 0 \hspace{1cm} (h \neq k)$$  \hspace{1cm} (A34)

$$\frac{\partial^2 \log L}{\partial \Pi_h \partial \eta_j'} = -Z_h' \left(\Omega_h^{-1} \otimes I_{M_h}\right)X_h'x_h = -M_h x_h' \Omega_h^{-1}$$  \hspace{1cm} (A35)

From (A9) and (A26):

$$\frac{\partial^2 \log L}{\partial \eta \partial \eta'} = -\sum_{h=1}^H X_h' \left(\Omega_h^{-1} \otimes I_{M_h}\right)X_h$$

$$= -\sum_{h=1}^H x_h' x_h \Omega_h^{-1}$$  \hspace{1cm} (A36)

It can be shown that the expectation of the cross partial derivatives with respect to the $\pi_{ij}$ or the $\eta_i$, and the elements of $\Omega_h$, is zero. Thus, the information matrix is block diagonal, and, providing we are not interested in the standard errors of the maximum likelihood estimator for $\Omega_h$, we can confine ourselves to the derivatives (A33)-(A36).

Specifically, let

$$\theta' = (\Pi', \Pi_2', \ldots, \Pi_H', \eta')$$  \hspace{1cm} (A37)

then
\[
-\frac{\partial^2 \log L}{\partial \theta \partial \theta'} = \begin{bmatrix}
M_1 \Omega_1^{-1} & 0 & \ldots & 0 & M_1 \bar{x}_1 \Omega_1^{-1} \\
0 & M_2 \Omega_2^{-1} & \ldots & 0 & M_2 \bar{x}_2 \Omega_2^{-1} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & M_H \Omega_H^{-1} & M_H \bar{x}_H \Omega_H^{-1} \\
M_1 \bar{x}_1 \Omega_1^{-1} & M_2 \bar{x}_2 \Omega_2^{-1} & \ldots & M_H \bar{x}_H \Omega_H^{-1} & \sum_{b=1}^{H} x_b' x_b \Omega_b^{-1}
\end{bmatrix}
\]  

(A38)

Let

\[
D = \sum_{h=1}^{H} \left( (x_h' x_h - M_h \bar{x}_h^2) \Omega_h^{-1} \right)
\]  

(A39)

\[
\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_H)'
\]  

(A40)

\[
A = \begin{bmatrix}
M_1^{-1} \Omega_1 & \\
& M_2^{-1} \Omega_2 & \ddots & \\
& & \ddots & \\
& & & M_H^{-1} \Omega_H
\end{bmatrix}
\]  

(A41)

Using results on the partitioned inverse of a matrix, and using \( V(.) \) to denote asymptotic covariance matrix, it can be shown that

\[
V(\hat{\theta}) = \left[ -\frac{\partial^2 \log L}{\partial \theta \partial \theta'} \right]^{-1}
\]

\[
= \begin{bmatrix}
A + \bar{x} \bar{x}' \otimes D^{-1} & -\bar{x} \otimes D^{-1} \\
-\bar{x}' \otimes D^{-1} & D^{-1}
\end{bmatrix}
\]  

(A42)

The relevant variance components from (A42) are

\[
V(\hat{\Pi}_h) = M_h^{-1} \Omega_h + \bar{x}_h^2 D^{-1}
\]  

(A43)

and

\[
V(\hat{\eta}) = D^{-1}
\]  

(A44)

*Asymptotic Covariance Matrix for Original Parameters*

The original parameters of interest are defined in equations (A4) - (A7). From (A4)
\[ \text{V}(\hat{\beta}) = z'D^{-1}z \quad \text{(A45)} \]

where \( z = (1, 1, \ldots, 1)' \). Let \( \beta = (b_1, b_2, \ldots, b_n)' \). Then,

\[ \text{V}(\hat{\beta}) = \left( \frac{\partial \beta}{\partial \eta'} \right) D^{-1} \left( \frac{\partial \beta}{\partial \eta'} \right)' \quad \text{(A46)} \]

Now,

\[ \frac{\partial b_i}{\partial \eta_j} = \begin{cases} \frac{1}{1 - b} \left( 1 + \frac{\eta_i}{1 - b} \right) & \text{for } i = j \\ \frac{1}{1 - b} \left( \frac{\eta_j}{1 - b} \right) & \text{for } i \neq j \end{cases} \quad \text{(A47)} \]

Let

\[ C = I_n + \frac{1}{1 - b} \eta z' \quad \text{(A48)} \]

From (A47) and (A48) it follows that

\[ \frac{\partial \beta}{\partial \eta'} = \frac{1}{1 - b} C \quad \text{(A49)} \]

and

\[ \text{V}(\hat{\beta}) = \frac{1}{(1 - b)^2} CD^{-1}C' \quad \text{(A50)} \]

Consider now the covariance matrix for the \( \hat{\alpha}_{ih} \). Let \( \alpha_h = (a_{1h}, a_{2h}, \ldots, a_{nh})' \). We have

\[ \text{V}(\hat{\alpha}_h) = \left[ \frac{\partial \alpha_h}{\partial \Pi'_h} \frac{\partial \alpha_h}{\partial \eta'} \right] \text{V}(\hat{\Pi}_h) \left[ \frac{\partial \alpha_h}{\partial \Pi'_h} \frac{\partial \alpha_h}{\partial \eta'} \right]' \quad \text{(A51)} \]

Now,

\[ \frac{\partial \alpha_h}{\partial \Pi'_h} = C \quad \text{and} \quad \frac{\partial \alpha_h}{\partial \eta'} = a_h C \quad \text{(A52)} \]
Thus,

\[
V(\alpha) = \begin{bmatrix}
C & a_h C \\
-a_h D & D
\end{bmatrix}
\begin{bmatrix}
M_h^{-1}\Omega_h + \bar{x}_h^2 D^{-1} & -\bar{x}_h D^{-1} \\
-\bar{x}_h D^{-1} & D^{-1}
\end{bmatrix}
\begin{bmatrix}
C' \\
a_h C'
\end{bmatrix}
\]

\[
= C\left[M_h^{-1}\Omega_h + (\bar{x}_h - a_h)^2 D^{-1}\right]C' \tag{A53}
\]

Noting that \( \hat{\alpha}_h = z' \hat{\alpha}_{\bar{h}} \), we also have

\[
V(\hat{\alpha}_h) = z'C\left[M_h^{-1}\Omega_h + (\bar{x}_h - a_h)^2 D^{-1}\right]C'z \tag{A54}
\]

Finally, let us consider the variance of the commodity scale estimates \( \hat{s}_{ih} = \hat{\alpha}_{i h} / \hat{\alpha}_{i r} \). In this regard,

\[
V(\hat{s}_{ih}) = \left(\frac{\partial \hat{s}_{ih}}{\partial a_{ih}}\right)^2 V(\hat{\alpha}_{ih}) + \left(\frac{\partial \hat{s}_{ih}}{\partial a_{ir}}\right)^2 V(\hat{\alpha}_{ir}) + 2\left(\frac{\partial \hat{s}_{ih}}{\partial a_{ih}}\right)\left(\frac{\partial \hat{s}_{ih}}{\partial a_{ir}}\right) \text{cov}(\hat{\alpha}_{ih}, \hat{\alpha}_{ir})
\]

\[
= \frac{1}{a_{ir}^2} V(\hat{\alpha}_{ih}) + \frac{a_{ih}^2}{a_{ir}^4} V(\hat{\alpha}_{ir}) - \frac{2a_{ih}}{a_{ir}^3} \text{cov}(\hat{\alpha}_{ih}, \hat{\alpha}_{ir}) \tag{A55}
\]

The elements \( V(\hat{\alpha}_{ih}) \) and \( V(\hat{\alpha}_{ir}) \) are given by the diagonal elements in (A53) and its counterpart for the reference household. The elements \( \text{cov}(\hat{\alpha}_{ih}, \hat{\alpha}_{ir}) \) are given by the diagonal elements of

\[
\text{cov}(\hat{\alpha}_{h}, \hat{\alpha}_{r}) = \begin{bmatrix}
\frac{\partial \alpha_h}{\partial \Pi_h} & \frac{\partial \alpha_h}{\partial \Pi_r} & \frac{\partial \alpha_r}{\partial \Pi_h} & \frac{\partial \alpha_r}{\partial \Pi_r} \\
\end{bmatrix}
\begin{bmatrix}
\hat{\Pi}_h \\
\hat{\Pi}_r \\
\hat{\eta} \\
\hat{\eta}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0 & C & a_h C \\
\end{bmatrix}
\begin{bmatrix}
\hat{\Pi}_h \\
\hat{\Pi}_r \\
\hat{\eta}
\end{bmatrix}
\begin{bmatrix}
C' \\
0 \\
a_h C'
\end{bmatrix}
\]

\[
= (\bar{x}_h - a_h)(\bar{x}_r - a_r)CD^{-1}C' \tag{A56}
\]
Table 1. Sample Characteristics

<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>Proportions of Expenditures</th>
<th>Household Type (no. of adults, no. of children)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1,0)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>Sample Size</td>
<td>1372</td>
<td>132</td>
</tr>
<tr>
<td>Age of Household Head</td>
<td>52.68</td>
<td>33.52</td>
</tr>
<tr>
<td>Weekly Household Income</td>
<td>306.73</td>
<td>274.54</td>
</tr>
<tr>
<td>(246.53)</td>
<td>(172.51)</td>
<td>(166.09)</td>
</tr>
<tr>
<td>Housing</td>
<td>0.19</td>
<td>0.20</td>
</tr>
<tr>
<td>Fuel &amp; Power</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Food</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>Alcohol &amp; Tobacco</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Clothing &amp; Footwear</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Household Furn. &amp; Equip</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>Medical &amp; Health Care</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Transport</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>Recreation &amp; Entertainment</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>Personal Care</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Other</td>
<td>0.06</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Notes:  
(1) Adults are all those aged 17 or over and children are all those aged 16 and below.  
(2) *Total sample size is 5532.
<table>
<thead>
<tr>
<th>Commodity Type</th>
<th>$\eta_i$</th>
<th>$\pi_{sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1,0)</td>
</tr>
<tr>
<td>Housing</td>
<td>0.0577</td>
<td>31.4384</td>
</tr>
<tr>
<td></td>
<td>(0.0024)</td>
<td>(1.5904)</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.1876)</td>
</tr>
<tr>
<td>Food</td>
<td>0.0347</td>
<td>32.4681</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.8173)</td>
</tr>
<tr>
<td>Alcohol &amp; Tobacco</td>
<td>0.0115</td>
<td>10.0246</td>
</tr>
<tr>
<td></td>
<td>(0.0091)</td>
<td>(0.6507)</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.8410)</td>
</tr>
<tr>
<td></td>
<td>(0.0029)</td>
<td>(1.6025)</td>
</tr>
<tr>
<td>Medical &amp; Health Care</td>
<td>0.0126</td>
<td>6.5825</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.7026)</td>
</tr>
<tr>
<td>Transport</td>
<td>0.0455</td>
<td>23.1214</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(2.1326)</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(1.5089)</td>
</tr>
<tr>
<td>Personal Care</td>
<td>0.0052</td>
<td>2.9486</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.2490)</td>
</tr>
<tr>
<td>Others</td>
<td>0.0385</td>
<td>4.0790</td>
</tr>
<tr>
<td></td>
<td>(0.0018)</td>
<td>(1.2281)</td>
</tr>
</tbody>
</table>

Note: The estimated standard errors are in brackets.
<table>
<thead>
<tr>
<th>Commodity Type</th>
<th>$b_1$</th>
<th>Subsistence Expenditures ($a_{ih}$)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1,0)</td>
<td>(1,1)</td>
<td>(1,2)</td>
<td>(1,3)</td>
<td>(2,0)</td>
<td>(2,1)</td>
<td>(2,2)</td>
</tr>
<tr>
<td>Housing</td>
<td>0.1721</td>
<td>44.6461</td>
<td>55.6935</td>
<td>62.2498</td>
<td>69.3904</td>
<td>54.3326</td>
<td>80.8232</td>
<td>82.4249</td>
</tr>
<tr>
<td></td>
<td>(0.0041)</td>
<td>(1.6292)</td>
<td>(3.9026)</td>
<td>(4.4800)</td>
<td>(9.9430)</td>
<td>(1.8154)</td>
<td>(4.1615)</td>
<td>(3.4164)</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.1717)</td>
<td>(0.5242)</td>
<td>(0.6003)</td>
<td>(1.2544)</td>
<td>(0.1994)</td>
<td>(0.3794)</td>
<td>(0.3524)</td>
</tr>
<tr>
<td>Food</td>
<td>0.1035</td>
<td>40.4086</td>
<td>55.4349</td>
<td>72.0089</td>
<td>80.7954</td>
<td>76.2837</td>
<td>94.2910</td>
<td>108.5187</td>
</tr>
<tr>
<td></td>
<td>(0.0023)</td>
<td>(0.8475)</td>
<td>(2.8826)</td>
<td>(3.3039)</td>
<td>(7.6824)</td>
<td>(1.0129)</td>
<td>(2.1782)</td>
<td>(2.0471)</td>
</tr>
<tr>
<td>Alcohol &amp; Tobacco</td>
<td>0.0342</td>
<td>12.6534</td>
<td>10.2387</td>
<td>8.5610</td>
<td>7.6153</td>
<td>22.1017</td>
<td>20.9678</td>
<td>19.0782</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.6188)</td>
<td>(1.0512)</td>
<td>(1.2132)</td>
<td>(1.4246)</td>
<td>(0.7419)</td>
<td>(1.2963)</td>
<td>(1.2491)</td>
</tr>
<tr>
<td>Clothing &amp; Footwear</td>
<td>0.0701</td>
<td>10.6531</td>
<td>18.1682</td>
<td>18.4018</td>
<td>28.1081</td>
<td>20.0279</td>
<td>25.5854</td>
<td>27.9937</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>(0.8219)</td>
<td>(2.7734)</td>
<td>(2.7713)</td>
<td>(5.7923)</td>
<td>(1.0271)</td>
<td>(1.9432)</td>
<td>(1.7799)</td>
</tr>
<tr>
<td>Household Furnishings &amp; Equipment</td>
<td>0.1322</td>
<td>27.0147</td>
<td>32.7264</td>
<td>38.0498</td>
<td>40.1224</td>
<td>49.4040</td>
<td>71.6005</td>
<td>56.8178</td>
</tr>
<tr>
<td></td>
<td>(0.0047)</td>
<td>(1.5508)</td>
<td>(3.5949)</td>
<td>(5.0889)</td>
<td>(9.7593)</td>
<td>(2.2763)</td>
<td>(5.9201)</td>
<td>(3.9197)</td>
</tr>
<tr>
<td>Medical &amp; Health Care</td>
<td>0.0376</td>
<td>9.4661</td>
<td>8.2428</td>
<td>11.8985</td>
<td>8.9702</td>
<td>17.6169</td>
<td>22.2135</td>
<td>22.5436</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.6868)</td>
<td>(1.3382)</td>
<td>(1.9893)</td>
<td>(2.4079)</td>
<td>(0.5132)</td>
<td>(1.3274)</td>
<td>(0.7944)</td>
</tr>
<tr>
<td>Transport</td>
<td>0.1358</td>
<td>33.5432</td>
<td>36.5327</td>
<td>39.7687</td>
<td>49.8903</td>
<td>64.0183</td>
<td>65.0505</td>
<td>76.2271</td>
</tr>
<tr>
<td></td>
<td>(0.0051)</td>
<td>(2.1086)</td>
<td>(4.4341)</td>
<td>(5.5586)</td>
<td>(14.7220)</td>
<td>(2.4982)</td>
<td>(4.2747)</td>
<td>(3.9894)</td>
</tr>
<tr>
<td>Recreation &amp; Entertainment</td>
<td>0.1745</td>
<td>25.9738</td>
<td>27.8446</td>
<td>24.7377</td>
<td>39.8047</td>
<td>48.3563</td>
<td>49.6488</td>
<td>61.9587</td>
</tr>
<tr>
<td></td>
<td>(0.0047)</td>
<td>(1.4963)</td>
<td>(4.3098)</td>
<td>(3.1839)</td>
<td>(9.7877)</td>
<td>(2.4763)</td>
<td>(4.3307)</td>
<td>(4.6053)</td>
</tr>
<tr>
<td>Personal Care</td>
<td>0.0156</td>
<td>4.1486</td>
<td>5.9926</td>
<td>7.4373</td>
<td>5.6231</td>
<td>7.6723</td>
<td>9.1436</td>
<td>9.8783</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.2291)</td>
<td>(0.6766)</td>
<td>(1.0986)</td>
<td>(1.0526)</td>
<td>(0.3116)</td>
<td>(0.6180)</td>
<td>(0.4776)</td>
</tr>
<tr>
<td>Others</td>
<td>0.1150</td>
<td>12.9057</td>
<td>23.3362</td>
<td>20.2497</td>
<td>18.4044</td>
<td>22.8208</td>
<td>31.8058</td>
<td>40.8624</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(1.2465)</td>
<td>(3.4371)</td>
<td>(2.2069)</td>
<td>(3.1771)</td>
<td>(1.4203)</td>
<td>(2.5751)</td>
<td>(3.5113)</td>
</tr>
<tr>
<td>Total</td>
<td>1.0001</td>
<td>228.9851</td>
<td>284.5588</td>
<td>315.3233</td>
<td>361.2065</td>
<td>393.8838</td>
<td>484.7969</td>
<td>521.4062</td>
</tr>
</tbody>
</table>

Note: The estimated standard errors are in brackets.
Table 4. Estimates of Commodity-Specific Scales

<table>
<thead>
<tr>
<th>Commodity Type</th>
<th>Commodity Specific Scales ($s_{lh}$)</th>
<th>Household Type (no. of adults, no. of children)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1,0)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>Housing</td>
<td>0.82</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Fuel &amp; Power</td>
<td>0.67</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Food</td>
<td>0.53</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Alcohol &amp; Tobacco</td>
<td>0.57</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Clothing &amp; Footwear</td>
<td>0.53</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>Household Furnishings &amp; Equipment</td>
<td>0.55</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Medical &amp; Health Care</td>
<td>0.54</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Transport</td>
<td>0.52</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Recreation &amp; Entertainment</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Personal Care</td>
<td>0.54</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Others</td>
<td>0.57</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.16)</td>
</tr>
</tbody>
</table>

Note: The estimated standard errors are in brackets.
<table>
<thead>
<tr>
<th>Commodity Type</th>
<th>Year*</th>
<th>Commodity- Specific Scales ($s_{bh}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Household Type (no. of adults, no. of children)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1,0)</td>
</tr>
<tr>
<td>Housing</td>
<td>1984</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.82</td>
</tr>
<tr>
<td>Fuel &amp; Power</td>
<td>1984</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.67</td>
</tr>
<tr>
<td>Food</td>
<td>1984</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.53</td>
</tr>
<tr>
<td>Alcohol &amp; Tobacco</td>
<td>1984</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.57</td>
</tr>
<tr>
<td>Clothing &amp; Footwear</td>
<td>1984</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.53</td>
</tr>
<tr>
<td>Household Furnishings &amp; Equipment</td>
<td>1984</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.55</td>
</tr>
<tr>
<td>Medical &amp; Health Care</td>
<td>1984</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.54</td>
</tr>
<tr>
<td>Transport</td>
<td>1984</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.52</td>
</tr>
<tr>
<td>Recreation &amp; Entertainment</td>
<td>1984</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.54</td>
</tr>
<tr>
<td>Personal Care</td>
<td>1984</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.54</td>
</tr>
<tr>
<td>Others</td>
<td>1984</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.57</td>
</tr>
</tbody>
</table>

*1984 Scales derived from the ELES parameters estimates presented in Binh and Whiteford (1990) which used 1984 HES data.
1988a Scales derived using the Kakwani procedure applied to data from 1988 Household Expenditure Survey.
1988b Scales derived using the proposed MLE procedure applied to data from 1988 Household Expenditure Survey.
## Table 6. Estimates of General Scales

<table>
<thead>
<tr>
<th>Reference Income**</th>
<th>Year*</th>
<th>General Scales ( s_k )</th>
<th>Household Type (no. of adults, no. of children)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1,0)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>Low Income ($325 p.w.)</td>
<td>1984</td>
<td>0.53</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.59</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.58</td>
<td>0.72</td>
</tr>
<tr>
<td>Medium Income ($450 p.w.)</td>
<td>1984</td>
<td>0.52</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.58</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.58</td>
<td>0.72</td>
</tr>
<tr>
<td>High Income ($700 p.w.)</td>
<td>1984</td>
<td>0.52</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>1988a</td>
<td>0.58</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>1988b</td>
<td>0.58</td>
<td>0.72</td>
</tr>
</tbody>
</table>


1988a Scales derived using the Kikwadi procedure applied to data from 1988 Household Expenditure Survey.

1988b Scales derived using the proposed MLE procedure applied to data from 1988 Household Expenditure Survey.

** The scales have been evaluated using the listed incomes as reference levels.
References:


A Note on A Bayesian Estimator in an Autocorrelated Error Model. William Griffiths and Dan Dao, No. 3 - April 1979.


Bayesian Econometrics and How to Get Rid of Those Wrong Signs. William E. Griffiths, No. 31 - November 1987.


