BAYESIAN ESTIMATION OF SOME AUSTRALIAN ELES-BASED EQUIVALENCE SCALES

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Bayesian Estimation of Some Australian ELES-Based Equivalence Scales

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Data from the 1988-89 Household Expenditure Survey of Australia are used to obtain estimates of commodity-specific equivalence scales for 11 commodity groups and 8 types of household compositions. General equivalence scales are also estimated for the different household types. The scales are derived from an extended linear expenditure system that is characterised by intercepts that differ over commodities and household types and slope coefficients that differ only over commodities. The scales are non-linear functions of these parameters. Gibbs sampling is used to generate observations from the scales’ posterior densities from which Bayesian estimates are obtained.

JEL Classification: C11, C31, D12
1 Introduction

Equivalence scales are typically defined as ratios that show the income or expenditure requirement of one unit relative to a base such that both units are on the same level of welfare. Household equivalence scales thus act as deflators that offer more sophistication than per-capita head counting when household budgets are adjusted to permit welfare comparisons across households with different demographic profiles.

It has become standard practice to employ equivalence scales in studies that measure and compare living standards of say, households, to account for economies of scales and/or differences in the needs of each individual member. This is certainly the case in many recent country studies on income inequality and poverty. See Anand (1983) for Malaysia, Tsakloglou (1993) for Greece, Slesnick (1994) for the U.S. and Meagher and Dixon (1986), Kakwani (1986) and Podder (1993) for Australia. Equivalence scales are also employed by many governments to ascertain the appropriateness of public policies such as the progressive taxation system or the structure of social welfare benefit schemes. The use of equivalence scales in income maintenance programs, for instance, generally results in large benefits accruing to families with more and older children compared to families with fewer and younger children.

In this paper, we focus on the estimation of household equivalence scales derived from the extended linear expenditure system (ELES) of Lluch (1973) that was later modified by Kakwani (1977) for equivalence scale estimation. In particular, we employ the Bayesian approach to estimation of the scales and contrast our results from those derived using a more traditional maximum likelihood (ML) estimation procedure. The advantage of the Bayesian approach lies in the fact that one is able to derive finite sample posterior distributions of commodity-specific and general scales whereas ML estimates have to rely on the asymptotic properties of the non-linear functions of the original parameters. In the case of general scales, standard errors that are typically difficult to derive from ML procedures are easily estimated using the Bayesian approach.

Other recent studies that have estimated equivalence scales for Australia are those of Binh and Whiteford (1990) and Bradbury (1994). Binh and Whiteford (1990) base
their estimates on the 1984 Australian Household Expenditure Survey (HES) and use an ELES and the same commodity groupings as we employ. Our work can be viewed as an update of their estimates using the 1988-89 HES, with more general stochastic assumptions for the model and an alternative estimation methodology. Bradbury (1994) develop a generalised translation model, a generalisation of Pollack and Wales (1981) that distinguishes between child and adult goods. Using the 1988-89 HES, he compares estimates obtained using this model with those from two other models, including the ELES.

The plan of this paper is as follows. Section 2 presents the model and the derivation of the equivalence scales based on its parameters. The Bayesian procedure is outlined in Section 3. The derivation of the conditional posterior probability density functions (pdfs) that are required for Gibbs sampling are detailed in Section 4. A brief discussion of the data and the results are presented in Section 5 and conclusions are drawn in Section 6.

2 The Model

Utility-based equivalence scale models were first promoted by Barten (1964) who introduced commodity-specific equivalence scales to deflate commodity quantities for different household types and at the same time made room for possible substitution between goods, a feature not found in the earlier models. The utility function for this model is generally represented by

\[ u^h = U^h \left( \frac{q_1}{s_1}, \frac{q_2}{s_2}, ..., \frac{q_n}{s_n} \right) \]

where \( q_i / s_i \) refers to the household per capita equivalent consumption of commodity \( i \) and the \( s_i \)'s are the commodity-specific scales which are usually functions of the household's demographic composition. Later contributions are reviewed and expanded by Pollack and Wales (1981) and Bradbury (1994).

In this context, we use the extended linear expenditure system (ELES) of Lluch (1973) which we now outline beginning with the Klein-Rubin utility function.
\[ u_h = \sum_{i=1}^{n} \ln \left( \frac{q_{ih}}{s_{ih}} - c_i \right) \]  \hspace{1cm} (1)

where

- \( q_{ih} = \) is the quantity of the \( i^{th} \) commodity consumed by the \( h \)-type household;
- \( s_{ih} = \) is the \( i^{th} \) commodity-specific equivalence scale for the \( h \)-type household;
- \( b_i = \) is the marginal budget share for the \( i^{th} \) commodity satisfying the constraints \( 0 < b_i < 1; \sum_{i=1}^{n} b_i = 1; \)
- \( c_i = \) is a parameter which, if interpreted as the subsistence quantity of the \( i^{th} \) commodity, satisfies the constraint \( c_i > 0 \).

Household types (\( h = 1, 2, \ldots, H \)) are defined in terms of the number of adults and children in the household.

Maximising (1) subject to the budget constraint

\[ \sum_{i=1}^{n} p_i q_{ih} = u_h \]  \hspace{1cm} (2)

where

- \( p_i = \) is the price of the \( i^{th} \) commodity and
- \( u_h = \) is the total expenditure for the \( h \)-type household

leads to the expenditure system

\[ v_{ih} = a_{ih} + b_i(u_h - a_h) \]  \hspace{1cm} (3)

where

- \( v_{ih} = p_i q_{ih} \) is expenditure on the \( i^{th} \) commodity by the \( h \)-type household;
- \( a_{ih} = p_i c_i s_{ih} \) is subsistence expenditure for the \( i^{th} \) commodity and \( h \)-type household, and
- \( a_h = \sum_{i=1}^{n} a_{ih} \) is total subsistence expenditure for a \( h \)-type household.
A feature of this system is linearity, which may not be an appropriate specification for many cases. Also, the utility function from which it is derived is directly additive; as shown in Deaton (1974, 1975), this is a restrictive assumption, particularly in studies that use a detailed disaggregation of commodities. Despite these facts, we have chosen to work with the ELES, mainly because it has been the major vehicle for deriving equivalence scales from cross-section data (which is our data source) particularly in Australia (Kakwani (1977), Binh and Whiteford (1990) and Bradbury (1994)). In the process, we mitigate the assumption of additive utility by restricting estimation to 11 broad commodity groups. In any case, the relative simplicity of the ELES is convenient for our objective of introducing a new methodology, and this study is a useful addition to the available empirical evidence on equivalence scales.

Equivalence scales are defined relative to a reference household whose scales $s_{ir}$ are set equal to unity. Thus, in terms of the parameters in equation (3), the commodity specific scales are defined by

$$s_{ih} = \frac{a_{ih}}{a_{ir}}$$  \hspace{1cm} (4)

Also of interest is the general household equivalence scale which, for a household of type $h$, is defined as the ratio of the income of that household to that of the income of the reference household, such that the indirect utility functions of the two households are the same. It can be shown that these general scales are given by

$$s_h = \frac{a_h}{x_r} + \prod_{i=1}^{n} (s_{ih})^h \left[ 1 - \frac{a_r}{x_r} \right]$$  \hspace{1cm} (5)

where $x_r$ is the income of the reference household (Binh and Whiteford, 1990). Our main focus is on estimating the $s_{ih}$ and $s_h$, although there is a secondary interest in the $a_{ih}$ and $b_i$.

One of the difficulties with estimating equation (3) is that not all the $a_{ih}$ parameters are identified. Following Lluch (1973), Kakwani (1977) overcame this problem in the context of equivalence scale estimation by using an extended linear expenditure system which includes a micro-consumption function given by

$$v_h = (1 - b)a_h + bx_h$$  \hspace{1cm} (6)
where

\[ x_h \] is net income for an h-type household and

\[ b \] is a common marginal propensity to consume.

Substituting (6) into (3) leads to the system of equations

\[ v_{ih} = \theta_{ih} + \eta_i x_h \]  

where

\[ \theta_{ih} = a_{ih} - b_i b a_h \]  

\[ \eta_i = b_i b \]  

Note that (7) defines a system of \( nH \) equations, one for each combination of commodity and household type. Each equation has a different intercept, but is characterised by a slope coefficient which does not vary over household type. Our problem now is to estimate \( \theta_{ih} \) and \( \eta_i \). These estimates lead us to estimates of the \( a_{ih} \) and \( b_i \) which in turn lead us to estimate the \( s_{ih} \) and \( s_h \). The earlier parameters can be found from \( (\theta_{ih}, \eta_i) \) through the following relationships

\[ b = \sum_{i=1}^{n} \eta_i \]  

\[ a_h = \frac{\sum_{i=1}^{n} \theta_{ih}}{1 - b} \]  

\[ b_i = \frac{\eta_i}{b} \]  

\[ a_{ih} = \theta_{ih} + b_i b a_h \]  

Suppose we have \( M_h \) observations on expenditures \( v_{ih} \) \((i = 1, 2, \ldots, n)\) and income \( x_h \) for the h-type household. Introducing conventional seemingly unrelated regression (SUR) statistical assumptions (Zellner, 1962), the model for the h-type household can be written as
or

\[
\begin{bmatrix}
  v_{1h} \\
  v_{2h} \\
  \vdots \\
  v_{nh}
\end{bmatrix} =
\begin{bmatrix}
  z_h \\
  z_h \\
  \vdots \\
  z_h
\end{bmatrix}
\begin{bmatrix}
  \theta_{1h} \\
  \theta_{2h} \\
  \vdots \\
  \theta_{nh}
\end{bmatrix} +
\begin{bmatrix}
  x_h \\
  x_h \\
  \vdots \\
  x_h
\end{bmatrix}
\begin{bmatrix}
  \eta_1 \\
  \eta_2 \\
  \vdots \\
  \eta_n
\end{bmatrix} +
\begin{bmatrix}
  \epsilon_{1h} \\
  \epsilon_{2h} \\
  \vdots \\
  \epsilon_{nh}
\end{bmatrix}
\]

or

\[V_h = Z_h \Theta_h + X_h \eta + E_h \quad (14)\]

where

\[v_{ih} \text{ is now written as an } (M_h \times 1) \text{ vector of observations on expenditure for the } i^{th} \text{ commodity and the } h^{th} \text{-type household; }\]

\[z_h \text{ is an } (M_h \times 1) \text{ vector of ones; }\]

\[x_h \text{ becomes an } (M_h \times 1) \text{ vector of observations on income for households of type } h;\]

\[e_{ih} \text{ is an } (M_h \times 1) \text{ vector of errors. }\]

\[V_h \text{ is of dimension } (nM_h \times 1);\]

\[Z_h = I_n \otimes z_h \text{ is an } (nM_h \times n) \text{ matrix of dummy variables; }\]

\[X_h = I_n \otimes x_h \text{ is an } (nM_h \times n) \text{ matrix of incomes; }\]

\[\Theta_h, \eta \text{ are } (n \times 1) \text{ vectors of unknown parameters; }\]

\[E_h \text{ is an } (nM_h \times 1) \text{ vector of errors which we assume are distributed as }\]

\[E_h \sim N[0, \Omega_h \otimes I_{M_h}] \quad (15)\]

where \(\Omega_h\) is a \((n \times n)\) error covariance matrix.

### 3 Bayesian Estimation

Application of the Bayesian approach to estimation begins with specification of a joint prior distribution for the unknown parameters. In what follows, we specify what are
traditional non-informative priors for the SUR model. Non-informative priors ignore the existence of what could be considerable prior information, but they do have the advantage of objective data-based reporting of posterior information. Our case of non-informative priors does not imply a lack of substantial prior information. Indeed, obvious prior inequalities on equivalence scales from different household compositions, as well as inequalities implied by the subsistence parameters, can be set up along the lines in Griffiths and Chotikapanich (1996).

The following generic notation will be useful:

\[
\Omega = \{\Omega_h \mid \text{for all } h\}
\]
\[
\Theta = \{\Theta_h \mid \text{for all } h\}
\]
\[
V = \{V_h \mid \text{for all } h\}
\]

The traditional non-informative or diffuse prior (Judge, et.al. 1985, p.478) for \(\Omega_h\) is

\[
g(\Omega_h) \propto |\Omega_h|^{-\frac{n+k}{2}}
\]

(16)

Treating all the \(\Omega_h\) is as a prior independent, the combined prior pdf for all error covariance matrices is

\[
g(\Omega) \propto \prod_{h=1}^{H} \Omega_h^{-\frac{n+k}{2}}
\]

(17)

For the location parameters \(\Theta_h\) and \(\eta\) which can take on any value on the real line, it is customary to assign priors that are proportional to a constant. Therefore, we have

\[
g(\eta) \propto \text{constant}
\]

(18)

\[
g(\Theta_h) \propto \text{constant}
\]

(19)

A priori independence implies

\[
g(\Theta) = \prod_{h=1}^{H} g(\Theta_h) \propto \text{constant}
\]

(20)
Finally, assuming prior independence of $\Theta$, $\eta$ and $\Omega$, the joint prior pdf will simply be the product of the priors specified in (17) (18) and (20). Thus, we have

$$g(\Theta, \eta, \Omega) = g(\Theta)g(\eta)g(\Omega)$$

(21)

$$\propto \prod_{h=1}^{H} |\Omega_h|^{-\frac{M_h}{2}}$$

(22)

as the joint prior density function.

For specification of the likelihood function, we first note that the pdf of the expenditure vector for all households of type $h$ is given by

$$f(v_h|\Theta_h, \eta, \Omega_h) \propto |\Omega_h|^{-\frac{M_h}{2}} \exp \left\{ -\frac{1}{2} \sum_{h=1}^{H} (V_h - Z_h \Theta_h - X_h \eta)' (\Omega_h^{-1} \otimes I_{M_h}) (V_h - Z_h \Theta_h - X_h \eta) \right\}$$

(23)

Then, the complete likelihood function is given by

$$f(\Theta, \eta, \Omega|V) = \prod_{h=1}^{H} f(V_h|\Theta_h, \eta, \Omega_h)$$

$$\propto \prod_{h=1}^{H} |\Omega_h|^{-\frac{M_h}{2}} \exp \left\{ -\frac{1}{2} \sum_{h=1}^{H} (V_h - Z_h \Theta_h - X_h \eta)' (\Omega_h^{-1} \otimes I_{M_h}) (V_h - Z_h \Theta_h - X_h \eta) \right\}$$

(24)

A convenient and efficient iterative procedure for obtaining maximum likelihood estimates that maximize this function has been suggested by Griffiths and Valenzuela (1995). Here we concentrate on Bayesian estimation.

Combining (22) and (23) via Bayes' Theorem yields the joint posterior pdf
This pdf is the source of all our post-sample information on the parameters $\Theta$, $\eta$ and $\Omega$. What is of particular interest is the information on the equivalence scales $s_{ih}$ and $s_h$ that we can derive from the information on $\Theta$ and $\eta$. To tackle this problem analytically, we would need to derive the marginal posterior pdfs $f(s_{ih} \mid V)$, $(i = 1, 2, ..., n; h = 1, 2, ..., H)$ and $f(s_h \mid V)$, $(h = 1, 2, ..., H)$, through variable transformation and by integrating unwanted parameters out of the joint posterior pdf. This task is a daunting one. Properties of the inverted Wishart distribution can be used to integrate the $\omega_{ih}$ from (25), but the resulting marginal posterior pdf for $(\Theta, \eta)$ is not of a recognizable form, and to then employ a transformation to $(s_{ih}, s_h)$ would not be a rewarding experience. A numerical approach is much more promising. If we can draw a sample of values of the parameters from the posterior pdf in (25), then corresponding values of the $s_{ih}$ and $s_h$ can be computed. This sample of values is equivalent to sampling from the marginal posterior pdfs $f(s_{ih} \mid V)$ and $f(s_h \mid V)$ for each commodity parameter and household type. Thus, the sampled values can be used to estimate posterior means and standard deviations for the various equivalence scales, as well as provide information for approximate plots of the posterior pdfs.

Drawing values from the joint posterior pdf in (25) is achieved conveniently using Gibbs sampling. Once conditional posterior pdfs for each of the parameters have been derived, one can sample iteratively from these pdfs. After a suitable 'burn-in' period, the draws represent draws from the joint posterior pdf. The application of Gibbs sampling to SUR systems of different kinds has been considered by Percy (1992), Chib and Greenberg (1995) and Griffiths, Thomson and Coelli (1996). Our model does not fit neatly into any of these earlier studies, and so, we proceed by deriving the required conditional posterior pdfs.
4 Conditional Posterior Pdfs

Note that the joint posterior pdf can be conveniently factored as

\[
f(\Theta, \eta, \Omega | V) \propto |\Omega|^{-\frac{n+1}{2}} \exp \left\{ -\frac{1}{2}(V_h - Z_h \Theta_h - X_h \eta)'(\Omega_h^{-1} \otimes I_{M_h})(V_h - Z_h \Theta_h - X_h \eta) \right\} \]

\[
\prod_{k \neq h} |\Omega|^{-\frac{n+1}{2}} \exp \left\{ -\frac{1}{2}\sum_{k \neq h} (V_k - Z_k \Theta_k - X_k \eta)'(\Omega_k^{-1} \otimes I_{M_k})(V_k - Z_k \Theta_k - X_k \eta) \right\}
\]

(26)

This factorization is a convenient one for obtaining the conditional posterior pdfs for \(\Omega_h\) and \(\Theta_h\). Let

\[
\Omega_h^* = \{ \Omega_k | \text{ for all } k \neq h \}
\]

\[
\Theta_h^* = \{ \Theta_k | \text{ for all } k \neq h \}
\]

From (26) we have (Judge, et.al. 1985, p.479)

\[
f(\Omega_h | \Theta, \eta, \Omega_h^*, V) \propto |\Omega_h|^{-\frac{n+1}{2}} \exp \left\{ -\frac{1}{2} \text{tr} \left( A_h \Omega_h^{-1} \right) \right\}
\]

(27)

where \(A_h\) is a matrix with \((i,j)\)th element equal to \(a_{ij} = (u_{ih} - z_i \theta_{ih} - x_i \eta_i)'(u_{jh} - z_j \theta_{jh} - x_j \eta_j)\). The conditional density in (27) has an inverted Wishart distribution from which random generation of observations is straightforward (Anderson, 1984, p.238).

Also, from (26) we can write the conditional posterior pdf for \(\Theta_h\) as

\[
f(\Theta_h | \Theta_h^*, \eta, \Omega, V) \propto \exp \left\{ -\frac{1}{2}(V_h - Z_h \Theta_h - X_h \eta)'(\Omega_h^{-1} \otimes I_{M_h})(V_h - Z_h \Theta_h - X_h \eta) \right\}
\]

\[
= \exp \left\{ -\frac{1}{2}(\Theta_h - \hat{\Theta}_h)'Z_h'(\Omega_h^{-1} \otimes I_{M_h})Z_h(\Theta_h - \hat{\Theta}_h) \right\}
\]

(28)

where

\[
\hat{\Theta}_h = [Z_h'(\Omega_h^{-1} \otimes I_{M_h})Z_h]^{-1} Z_h'(\Omega_h^{-1} \otimes I_{M_h})(V_h - X_h \eta)
\]

\[
= (I_n \otimes M_h^{-1} z_h')(V_h - X_h \eta)
\]

(29)
Equation (28) suggests that the conditional posterior density for $\Theta_h$ is a multivariate normal density function with mean $\hat{\Theta}_h$ and covariance matrix $(M_h)^{-1}\Omega_h$. That is,

$$f(\Theta_h | \Theta_h', \eta, \Omega, V) \sim N\left[\hat{\Theta}_h, \left[Z_h'(\Omega_h^{-1} \otimes I_{M_h})Z_h\right]^{-1}\right]$$

$$\sim N\left[\hat{\Theta}_h, \frac{1}{M_h}\Omega_h\right] \quad (30)$$

To derive the conditional posterior for $\eta$, we again consider the joint posterior density in (25). Conditioning on all parameters except $\eta$ implies that

$$f(\eta | \Theta, \Omega, V) \propto \exp\left\{-\frac{1}{2} \sum_{h=1}^{H} (V_h - Z_h \Theta_h - X_h \eta)' (\Omega_h^{-1} \otimes I_{M_h}) (V_h - Z_h \Theta_h - X_h \eta)\right\} \quad (31)$$

If we let $V_h^o = V_h - Z_h \Theta_h$, the summation term in (31) can be written as

$$\sum_{h=1}^{H} V_h^o (\Omega_h^{-1} \otimes I_{M_h}) V_h^o + \eta' \left[\sum_{h=1}^{H} X'_h (\Omega_h^{-1} \otimes I_{M_h}) X_h\right] \eta - 2\eta' \left[\sum_{h=1}^{H} X'_h (\Omega_h^{-1} \otimes I_{M_h}) V_h^o\right] \quad (32)$$

To enable further simplification, let

$$W = \sum_{h=1}^{H} X'_h (\Omega_h^{-1} \otimes I_{M_h}) X_h = \sum_{h=1}^{H} (x'_h x_h) \Omega_h^{-1} \quad (33)$$

$$Q = \sum_{h=1}^{H} X'_h (\Omega_h^{-1} \otimes I_{M_h}) V_h^o = \sum_{h=1}^{H} (\Omega_h^{-1} \otimes x'_h) V_h^o \quad (34)$$

and define

$$\hat{\eta} = \left[\sum_{h=1}^{H} X'_h (\Omega_h^{-1} \otimes I_{M_h}) X_h\right]^{-1} \sum_{h=1}^{H} X'_h (\Omega_h^{-1} \otimes I_{M_h}) V_h^o \quad (35)$$

$$= W^{-1} Q \quad (36)$$

It then follows that
\[
f(\eta \mid \Theta, \Omega, V) \propto \exp\left\{-\frac{1}{2}(\eta - \hat{\eta})'W(\eta - \hat{\eta})\right\}
\]
\[
\propto \exp\left\{-\frac{1}{2}(\eta - \hat{\eta})'\left(\sum_{h=1}^{H}(x_h'x_h)\Omega_h^{-1}\right)(\eta - \hat{\eta})\right\} \tag{37}
\]

which suggests that \(f(\eta \mid \Theta, \Omega, V)\) is a multivariate normal distribution with mean \(\hat{\eta}\) and covariance matrix \(W\). That is,

\[
f(\eta \mid \Theta, \Omega, V) \sim N\left[\hat{\eta}, \left(\sum_{h=1}^{H}(x_h'x_h)\Omega_h^{-1}\right)^{-1}\right] \tag{38}
\]

Given the conditional posterior pdfs in (27), (30) and (38), the Gibbs sampler can be used to generate observations on \(\Omega_1, \Omega_2, \ldots, \Omega_H, \Theta_1, \Theta_2, \ldots, \Theta_H, \eta\) using the following steps:

1. Given some initial values for \(\theta_{ih}\) and \(\eta_i\), compute, for each \(h\),

\[
[A_h]_{ij} = (v_{ih} - z_{ih}^T - x_h \eta_i)'(v_{jh} - z_{jh}' - x_h \eta_j)
\]

2. Draw values \(\Omega_1, \Omega_2, \ldots, \Omega_H\) from respective inverted Wishart distributions with parameter matrices \(A_1, A_2, \ldots, A_H\) and degrees of freedom \(M_1, M_2, \ldots, M_H\).

3. Compute \(\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_H\) as defined in (29) and, given the \(\Omega_h\) drawn in step 2, draw values \(\Theta_h, h = 1, 2, \ldots, H\) from \(N(\hat{\Theta}_h, M_h^{-1}\Omega_h)\) distributions.

4. Using the values for \(\Omega_h\) and \(\Theta_h\) drawn in steps 2 and 3, respectively, compute \(W\) and \(\hat{\eta}\) as defined in (33) and (35).

5. Draw a value for \(\eta\) from a \(N(\hat{\eta}, W^{-1})\) distribution.

6. Return to step 1 using the \(\theta_{ij}\) and \(\eta_i\) drawn in steps 3 and 5, respectively, and continue to proceed iteratively through all the steps, until a large sample has been generated.
5 Application, Data and Results

The Bayesian procedure described in the previous section is applied here using data from the 1988-89 Australian Household Expenditure Survey. We have the following 11 commodity groupings:

1. Housing
2. Fuel and Power
3. Food
4. Alcohol and Tobacco
5. Clothing and Footwear
6. Household Furnishings and Equipment
7. Medical and Health Care
8. Transport
9. Recreation and Entertainment
10. Personal Care
11. Others

The sample used is restricted to households of related persons with one or two adults and at most three children, resulting in \( H = 8 \) different household types. Adults are all those aged 17 or older and children refer to all those aged 16 or below. The data comprise a total of 5532 sample households which make up 77 percent of the total sample households in the HES. The household types and their distribution in the sample are presented in Table 1.

For Bayesian estimation, we generated 18000 sets of observations for each parameter element in \( \Omega_h, \eta \) and \( \Theta_h \) for the 8 household types. Observations from the first 3000 runs in the iterations provided the 'burn-in' period of the Gibbs sampler.
and hence were discarded, leaving 15000 observations in the final estimation sample. These observations were used to estimate the posterior means and standard deviations of all the commodity-specific and general scales as well as to provide points for drawing marginal posterior pdfs for some selected scales. All calculations were carried out using the econometric package SHAZAM.

Checks of convergence of the generated Gibbs sequence were conducted through diagrams. The estimated scale values for the first and last 1000 observations of the generated series were plotted against each other. In Fig.1 are the plots for the generated values of the clothing scales. The last 1000 observations do not appear to differ much from the first 1000 sample points. This trend is typical of the diagrams for the other scales (not shown here) including that of the general scales and points to a fast convergence rate for all the series generated.

Table 2 shows the posterior means and standard deviations of the scales from Bayesian estimation. As expected, for most commodities there is an increase in the per household equivalent expenditure as household size increases; these increases generally occur at a decreasing rate indicating economies of scale for additional children. There are some exceptions to these observations. The estimated scales for Alcohol and Tobacco decline as the number of children in the household increases. Also, the scales for Medical and Health Care and Others commodity groups exhibit no defined trend for 1-adult households. A more thorough investigation of expenditure patterns of households may be required for us to provide definitive explanations for such deviations but it may be possible that the presence of children in the household tends to influence expenses away from 'adult goods' under which alcohol, tobacco and some other miscellaneous goods are classified. Tables 3 contain ML estimates of the commodity specific scales obtained through the iterative procedure detailed in Griffiths and Valenzuela (1995). In comparing Tables 2 and 3, most obvious is the striking similarity of the point estimates from both methods; in most cases, they differ only from the third decimal point. The two sets of estimates appear to be highly reliable in that they possess small values of standard deviations, although the Bayesian standard errors are, in general, consistently higher than the ML standard errors. This result is expected because the ML estimates can be viewed as means and standard
deviations from a conditional (rather than unconditional) posterior pdf, whereas the Bayesian estimates are from unconditional pdfs. Across commodities, the estimated scales for Clothing and Footwear, Recreation and Entertainment and Others invariably have the largest variances while those for Food and Alcohol and Tobacco have the smallest variances. Across households, the scales for the type (1,0) consistently have the smallest variances while those for the type (1,3) always exhibit the largest variances. This result can be attributed to the relative sizes of the samples; there are more than a thousand households of type (1,0) while there are only 42 households of type (1,3). (See Table 1). Between the two sets of estimates, it is then not surprising to note that the largest differences in the scale values were observed for household type (1,3) and the smallest differences were those of household type (1,0).

The estimates for the general scales are presented in Table 4. It may be recalled that general scales depend on a chosen level of income of the reference household $z_r$. In Table 4, the ML estimates are based on $z_r = 450$ which is medium income level. On the other hand, the Bayesian estimates are based on a randomly selected $z_r$ level satisfying $z_r < a_A$. As can be seen, the Bayesian and ML estimates are again very close to each other. Bayesian scales for the 2-adult households are smaller in magnitude while those for the 1-adult households are slightly larger, compared to the ML estimates. While it is theoretically possible to derive standard errors for estimates from both procedures, doing so for the ML estimates is cumbersome because of the large of the number of parameters involved and their various interrelationships. With the Bayesian estimates, posterior standard deviations are straightforwardly estimated.

The posterior pdfs of some of the scales are presented in Figure 2-5. Normal conditional posterior pdfs implied by the ML estimates are plotted alongside the Bayesian scales. It is immediately evident that the ML-based posteriors approximate those of the Bayesian posterior pdfs. It is also immediately obvious that the food scales posterior pdf for the (1,0) household type has a relatively small variance while that of the household type (1,3) has a relatively large variance. In general, the pdfs shift to the right with the addition of children in the household. The magnitudes of the shifts however differ across the commodities. For example, with food, the effects of increasing the number of adults and children in the household has a clear distinct
effect; overlapping of the posterior pdfs is minimal. With clothing, the location of the pdfs is as expected but considerable overlapping of the pdfs means that we are more uncertain about the relative magnitudes of the scales.

Across commodities (for which the posterior pdfs have been plotted), the posterior pdfs of the food scales are observed to have less variability compared to those of the clothing or housing scales. The posteriors for food also indicate lesser gains in economies of scale compared to housing or clothing; this observation conforms with our expectations.

The posterior pdfs for the general scales in Fig.5 yield patterns that are consistent with the observations for the posterior pdfs of the three commodity groups analysed earlier. That is, the pdfs shifts to the right at a diminishing rate with the addition of children in the household. The least variance was observed for household type (1,0) while the largest variance was observed for household type (1,3). It is also noted that the standard errors for the general scales are larger than those for the commodity-specific scales.

6 Conclusion

In this paper, we have demonstrated how Bayesian techniques can be used to estimate equivalence scales. It is relatively straightforward to use numerical methods to obtain posterior pdfs and summary measures such as the posterior means and standard deviations. Unlike ML estimation, posterior pdfs let us make finite sample inferences about non-linear functions of the original parameters like the equivalence scales. Also, Bayesian estimates take account of the uncertainty associated with the error covariance matrix estimation in that they are not conditional on point estimates of the error covariances, as ML estimates are. The resulting Bayesian estimates of means and standard deviations of the equivalence scales are plausible and lend easily to economic interpretation.

It is also convenient to be able to present information diagramatically through plots of posterior pdfs. Through such diagrams, we were able to show clearly the
relationships between the different scales in terms of both the magnitudes and reliability of the derived information; such relationships may not be so obvious when analysing point estimates alone.

In this study, we have also observed that because our sample size is large, there is not a great deal of difference between the Bayesian and ML estimates of the commodity-specific scales. However, we were unable to obtain indications of reliability of the general scales from ML estimation because of the large number of parameters involved. Drawing such information proved straightforward using the Bayesian approach. A useful future extension of this study would be the imposition of prior information in the form of obvious inequalities between scales and on subsistence parameters.
Table 1. Distribution of Sample Households

<table>
<thead>
<tr>
<th>Household Type (no. of adults, no. of children)</th>
<th>Number of Households</th>
<th>Percent of Sample</th>
</tr>
</thead>
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<tr>
<td>(2,0)</td>
<td>2074</td>
<td>38%</td>
</tr>
<tr>
<td>(2,1)</td>
<td>532</td>
<td>10%</td>
</tr>
<tr>
<td>(2,2)</td>
<td>889</td>
<td>16%</td>
</tr>
<tr>
<td>(2,3)</td>
<td>388</td>
<td>7%</td>
</tr>
<tr>
<td>(1,0)</td>
<td>1372</td>
<td>25%</td>
</tr>
<tr>
<td>(1,1)</td>
<td>132</td>
<td>2%</td>
</tr>
<tr>
<td>(1,2)</td>
<td>103</td>
<td>2%</td>
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<tr>
<td>(1,3)</td>
<td>42</td>
<td>8%</td>
</tr>
<tr>
<td>Commodity Type</td>
<td>Commodity Specific Scales</td>
<td>Household Type (no. of adults, no. of children)</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>---------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>(1,0)</td>
<td>(1,1)</td>
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<td>Housing</td>
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<td>(0.04040)</td>
<td>(0.08350)</td>
</tr>
<tr>
<td>Fuel &amp; Power</td>
<td>0.67342</td>
<td>0.92021</td>
</tr>
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<td>(0.01893)</td>
<td>(0.05229)</td>
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<td>(0.04096)</td>
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<td>(0.15584)</td>
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<td>0.66360</td>
</tr>
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<td>(0.04031)</td>
<td>(0.08347)</td>
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<td>Medical &amp; Health Care</td>
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<td>0.46764</td>
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<td>(0.06528)</td>
<td>(0.17297)</td>
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</tbody>
</table>

|                                | (1,2)                     | (1,3)                                        |
|                                | 1.14785                   | 1.27995                                      |
|                                | (0.09622)                 | (0.22548)                                    |
|                                | 1.06296                   | 1.11025                                      |
|                                | (0.06003)                 | (0.13720)                                    |
|                                | 0.94421                   | 1.05883                                      |
|                                | (0.04761)                 | (0.12146)                                    |
|                                | 0.38910                   | 0.34489                                      |
|                                | (0.05993)                 | (0.07912)                                    |
|                                | 0.92086                   | 1.41036                                      |
|                                | (0.15551)                 | (0.35590)                                    |
|                                | 0.77333                   | 0.81383                                      |
|                                | (0.11627)                 | (0.24227)                                    |
|                                | 0.67554                   | 0.50716                                      |
|                                | (0.12328)                 | (0.16573)                                    |
|                                | 0.62282                   | 0.78055                                      |
|                                | (0.09542)                 | (0.28149)                                    |
|                                | 0.51331                   | 0.82662                                      |
|                                | (0.07593)                 | (0.24889)                                    |
|                                | 0.79237                   | 0.73424                                      |
|                                | (0.15972)                 | (0.16776)                                    |
|                                | 0.88952                   | 0.80893                                      |
|                                | (0.11805)                 | (0.17583)                                    |
|                                | 1.00000                   | 1.00000                                      |
|                                | 1.00000                   | 1.00000                                      |
|                                | 1.00000                   | 1.00000                                      |
|                                | 1.00000                   | 1.00000                                      |
|                                | 1.00000                   | 1.00000                                      |
|                                | 1.00000                   | 1.00000                                      |

|                                | Household Type (no. of adults, no. of children) |
|                                | (1,0)                     | (1,1)                                        |
|                                | 1.48807                   | 1.51560                                      |
|                                | (0.09238)                 | (0.07938)                                    |
|                                | 1.21552                   | 1.34328                                      |
|                                | (0.04019)                 | (0.03851)                                    |
|                                | 1.23641                   | 1.42314                                      |
|                                | (0.03288)                 | (0.03215)                                    |
|                                | 0.94899                   | 0.86359                                      |
|                                | (0.06648)                 | (0.06332)                                    |
|                                | 1.27952                   | 1.40232                                      |
|                                | (0.11570)                 | (0.11224)                                    |
|                                | 1.45087                   | 1.15187                                      |
|                                | (0.13807)                 | (0.09439)                                    |
|                                | 1.28029                   | 1.32548                                      |
|                                | (0.05732)                 | (0.11088)                                    |
|                                | 1.26139                   | 1.31282                                      |
|                                | (0.08378)                 | (0.07144)                                    |
|                                | 1.01755                   | 1.19239                                      |
|                                | (0.07744)                 | (0.07681)                                    |
|                                | 1.02866                   | 1.36633                                      |
|                                | (0.10475)                 | (0.12984)                                    |
|                                | 1.28465                   | 1.36945                                      |
|                                | (0.11507)                 | (0.14589)                                    |
|                                | 1.19372                   | 1.29070                                      |
|                                | (0.09478)                 | (0.08032)                                    |
|                                | 1.79408                   | 2.08332                                      |
|                                | (0.19043)                 | (0.27199)                                    |

Note: The estimated standard errors are in brackets.
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<th>(1.1)</th>
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Note: The estimated standard errors are in brackets.
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<th>Bayesian Estimates</th>
<th>ML Estimates</th>
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<tr>
<td>(2,0)</td>
<td>1.00000</td>
<td>1.00000</td>
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<td>(2,1)</td>
<td>1.2380 (0.05705)</td>
<td>1.23164</td>
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<td>(2,2)</td>
<td>1.33410 (0.05458)</td>
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<td>(2,3)</td>
<td>1.47000 (0.07253)</td>
<td>1.46043</td>
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<td>(1,0)</td>
<td>0.58189 (0.02198)</td>
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<td>(1,1)</td>
<td>0.72262 (0.05323)</td>
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<td>0.78616 (0.05633)</td>
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<td>(1,3)</td>
<td>0.89943 (0.15416)</td>
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Note: The estimated standard errors are in brackets.
Fig. 1. Plots of first and last 1000 sample points from the generated series for clothing scales.
Fig. 1. (cont.) Plots of first and last 1000 sample points from the generated series for clothing scales.
Fig. 2. Posterior Distributions of Food Scales for each Household Type (no. of adults, no. of children)

Fig. 3. Posterior Distributions of Clothing Scales for each Household Type (no. of adults, no. of children)
Fig. 4. Posterior Distributions of Housing Scales for each Household Type (no. of adults, no. of children)

ML posteriors
Bayesian posteriors

Fig. 5. Posterior Distributions of General Scales for each Household Type (no. of adults, no. of children)

ML posteriors
Bayesian posteriors
References:


WORKING PAPERS IN ECONOMETRICS
AND APPLIED STATISTICS


A Note on A Bayesian Estimator in an Autocorrelated Error Model. William Griffiths and Dan Dao, No. 3 - April 1979.


Bayesian Econometrics and How to Get Rid of Those Wrong Signs. William E. Griffiths, No. 31 - November 1987.


