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No. 53 - October 1991

ISSN 0 157-0188
ISBN 0 85834 939 6
ESTIMATION OF AUSTRALIAN WOOL AND LAMB PRODUCTION TECHNOLOGIES: AN ERROR COMPONENTS APPROACH

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Abstract

Although the theory of producer behaviour under uncertainty is well developed and it is a widely held belief that the agricultural sector is subject to significant production and price uncertainty, it is surprising that there are relatively few empirical studies of agricultural producer behaviour which take formal account of uncertainty. The present paper attempts to partially fill this gap in the literature by formulating and estimating a model of producer behaviour under uncertainty concerning both output price and the level of production obtained from given inputs. The empirical model of three lamb and wool producing sectors involves the estimation of a system of input cost share and cost equations whose disturbances have an error components structure. Estimation by the method of maximum likelihood is achieved by exploiting the structure of the covariance matrix for the disturbances. The empirical results are evaluated in terms of input demand elasticities and estimates of the stochastic components of production are presented.
1. Introduction

Several authors, including Baron (1970) and Newbery and Stiglitz (1981), have developed theoretical models of producer behaviour which accommodate two types of production uncertainty commonly encountered in agriculture. First, the level of output associated with a particular level of inputs is uncertain (the production technology is stochastic). Second, firms make their production decisions before output prices are known. In this paper we develop an empirical model of agricultural production response which accommodates these two types of uncertainty.

A large number of econometric studies of agricultural production response have concentrated on the estimation of production functions (e.g. Manderson (1966), Anderson and Griffiths (1981, 1982), Duncan (1972), Griffiths and Anderson (1982), Wan et al (1989)) or production frontiers (e.g. Powell and Gruen (1967), Battese and Corra (1977), Kelly (1977)). While some authors have used functional forms which reflect a stochastic production technology (e.g. Wan et al (1989)), these models do not reflect uncertainties associated with output price. This is not surprising, since the maximum level of output associated with a particular level of inputs is theoretically independent of output price.

Several other econometric studies of agricultural production response have concentrated on the estimation of output supply functions. These include, for example, the studies of Malecky (1975), Fisher (1975), Griffiths and Anderson (1978) and Fisher and Munro (1983). Most of these studies make some allowance for output price uncertainty, usually by specifying output as a function of some distributed lag of past prices. Importantly, however, most of these studies implicitly assume that the production technology, as distinct from the output supply function, is deterministic.

More recently, econometricians have found it more convenient to estimate agricultural production technologies using the dual approach. The dual approach is considered more convenient because data on profits, costs and prices are generally more accurate and readily available than data on input and output quantities. Cost functions for agricultural industries have been estimated by, for example, McKay et al (1980), Ray (1982) and Watts and Quiggin (1984). Profit functions have been estimated by, for example, Lawrence and Hone (1981), McKay et al (1982, 1983), Fisher and Wall (1990) and Fulghiniti and Perrin (1990). These types of studies generally do not make any allowance for a stochastic production technology, nor do they typically make allowance for the possibility that output prices are uncertain at the time input decisions are made.

In this paper we attempt to redress these oversights by formulating and estimating the parameters of a model of producer behaviour under uncertainty concerning both the output price and the level of output obtained from given inputs. Producers are assumed to choose input quantities at known market prices to maximise the expected utility of profits. Assuming that the stochastic production technology is
multiplicative in its deterministic and stochastic components and that the technology exhibits constant returns to scale, we are able to demonstrate that inputs are chosen to minimise the cost of producing "planned output" and that the resulting input cost share and cost functions can be expressed in terms of actual outputs. The stochastic component of the technology becomes incorporated in the disturbance term for the cost equation.

Our model of producer behaviour under uncertainty is applied to data obtained from the Australian Sheep Industry Surveys conducted by the Australian Bureau of Agricultural and Resource Economics. Three different sectors of the industry (merino woolgrowing, non-merino woolgrowing and prime lamb producing) are distinguished to allow for differences in technology. The resulting empirical model becomes a multivariate error components model due to the time specific stochastic component of the technology. The model is estimated by the method of maximum likelihood taking into account the grouped nature of the data. The results indicate that the stochastic component of the production technology is significant and accounts for fluctuations in production from about 85% to 117% of planned output (using 95% confidence interval estimates). The various elasticities are broadly consistent with estimates from other studies.

The plan of the paper is as follows. Section 2 outlines the underlying economic model, the associated econometric model, and the method used to estimate the models parameters. The data consists of grouped panel observations and is briefly described in Section 3. The estimation results are described in Section 4. The results include estimates of the parameters of the model, estimates of the rate of technological change, elasticity estimates and estimates of the values of random variables which give rise to production uncertainty.

2. The Model

2.1 Cost Minimisation Under Uncertainty

In this subsection the economic model underlying our empirical specification is outlined. The basic assumption is that there are uncertainties about the price of output and the level of output associated with a particular level of inputs, and that firms choose quantities of inputs before these uncertainties are resolved. It is further assumed that firms maximise the expected utility of profits in which case the producer maximisation problem is given by:

\[ \max_{x \geq 0} \mathbb{E}(U(pq-w^Tx)) \]

where \( U(\cdot) \) is a utility function which satisfies the von Neumann-Morgenstern axioms, \( q \) represents output, \( p \) represents the price of output, \( x \) is an (Ix1) vector of inputs and \( w \) is an (Ix1) vector of input.
prices. In models of this type the production technology is commonly described by way of a stochastic production function of the form:

\[ q = f(x) \theta \]

where \( \theta \) is a non-negative random variable giving rise to output uncertainty (representing, for example, the random effects of weather and disease).

An important implication of the assumption that the production function is multiplicative in its stochastic and deterministic components is that revenue may be expressed as \( pq = p\theta f(x) = rf(x) \), where \( r \equiv p\theta \). Since randomness is always generated by way of the product \( r \equiv p\theta \), it is possible and convenient to express firm income as the product of one random component, \( r \), and one deterministic component, \( \bar{q} = q/\theta = f(x) \). This deterministic component is interpreted as a 'planned output' and the random component \( r \) is interpreted as a 'random return to planned output', or more loosely, a 'random return to production'. Income is equal to \( r\bar{q} \) which, conveniently, is no different to the type of expression obtained in the alternative case where a firm faces price risk with a deterministic technology.

A further implication of the model is that the expected utility maximising firm will choose its input vector to minimise the cost of producing the 'planned output' level, \( \bar{q} \). This yields the cost function

\[ C(w, \bar{q}) \equiv \min_{x} \{ w'x: f(x) \geq \bar{q}, x \geq 0 \} \]

which indicates the minimum factor cost of producing planned output level \( \bar{q} \) when the input price vector is \( w \). A formal proof of this result is presented in Appendix A. The result implies that the problem of choosing input quantities to maximise expected utility of profits under price and multiplicative technology uncertainties can be divided into two parts: the first is a deterministic problem of choosing inputs to minimise the cost of producing planned output, \( \bar{q} \); and the second is to choose \( \bar{q} \) to maximise expected utility.

This separation of the deterministic cost minimisation problem from the maximum expected utility choice of planned output under uncertainty is of great importance when it comes to estimating the parameters of the technology. Although firms choose inputs to solve (1), we can use the cost function \( C(w, \bar{q}) \) as the basis for our empirical work and not have to make any assumptions about the functional form of the utility function \( U(\cdot) \). The strategy adopted is to use Shephard's Lemma to derive the factor share equations \( s_{i} = \partial \ln(C(w, \bar{q})) / \partial \ln(w_{i}) \), then to estimate the parameters of the cost function and the factor share equations using observations on costs and factor cost shares \( s_{i} = w_{i}x_{i} / \Sigma w_{j}x_{j} \). Of course, one difficulty with this strategy is that planned output, \( \bar{q} \), is not observable.
However, actual output, $q = \theta \bar{q}$, is observable and this relationship is used below to make the strategy operational.

If it is assumed that the technology exhibits constant returns to scale, the cost function factors as $C(w, \bar{q}) = c(w)\bar{q}$ where $c(w)$ is the unit cost function. Replacing planned output $\bar{q}$ by $q/\theta$, the cost of production may be expressed as $C(w, \bar{q}) = c(w)q/\theta$. The non-observable random term $\theta$ can then be treated as a random disturbance for the purposes of estimation.

2.2 Functional Form and Stochastic Specification

To make the model empirically operational we need to choose a functional form for the unit cost function $c(w)$, embed the resulting cost and factor cost share equations in a stochastic framework, incorporate technological differences into the model and, finally, specify the distribution for $\theta$. We deal with each of these matters in turn.

The production technology is assumed to be described by a constant returns to scale translog cost function. The translog cost function is a reasonable choice in the sense that it can serve as a local, second-order approximation to an arbitrary cost function. The constant returns to scale assumption is theoretically plausible in the presence of risk and is adopted for purposes of analytical tractability. The basic model consists of the cost function and an associated subset of cost share equations. The basic system of equations can be written:

$$s_i = \alpha_i + \sum_{j=1}^{I} \alpha_{ij} \ln(w_j)$$

$$i = 1, ..., I.$$  

$$\ln(C) = \alpha_0 + \beta_0 t + \sum_{i=1}^{I} \alpha_i \ln(w_i) + \sum_{i=1}^{I} \sum_{j=1}^{I} \alpha_{ij} \ln(w_i) \ln(w_j) + \ln(q)$$

where the parameters obey the restrictions

$$\sum_{i=1}^{I} \alpha_i = 1$$

$$\sum_{j=1}^{I} \alpha_{ij} = 0 \quad (i, j = 1, ..., I)$$

$$\alpha_{ij} = \alpha_{ji} \quad (i, j = 1, ..., I)$$

and where $C$ represents total costs, $s_i$ represents the cost share of the $i^{th}$ input, $w_i$ represents the price of input $i$, $\bar{q}$ represents planned output and $t$ is a time trend.

The restrictions (6) ensure that the cost function is linearly homogenous in $w$. Economic theory also requires the cost function to be nondecreasing and concave in factor prices but these requirements are
not imposed on our translog model; rather, these requirements are checked after the parameters of the model have been estimated. The introduction of a time trend in the cost function serves the purpose of dealing with exogenous technical change. The assumed form of technical change is Hicks neutral with the implication that factor shares are unaffected by technical change. The coefficient of \( t, \beta_0 \), may be interpreted as the percentage change in the unit cost of production per year (unit of time).

Apart from allowing for changes in the technology over time our model also allows for differences in technology over geographical space. In particular, we distinguish between three sectors of sheep properties based upon the nature of the enterprise: merino woolgrowing, non-merino woolgrowing and prime lamb producing sectors. Each can be expected to have a different technology. Rather than limit the extent of these differences we allow all of the parameters of the cost and share equations to be different in the three sectors. Below, subscripts \( k \) are used to differentiate the technologies of merino woolgrowers (\( k=1 \)), non-merino woolgrowers (\( k=2 \)) and prime lamb producers (\( k=3 \)).

In order to formulate a model which can be used in the empirical work we need to introduce further subscripts to denote firms (\( n \)) and time (\( t \)) and to deal with the following two issues. First, the equations represented by (4) and (5) need to be embedded in an empirically workable stochastic framework by introducing a disturbance term. Second, the planned outputs (\( \bar{q}_{nt} \)) which appear on the right-hand side of (5) are unobserved. When these considerations are accommodated the econometric model is given by (7) and (8) below.

The system of equations represented by (4) and (5) is embedded in a stochastic framework by simply adding random disturbance terms to the right hand sides. It is well known that the share equations (4) sum identically to unity and that there is, consequently, no loss of generality in ignoring one of the share equations. Thus, ignoring the last share equation and introducing the disturbance vector \( e_{nt} = (e_{1nt}, \ldots, e_{1nt})' \), the resulting system of equations becomes:

\[
\begin{align*}
    s_{int} &= \alpha_{ik} + \sum_{j=1}^I \alpha_{ijk} \ln(w_{jnt}) + e_{int} \\
    \ln(C_{nt}) &= \alpha_{0k} + \beta_0 t + \sum_{i=1}^I \alpha_{ik} \ln(w_{i1nt}) + \sum_{i=1}^I \sum_{j=1}^I \alpha_{ijk} \ln(w_{int}) \ln(w_{jnt}) + \ln(\bar{q}_{nt}) + e_{int}
\end{align*}
\]

where \( k \) denotes the \( k \)th sector, \( n \) refers to a firm within that sector, and \( t \) denotes time in years. It is assumed that \( e_{nt} \) is an independent and identically distributed normal random vector with \( E\{e_{nt}\} = 0 \), \( E\{e_{nt} e_{ms}'\} = \Delta \) if \( n=m \) and \( t=s \), and \( E\{e_{nt} e_{ms}'\} = 0 \) if \( n \neq m \) or \( t \neq s \). The variance-covariance matrix \( \Delta \) is positive definite. The disturbances are introduced to allow for errors in optimisation and for influences not formally expressed in the model, and the assumptions imply homoskedasticity and independence over time.
The planned outputs which appear on the right-hand side of the cost function are replaced by \( q_{nt}/\theta_t \), where the subscript \( t \) on the random variable \( \theta \) indicates that \( \theta \) is a random variable which varies from time period to time period but not from firm to firm. After making this substitution the system of equations can be rewritten as:

\[
\begin{align*}
\text{(7)} & \quad s_{nt} = \alpha_{ik} + \sum_{j=1}^{l} \alpha_{ijk} \ln(w_{jnt}) + e_{int} \quad i = 1, \ldots, I-1, \\
\text{(8)} & \quad \ln[C_{nt}/q_{nt}] = \alpha_{0k} + \beta_{0k} \sum_{i=1}^{l} \alpha_{ik} \ln(w_{in}) + \sum_{i=1}^{l} \sum_{j=1}^{l} \alpha_{ijk} \ln(w_{iin}) \ln(w_{iin}) + u_{nt}
\end{align*}
\]

The disturbance term in the cost function (8) is \( u_{nt} = e_{int} - \ln(\theta_t) \). It is assumed that \( \theta_t \) is lognormally distributed so that \( \ln(\theta_t) \) and therefore the \( u_{nt} \) are normally distributed. The lognormality assumption ensures that realised outputs are nonnegative. It is further assumed that \( \text{E}\{\theta_t\} = 1 \). Consequently, \( \ln(\theta_t) \) has mean \(-0.5\sigma^2\) and variance \( \sigma^2 \), and \( \theta_t \) has variance \( \exp(\sigma^2) \).

The term \( \theta_t \) is the random component of output arising from disturbances due to climatic and other conditions. Our model assumes that these, primarily climatic, conditions are common to all firms in the sample. In other words, \( \theta_t \) reflects the climatic conditions common to all firms in the wool and lamb industry.

An important consequence of the common climatic effect, \( \theta_t \), is that \( \ln(\theta_t) \) appears as part of the disturbance in the cost equation of every firm in the sample. This disturbance, \( u_{nt} \), thus consists of an individualistic component, \( e_{int} \), and a common component, \( \ln(\theta_t) \), that varies over time but not over firm or sector. Thus, the system of equations represented by (7) and (8) falls into the class of multivariate error components models. As will be demonstrated below, the disturbance terms \( u_{nt} \) will be correlated over \( n \) but will retain independence over \( t \). To obtain efficient estimates of the parameters, data for firms from all sectors must be utilised in a single estimation that exploits the full covariance matrix of all disturbances.

2.3 Grouped Data and the Covariance Matrix

Before proceeding to derive the covariance matrix for the disturbances we have to deal with some issues related to the data used in the empirical work. The first is that, while equations (7) and (8) are structural functions representing the behaviour of a single firm, the data consist of observations on groups of firms. Fortunately each group in the data set is comprised of firms which are located in the same state and zone and are engaged in the same production activity. This implies that the firms in
each group have access to identical technologies and face the same input prices. If, in addition, firms in each group are equal in size, it is reasonably easy to average equations (7) and (8) over the firms in a group. The notation \( n_{gt} \) is used to represent the number of firms in group \( g \) in time period \( t \). The arithmetic averages of (7) and (8) over the \( n_{gt} \) firms in group \( g \) in period \( t \) are given by:

\[
\bar{x}_{igt} = \alpha_{ik} + \frac{1}{n_{gt}} \sum_{j=1}^{n_{gt}} \alpha_{ijk} \ln(w_{igt}) + \bar{\epsilon}_{igt} \quad i = 1, \ldots, I-1.
\]

and

\[
\bar{z}_{gt} = \alpha_{0k} + \beta_{0k} t + \frac{1}{n_{gt}} \sum_{j=1}^{n_{gt}} \alpha_{ijk} \ln(w_{igt}) + \frac{1}{n_{gt}} \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_{ijk} \ln(w_{igt}) \ln(w_{ijt}) + \bar{\mu}_{gt} \]

where

\[
\bar{x}_{igt} = \frac{1}{n_{gt}} \sum_{j=1}^{n_{gt}} x_{ijt} \]

is the average cost share of factor \( i \) for group \( g \) in time period \( t \),

\[
\bar{\epsilon}_{igt} = \frac{1}{n_{gt}} \sum_{j=1}^{n_{gt}} \epsilon_{ijt} \]

is a normally distributed random error term,

\[
\bar{z}_{gt} = \frac{1}{n_{gt}} \sum_{j=1}^{n_{gt}} \ln(C_{jt}/q_{jt}) \]

is the average of the logarithm of cost per unit of output for group \( g \) in time period \( t \),

and

\[
\bar{\mu}_{gt} = \frac{1}{n_{gt}} \sum_{j=1}^{n_{gt}} u_{jt} \]

is a normally distributed random error term.

The equations represented by (9) and (10) are structural equations representing the behaviour of a single group of firms. The data described in Section 3 are all but perfectly suitable for the estimation of the parameters of these equations. However a problem with the data concerns the construction of total costs. To be specific, the cost data used in the empirical work is calculated as the arithmetic average of the costs of individual firms. This is slightly inconsistent with the construction of the dependent variable in equation (10). This dependent variable is the arithmetic average of the dependent variable in equation (8), and is equivalent to the logarithm of the geometric average of the costs of individual firms. Because the arithmetic and geometric averages are likely to be roughly equivalent this discrepancy is overlooked.

There are two further issues which need to be dealt with before the estimation of the econometric model can proceed. First, it is necessary to impose upon the share equations the restrictions which ensure that the cost function is homogenous in factor prices. Second, it is necessary to derive the variance-
covariance properties of the disturbance terms. In dealing with both these issues it is convenient to rewrite the model using vectors and matrices and to be explicit about the numbers of inputs, namely I=4. For group g in time period t the restricted version of the model can be represented by the matrix equation (11) below. In addition, the complete set of observations can be represented by the matrix equation (13). The variance-covariance properties of the disturbance vector in equation (13) are summarised by equation (14). These equations are derived as follows.

The restricted version of equations (9) and (10) can be written as:

\begin{equation}
Y_{gt} = X_{gt}\beta + u_{gt}
\end{equation}

where

\begin{equation}
Y_{gt} = (\tilde{\xi}_{1gt}, ..., \tilde{\xi}_{3gt}, \tilde{\xi}_{gt}, ln(w_{4gt}))'
\end{equation}

\begin{equation}
X_{gt} = (X_{1gt}, X_{2gt}, X_{3gt})
\end{equation}

\begin{equation}
X_{kgt} = \begin{cases}
\text{a (4x11) matrix of zero's if group } g \text{ belongs to sector } k \\
0 1 0 0 \omega_{21} \omega_{31} \omega_{41} 0 0 0 0 \\
0 0 1 0 \omega_{12} 0 0 \omega_{32} \omega_{42} 0 0 \\
0 0 0 1 0 \omega_{13} 0 \omega_{23} 0 \omega_{43} 0 \\
1 \omega_{14} \omega_{24} \omega_{34} \omega_{12} \omega_{13} \omega_{14} \omega_{23} \omega_{24} \omega_{34} \gamma \\
\end{cases}
\end{equation}

\begin{equation}
\beta' = (\beta_1', \beta_2', \beta_3')'
\end{equation}

\begin{equation}
\beta_k = (\alpha_{0k} + 0.5\sigma^2, \alpha_{1k}, ..., \alpha_{3k}, \alpha_{12k}, ..., \alpha_{14k}, \alpha_{23k}, \alpha_{24k}, \alpha_{34k}, \beta_{0k})'
\end{equation}

\begin{equation}
u_{gt} = (\tilde{\xi}_{1gt}, ..., \tilde{\xi}_{3gt}, \tilde{u}_{gt}, 0.5\sigma^2)'
\end{equation}

\begin{equation}
\omega_{ij} = ln(w_{igt}) - ln(w_{jgt})
\end{equation}

and \begin{equation}
c_{ij} = -0.5(ln(w_{igt}) - ln(w_{jgt}))^2
\end{equation}

It can be easily demonstrated that the composite error vector $u_{gt}$ is normally distributed with mean vector zero and the following variance-covariance properties:

\begin{equation}
E(u_{gt}u_{ms}') = \begin{cases}
\Gamma' + (1/n_{gt})\Delta & \text{if } g=m \text{ and } t=s \\
\Gamma & \text{if } g\neq m \text{ and } t=s \\
0 & \text{if } t<s
\end{cases}
\end{equation}
where $F$ is a $(4 \times 4)$ positive semidefinite matrix with $\text{Var}(\ln(\theta_t)) = \sigma^2$ in the fourth column of the fourth row and zero's elsewhere.

The model represented by equations (11) and (12) is an error components model. Some interesting features of this particular model become apparent when the entire set of observations is written in matrix form. One way of doing this is to stack the observations first by group and then by time period. Using the notation $G(t)$ to represent the number of groups or observations in time period $t$, the entire set of 584 observations may be written as:

\[
\begin{bmatrix}
Y_{11} \\
Y_{21} \\
\vdots \\
Y_{G(1)1} \\
Y_{12} \\
\vdots \\
Y_{G(T)1} \\
\end{bmatrix} = 
\begin{bmatrix}
X_{11} \\
X_{21} \\
\vdots \\
X_{G(1)1} \\
X_{12} \\
\vdots \\
X_{G(T)1} \\
\end{bmatrix} \beta + 
\begin{bmatrix}
u_{11} \\
u_{21} \\
\vdots \\
u_{G(1)1} \\
u_{12} \\
\vdots \\
u_{G(T)1} \\
\end{bmatrix}
\]

or more compactly as

(13) \hspace{1cm} y = X\beta + u

where the definitions are obvious, although it is worth emphasising that $y$ and $u$ are $(2336 \times 1)$, $X$ is $(2336 \times 33)$ and $\beta$ is $(33 \times 1)$. It can be easily demonstrated that the error vector $u$ is normally distributed with mean vector zero and the following variance-covariance properties:

\[
E(uu') = 
\begin{bmatrix}
V_1 & 0 & \ldots & 0 \\
0 & V_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & V_T \\
\end{bmatrix} = \Omega
\]

where

\[
V_t = j_t j_t' \otimes \Gamma + \Lambda_t \otimes \Delta
\]

$j_t$ is a $(G(t) \times 1)$ vector of ones
Two features of the variance-covariance matrix $\Omega$ are of interest. First, $\Omega$ is block diagonal owing to the fact that disturbance vectors associated with different time periods are uncorrelated. Second, the dimensions of each block vary, owing to different numbers of groups in each time period.

2.4 Estimation of Parameters

The econometric model is given by equations (13) and (14). The unknown parameters appear in the form of the coefficient vector $\beta$ and the variance-covariance matrix $\Omega$. This section is concerned with the method used to obtain maximum likelihood estimates of these parameters.

The logarithm of the likelihood function associated with equations (13) and (14) is given by:

\[
L = -\frac{1}{2} \ln |\Omega| + \text{tr}(u^T \Omega^{-1} u)
\]

(15)

Since the covariance matrix $\Omega$ is of the order (2336x2336), maximum likelihood estimation of the model is only feasible if the determinant and inverse of $\Omega$ can be expressed in terms of the determinants and inverses of matrices of smaller dimensions. Three characteristics of the model make this possible. First, and foremost, the assumption of time independence of disturbances implies that $\Omega$ is block diagonal. Second, less than a third of the elements of the design matrix $X$ are nonzero. Third, the matrix of weights $\Lambda_t$ is diagonal. These characteristics of the model imply that the log-likelihood can be expressed in the form of equation (16) below. Equation (16) is derived as follows.

The covariance matrix $\Omega$ is block diagonal and it can be demonstrated (Hadley, 1973, p.109, 128) that:

\[
|\Omega| = |V_1| \ldots |V_T|
\]

and

\[
\Omega^{-1} = \begin{bmatrix}
    V_1^{-1} & 0 & \ldots & 0 \\
    0 & V_2^{-1} & \ldots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \ldots & V_T^{-1}
\end{bmatrix}
\]
In addition, a result of Magnus (1982, p. 243) may be used to demonstrate that:

\[ |V_t| = |A|^4 |\Delta|^{-1} |W_t| \]

and

\[ V_t^{-1} = \Lambda_t^{-1} \odot \Delta^{-1} + \Lambda_t^{-1} \Lambda_t^{-1} \odot \frac{1}{b_t} (W_t^{-1} \Delta^{-1}) \]

where

\[ W_t = \Delta + b_t \Gamma \]

and

\[ b_t = \sum_{j=1}^{G(t)} n_{jt} \rightarrow (0) \]

Together, these results imply that

\[
\ln |\Omega| = \sum_{t=1}^{T} \ln |V_t| \\
= \sum_{t=1}^{T} \left[ 4 \ln |A| + (G(t)-1) \ln |\Delta| + \ln |W_t| \right] \\
= \sum_{t=1}^{T} \left[ -4 \sum_{j=1}^{G(t)} \ln(n_{jt}) + (G(t)-1) \ln |\Delta| + \ln |W_t| \right]
\]

and

\[ u' \Omega^{-1} u = \sum_{t=1}^{T} \left[ u_t' [\Lambda_t^{-1} \odot \Delta^{-1} + \Lambda_t^{-1} \Lambda_t^{-1} \odot \frac{1}{b_t} (W_t^{-1} \Delta^{-1})] u_t \right] \\
= \sum_{t=1}^{T} \sum_{h=1}^{G(t)} \sum_{g=1}^{G(t)} u_t A_{ght} u_h \]

where

\[ A_{ght} = \begin{cases} 
    \frac{n_{gt} \Delta^{-1}}{b_t} + \frac{n_{gh} t}{b_t} (W_t^{-1} \Delta^{-1}) & \text{if } g=h \\
    \frac{n_{gt} n_{ht}}{b_t} (W_t^{-1} \Delta^{-1}) & \text{otherwise}
\end{cases} \]
It follows that the log-likelihood function may be written:

\begin{equation}
L = \text{constant} + 2 \sum_{t=1}^{T} \sum_{j=1}^{\text{nlj}} \ln(n_{jt}) - \frac{\ln|\Lambda|}{2} \sum_{t=1}^{T} (G(t)-1) - \frac{1}{2} \sum_{t=1}^{T} \ln|W_{t}| + \frac{1}{2} \sum_{t=1}^{T} G(t) G(t) - \sum_{t=1}^{T} \sum_{h=1}^{L} \sum_{g=1}^{L} u_{tg} A_{ght} u_{ht}
\end{equation}

This is the form of the log-likelihood function to be maximised. Equation (16) is preferred to (15) because evaluation of (16) only requires the calculation of determinants and inverses of matrices of relatively small dimensions. In practice the function is maximised with respect to the elements of the two lower triangular matrices \( L_c \) and \( L_d \). The matrices \( L_c \) and \( L_d \) are defined such that \( L_c L_c' = \Gamma \) and \( L_d L_d' = \Delta \). Maximisation with respect to \( L_c \) and \( L_d \) ensures that \( \Gamma \) and \( \Delta \) are positive semidefinite.

The likelihood function (16) was maximised iteratively. The iterations were controlled by a monitor program known as NLMON, which is an adaptation of a program originally written at the University of British Columbia by Wales (1976). At each iteration the likelihood function was maximised using the POWEL and FMIN algorithms. The iterative procedure was as follows:

1. Maximise (16) with respect to \( \beta \) and \( \Delta \) subject to the restriction that \( \Gamma = 0 \).
2. Maximise (16) with respect to \( \Delta \) and \( \Gamma \) conditional upon the estimate of \( \beta \) obtained in the previous step.
3. Maximise (16) with respect to \( \beta \) conditional upon the estimate of \( \Delta \) and \( \Gamma \) obtained in step 2 (i.e. apply Estimated Generalised Least Squares).
4. Repeat steps 2 and 3 until convergence (with tolerance 0.1E-04).

2.5 Estimation of Random Production Components

An important aspect of our model is the inclusion of uncertainty for producers through the random disturbance term, \( \theta_t \). This random disturbance helps determine the level of production actually realised after producers have committed themselves to their chosen input vectors. As previously indicated, we have in mind that \( \theta_t \) reflects primarily climatic conditions. It is of considerable interest to be able to provide estimates of \( \theta_t \). Accordingly, this section deals with the estimation of the random variables \( \theta_t \) (\( t = 1, \ldots, T \)). An estimator for \( \theta_t \) is \( \bar{\theta}_t \) and is given by equation (21) below. This estimator is found as follows.

A particular set of equations which expresses the relationship between costs, input prices and the planned output of a group of firms is the set of equations given by equation (11) above. It is possible
to obtain an estimator for $\theta_t$ by taking a weighted average of equation (11) over the G(t) groups in the data set in period t. The weights are the numbers of firms in each group ($n_{jt}$) and the procedure yields:

$$w_t = Z_t \beta + \nu_t$$

where

$$w_t = \frac{1}{b_t} \sum_{j=1}^{G(t)} n_{jt} y_{jt}$$

$$Z_t = \frac{1}{b_t} \sum_{j=1}^{G(t)} n_{jt} \chi_{jt}$$

and

$$\nu_t = \frac{1}{b_t} \sum_{j=1}^{G(t)} n_{jt} u_{jt}$$

It can be demonstrated that the error vector $\nu_t$ is normally distributed with mean vector zero and variance-covariance properties:

$$E\{\nu_t \nu_t'\} = \Gamma + \frac{1}{b_t} \Delta$$

$$E\{\nu_t a_t'\} = \Gamma$$

where

$$a_t = \left( \begin{array}{cccc} 0 & 0 & 0 & -\ln(\theta_t) - 0.5 \sigma^2 \end{array} \right)'$$

Estimation of $\theta_t$ is accomplished by way of the estimation of $a_t$. To be specific, if $\nu_t$ is known then the best predictor of $a_t$ is the conditional expectation of $a_t$ given $\nu_t$. It can be demonstrated using Morrison (1976, p.91-92) that:

$$E\{a_t | \nu_t\} = \Gamma (\Gamma + \frac{1}{b_t} \Delta)^{-1} \nu_t$$

Unfortunately the matrices $\Gamma$ and $\Delta$ and the vector $\nu_t$ are unobserved. However, a consistent estimator of $a_t$ is:

$$\bar{a}_t = \bar{\Gamma} (\bar{\Gamma} + \frac{1}{b_t} \bar{\Delta})^{-1} \bar{\nu}_t$$

where $\bar{\Gamma}$ and $\bar{\Delta}$ are the maximum likelihood estimates of $\Gamma$ and $\Delta$ obtained by maximising (16), and $\bar{\nu}_t$ is a weighted average of the associated residuals:
Finally, a consistent estimator of $\theta_t$ is given by:

$\tilde{\theta}_t = \exp(-b^T \bar{\bar{\mu}}_t - 0.5b^T \bar{\bar{\theta}} b)$

where $b = (0 \ 0 \ 0 \ 1)'$

Estimates of $\theta_t$ obtained using (21) are presented and discussed in Section 4 below.

3. Data

The wool and lamb producing industry was chosen for the empirical application because variations in levels of production and aggregate prices of sheep and wool products are relatively large (see, for example, Motha et al (1975)). The data consists of observations from the Australian Bureau of Agricultural and Resource Economics (ABARE) Australian Sheep Industry Surveys (ASIS). The ASIS surveys are annual surveys and are described in detail in several publications including BAE (1973). Our sample extends from 1952-53 to 1975-76 because it was only in these years that properties in the ASIS samples were classified by enterprise type (merino woolgrowing, non-merino woolgrowing and prime lamb producing).

Each observation in the data set is a record of the average physical and financial characteristics of a group of ASIS firms. Each firm in a particular group is engaged in the same production activity (merino woolgrowing, non-merino woolgrowing or prime lamb producing) in a particular agricultural zone (pastoral, wheat-sheep or high rainfall) in a particular state in a particular time period. Each firm derives more than 80% of its total returns from the sheep enterprise and less than 15% of its total returns from either the cattle or the cereal enterprise. In time-series and cross-section there is a total of 584 observations - 310 observations on merino woolgrowers, 117 observations on non-merino woolgrowers, and 157 observations on prime lamb producers. There is a different number of observations on different types of firms because some types of firm do not always exist in some zones in some states. An associated characteristic of the data which has important econometric implications is that the number of firms in each group varies by sector, zone, state and time period.

The ASIS data is used to construct observations on $q$ (output), $C$ (total cost) and $w$ (input prices). Inputs are grouped into one of four broad input categories, namely land, (physical) capital, livestock
and a group of other inputs including labour, equipment, supplies, services and overheads. The manner in which these variables are constructed is detailed in Appendix B.

4. Results

4.1 Parameter Estimates, Concavity and Technical Change

Estimates of the structural parameters $\beta$ are presented in Table 1 below. The figures in parentheses are estimates of the asymptotic standard errors and have been calculated using $\text{Var}(\hat{\beta}) = (X\hat{\Omega}^{-1}X)^{-1}$. All the estimates are significantly different from zero at the usual levels of significance.

For the estimated functions to be consistent with cost minimisation the estimated cost shares must be positive and the estimated cost functions themselves must be concave. These conditions were checked at several sets of representative input prices. Specifically, these conditions were checked at $K \times T = 69$ sets of weighted average input prices, calculated as the weighted arithmetic average of prices paid by all firms in sector $k$ in time period $t$, using input quantities as weights.

Calculation of the estimated cost shares at each set of input prices was straightforward. As expected, all the estimated cost shares were positive.

Concavity of the cost function was determined by checking for a negative semidefinite Hessian matrix. A necessary but not sufficient condition for a Hessian to be negative semidefinite is that each diagonal element, indicating the response of an input to a change in its own price, be non-positive. Of the 69 estimated Hessians, 40 satisfied this condition. All but one violation could be attributed to a positive response by the third input (livestock) to a change in its own price. A necessary and sufficient condition for the Hessian to be negative semi-definite is that all of the eigenvalues are non-positive and at least one of the eigenvalues vanishes. Using this criterion, 33 of the 69 Hessians are negative semidefinite.

As indicated above, the coefficient of the time variable in the cost function, $\beta_{0k}$, may be interpreted as the rate of technical progress as measured by the annual proportional reduction in unit costs. The estimates of $\beta_{0k}$ presented in the last row of Table 1 are all negative indicating technical progress. The rate of technical change is estimated to be 2.9% per annum in the merino woolgrowing sector, 3.9% in the non-merino woolgrowing sector, and 3.0% in the prime lamb producing sector. These results are consistent with rates of technical progress in non-agricultural sectors and are also consistent with the results of Duncan (1972) who estimated the annual rate of technological advance for agricultural firms in the arid zone of New South Wales to be 1.49%.
### TABLE 1: Estimated Parameters of the Technologies\textsuperscript{a}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Merino Woolgrowing</th>
<th>Non-Merino Woolgrowing</th>
<th>Prime Lamb Producing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-.616</td>
<td>-.887</td>
<td>-.1101</td>
</tr>
<tr>
<td></td>
<td>(.0568)</td>
<td>(.1351)</td>
<td>(.1298)</td>
</tr>
<tr>
<td>$\alpha_{1k}$</td>
<td>.246</td>
<td>.330</td>
<td>.317</td>
</tr>
<tr>
<td></td>
<td>(.0040)</td>
<td>(.02418)</td>
<td>(.0263)</td>
</tr>
<tr>
<td>$\alpha_{2k}$</td>
<td>.664</td>
<td>.584</td>
<td>.563</td>
</tr>
<tr>
<td></td>
<td>(.0165)</td>
<td>(.0492)</td>
<td>(.0421)</td>
</tr>
<tr>
<td>$\alpha_{3k}$</td>
<td>.446</td>
<td>.382</td>
<td>.385</td>
</tr>
<tr>
<td></td>
<td>(.0100)</td>
<td>(.0269)</td>
<td>(.0250)</td>
</tr>
<tr>
<td>$\alpha_{4k}$</td>
<td>-.356</td>
<td>-.296</td>
<td>.265</td>
</tr>
<tr>
<td></td>
<td>(.0176)</td>
<td>(.0546)</td>
<td>(.0380)</td>
</tr>
<tr>
<td>$\alpha_{11k}$</td>
<td>.022</td>
<td>.044</td>
<td>.044</td>
</tr>
<tr>
<td></td>
<td>(.0016)</td>
<td>(.0130)</td>
<td>(.0140)</td>
</tr>
<tr>
<td>$\alpha_{12k}$</td>
<td>.018</td>
<td>.024</td>
<td>.023</td>
</tr>
<tr>
<td></td>
<td>(.0006)</td>
<td>(.0051)</td>
<td>(.0033)</td>
</tr>
<tr>
<td>$\alpha_{13k}$</td>
<td>-.006</td>
<td>-.006</td>
<td>-.007</td>
</tr>
<tr>
<td></td>
<td>(.0004)</td>
<td>(.0030)</td>
<td>(.0030)</td>
</tr>
<tr>
<td>$\alpha_{14k}$</td>
<td>-.034</td>
<td>-.062</td>
<td>-.060</td>
</tr>
<tr>
<td></td>
<td>(.0009)</td>
<td>(.0076)</td>
<td>(.0085)</td>
</tr>
<tr>
<td>$\alpha_{22k}$</td>
<td>.114</td>
<td>.081</td>
<td>.073</td>
</tr>
<tr>
<td></td>
<td>(.0061)</td>
<td>(.0198)</td>
<td>(.0186)</td>
</tr>
<tr>
<td>$\alpha_{23k}$</td>
<td>-.008</td>
<td>-.009</td>
<td>-.010</td>
</tr>
<tr>
<td></td>
<td>(.0020)</td>
<td>(.0058)</td>
<td>(.0045)</td>
</tr>
<tr>
<td>$\alpha_{24k}$</td>
<td>-.123</td>
<td>-.096</td>
<td>-.086</td>
</tr>
<tr>
<td></td>
<td>(.0052)</td>
<td>(.0161)</td>
<td>(.0136)</td>
</tr>
<tr>
<td>$\alpha_{33k}$</td>
<td>.077</td>
<td>.067</td>
<td>.069</td>
</tr>
<tr>
<td></td>
<td>(.0041)</td>
<td>(.0127)</td>
<td>(.0118)</td>
</tr>
<tr>
<td>$\alpha_{34k}$</td>
<td>-.063</td>
<td>-.052</td>
<td>-.052</td>
</tr>
<tr>
<td></td>
<td>(.0024)</td>
<td>(.0070)</td>
<td>(.0069)</td>
</tr>
<tr>
<td>$\alpha_{44k}$</td>
<td>.220</td>
<td>.210</td>
<td>.198</td>
</tr>
<tr>
<td></td>
<td>(.0054)</td>
<td>(.0177)</td>
<td>(.0126)</td>
</tr>
<tr>
<td>$\beta_{0k}$</td>
<td>-.029</td>
<td>-.039</td>
<td>-.030</td>
</tr>
<tr>
<td></td>
<td>(.0028)</td>
<td>(.0053)</td>
<td>(.0042)</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Numbers in parentheses are standard errors.
4.2 Price Elasticity Estimates

Economic theory has little to say about the expected signs and magnitudes of the estimated coefficients in Table 1. However, economic theory does have something to say about the signs and magnitudes of certain elasticities. In this context, two types of elasticity are of interest: input price elasticities and Allen-Uzawa substitution elasticities.

In the case of the constant returns to scale translog cost function, the set of input price elasticities may be calculated as:

\[
e_{ijkt} = s_{jkt} \left[ 1 + \frac{c_{ijk} - \delta_{ij} s_{jkt}}{s_{jkt} + s_{jkt}} \right]
\]

where \( \delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{otherwise} \end{cases} \)

and \( s_{jkt} \) is the cost share of the \( j \)th input for firms in sector \( k \) in time period \( t \).

The own- and cross-price elasticity estimates reported in Table 2 below were calculated at a weighted average of input prices over all groups of firms over all time periods. The numbers in parentheses are estimated standard errors and have been calculated as linear approximations. Elasticities which are not significantly different from zero at the 5% level of significance (two-tailed test) are marked with an asterisk (*).

When \( i = j \) the \( e_{ijkt} \) can be expected to be negative. All the own-price elasticities reported in Table 1 are correctly signed.

When there are more than two factors of production and when \( i \neq j \) economic theory has little to say about the signs of the individual cross-price elasticities. Most estimates of the cross-price elasticities are positive, indicating that most inputs are substitutes. However, estimates of the cross-price elasticities between livestock and the final group of inputs (including labour, drenches, dips, licks and so on) are negative, indicating that these two groups of inputs are complements.

From Table 2 it can be seen that all inputs are inelastic with respect to their own price. Specifically, the own-price elasticities for land are approximately -0.6, the own-price elasticities for capital lie in the range -0.3 to -0.4, the own-price elasticities for livestock lie between 0.0 and -0.1, and the own-price elasticities for labour and other inputs are approximately -0.1.
### TABLE 2: Input Price Elasticities at Average Prices

<table>
<thead>
<tr>
<th>Sector (k)</th>
<th>Price of Land</th>
<th>Price of Capital</th>
<th>Price of Livestock</th>
<th>Price of Labour etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qty of Land</td>
<td>1</td>
<td>-0.634 (0.009)</td>
<td>0.509 (0.009)</td>
<td>0.022 (0.005)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.139 (0.002)</td>
<td>-0.315 (0.018)</td>
<td>0.067 (0.007)</td>
</tr>
<tr>
<td>Qty of Capital</td>
<td>1</td>
<td>0.018 (0.004)</td>
<td>0.195 (0.021)</td>
<td>-0.109 (0.017)</td>
</tr>
<tr>
<td>Qty of Livestock</td>
<td>1</td>
<td>0.018 (0.002)</td>
<td>0.056 (0.010)</td>
<td>-0.019 (0.005)</td>
</tr>
<tr>
<td>Qty of Labour etc.</td>
<td>1</td>
<td>0.015 (0.002)</td>
<td>0.490 (0.034)</td>
<td>0.032* (0.020)</td>
</tr>
<tr>
<td>Qty of Land</td>
<td>2</td>
<td>-0.557 (0.029)</td>
<td>0.490 (0.034)</td>
<td>0.032* (0.020)</td>
</tr>
<tr>
<td>Qty of Capital</td>
<td>2</td>
<td>0.224 (0.016)</td>
<td>-0.426 (0.048)</td>
<td>0.046 (0.018)</td>
</tr>
<tr>
<td>Qty of Livestock</td>
<td>2</td>
<td>0.065* (0.041)</td>
<td>0.202 (0.078)</td>
<td>-0.025* (0.066)</td>
</tr>
<tr>
<td>Qty of Labour etc.</td>
<td>2</td>
<td>0.012* (0.017)</td>
<td>0.112 (0.036)</td>
<td>-0.040 (0.016)</td>
</tr>
<tr>
<td>Qty of Land</td>
<td>3</td>
<td>-0.559 (0.032)</td>
<td>0.497 (0.034)</td>
<td>0.043 (0.019)</td>
</tr>
<tr>
<td>Qty of Capital</td>
<td>3</td>
<td>0.219 (0.015)</td>
<td>-0.443 (0.032)</td>
<td>0.062 (0.013)</td>
</tr>
<tr>
<td>Qty of Livestock</td>
<td>3</td>
<td>0.072 (0.033)</td>
<td>0.236 (0.051)</td>
<td>-0.148 (0.041)</td>
</tr>
<tr>
<td>Qty of Labour etc.</td>
<td>3</td>
<td>0.007* (0.021)</td>
<td>0.136 (0.033)</td>
<td>-0.035 (0.017)</td>
</tr>
</tbody>
</table>

* Numbers in parentheses are estimates of standard errors obtained by linear approximation.

* Not significantly different from zero at the 5% level of significance (two-tailed test).
The own-price elasticities for capital are a little closer to zero than the agricultural industry estimates reported by Ryan and Duncan (1974). They are also a little closer to zero than the sheep industry estimates obtained by McKay et al (1980) and Watts and Quiggin (1984). In fact McKay et al and Watts and Quiggin estimate the demand for capital to be slightly elastic. The own-price elasticities for land are slightly closer to zero but the own-price elasticities for livestock are quite similar to those reported by these other authors. For the remaining input group, which includes labour and other inputs, these different sets of elasticities are not comparable.

Interestingly, the demand for land appears to be quite sensitive to changes in the price of capital. In each sector the relevant cross-price elasticity estimate is approximately 0.5 (McKay et al also estimated this cross-price elasticity to be approximately 0.5). Interestingly, the change in the demand for land in response to a change in the price of capital is estimated to be somewhat greater than the change in the demand for capital in response to a change in the price of land. The relevant cross-price elasticity estimates lie between 0.1 and 0.2. Of course these cross-price elasticity estimates are still large in comparison with the cross-price elasticity estimates for most of the remaining inputs. The estimated cross-price elasticities for the remaining inputs are generally less than 0.2 in absolute value and tend to be more symmetric.

It is possible to calculate another set of input price elasticities which have the property that they are symmetric. These elasticities are the Allen-Uzawa substitution elasticities defined as:

$$\eta_{ijkt} = \frac{C_{ij}C_k(w, q_k)}{C_j}$$

and calculated as:

$$\eta_{ijkt} = \frac{\varepsilon_{ijkt}}{s_{jkt}}$$

The Allen-Uzawa elasticity estimates reported in Table 3 below were again calculated at a weighted average of input prices over all groups of firms over all time periods. Again, the numbers in parentheses are estimated standard errors and have been calculated as linear approximations. Elasticities which are not significantly different from either +1 or -1 (depending on the sign of the elasticity) at the 5% level of significance (two-tailed test) are marked with a hatch (#). Since the input cost shares are positive the input price and Allen-Uzawa substitution elasticities possess the same sign.

The estimated Allen-Uzawa elasticities of substitution between land and capital lie within the range 1.4 to 1.8, somewhat lower than the range of elasticities reported by McKay et al (1980) (2.2 to 3.0). The
<table>
<thead>
<tr>
<th>Sector (k)</th>
<th>Land</th>
<th>Capital</th>
<th>Livestock</th>
<th>Labour etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Land</strong></td>
<td>-8.232</td>
<td>1.806</td>
<td>0.229</td>
<td>0.189</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.039)</td>
<td>(0.053)</td>
<td>(0.027)</td>
</tr>
<tr>
<td><strong>Capital</strong></td>
<td>1</td>
<td>-1.117#</td>
<td>0.693</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td></td>
<td>(0.074)</td>
<td>(0.034)</td>
</tr>
<tr>
<td><strong>Livestock</strong></td>
<td>1</td>
<td>-1.123#</td>
<td></td>
<td>-0.192</td>
</tr>
<tr>
<td></td>
<td>(0.178)</td>
<td></td>
<td></td>
<td>(0.048)</td>
</tr>
<tr>
<td><strong>Labour etc.</strong></td>
<td>1</td>
<td></td>
<td></td>
<td>-0.096</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.021)</td>
</tr>
<tr>
<td><strong>Land</strong></td>
<td>2</td>
<td>-3.742</td>
<td>1.505</td>
<td>0.434</td>
</tr>
<tr>
<td></td>
<td>(0.197)</td>
<td>(0.108)</td>
<td>(0.271)</td>
<td>(0.113)</td>
</tr>
<tr>
<td><strong>Capital</strong></td>
<td>2</td>
<td>-1.308</td>
<td>0.621#</td>
<td>0.344</td>
</tr>
<tr>
<td></td>
<td>(0.148)</td>
<td></td>
<td>(0.238)</td>
<td>(0.110)</td>
</tr>
<tr>
<td><strong>Livestock</strong></td>
<td>2</td>
<td>-0.330</td>
<td>-0.330</td>
<td>-0.537</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.876)</td>
<td>(0.215)</td>
</tr>
<tr>
<td><strong>Labour etc.</strong></td>
<td>2</td>
<td></td>
<td></td>
<td>-0.185</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.101)</td>
</tr>
<tr>
<td><strong>Land</strong></td>
<td>3</td>
<td>-3.671</td>
<td>1.438</td>
<td>0.470</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(0.103)</td>
<td>(0.215)</td>
<td>(0.136)</td>
</tr>
<tr>
<td><strong>Capital</strong></td>
<td>3</td>
<td>-1.281</td>
<td>0.684</td>
<td>0.394</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td></td>
<td>(0.147)</td>
<td>(0.096)</td>
</tr>
<tr>
<td><strong>Livestock</strong></td>
<td>3</td>
<td>-1.641#</td>
<td></td>
<td>-0.388</td>
</tr>
<tr>
<td></td>
<td>(0.445)</td>
<td></td>
<td></td>
<td>(0.193)</td>
</tr>
<tr>
<td><strong>Labour etc.</strong></td>
<td>3</td>
<td></td>
<td></td>
<td>-0.263</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.126)</td>
</tr>
</tbody>
</table>

*Numbers in parentheses are estimates of standard errors obtained by linear approximation.

# Not significantly different from +1 (or -1) at the 5% level of significance (two-tailed test).
estimated elasticities of substitution between land and livestock lie within the range 0.2 to 0.5, and the estimated elasticities of substitution between capital and livestock lie within the range 0.6 to 0.7. McKay et al estimate that these two sets of elasticities lie within the ranges -0.7 to -1.5 and -0.4 to 0.1 respectively.

An interesting interpretation of the Allen-Uzawa elasticities is based on the result that:

\[
\frac{\partial \ln s_i}{\partial \ln w_j} \begin{cases} > 0 & \text{as } \eta_{ijkt} > 1 \\ = 0 & \\ < 0 & \text{as } \eta_{ijkt} < 1 \end{cases}
\]

for the case where \( j \neq i \). Equivalently, since \( s_i \) and \( w_j \) are non-negative:

\[
\frac{\partial s_i}{\partial w_j} \begin{cases} > 0 & \text{as } \eta_{ijkt} > 1 \\ = 0 & \\ < 0 & \text{as } \eta_{ijkt} < 1 \end{cases}
\]

That is, the Allen-Uzawa substitution elasticities reveal the direction of change in the cost share of factor \( i \) in response to a change in the price of factor \( j \). In Table 3, for example, it is evident that an increase in the price of capital tends to cause an increase in the cost share of land and tends to cause a decrease the cost shares of the remaining inputs.

4.3 Disturbances

Associated with the coefficient and elasticity estimates presented in Tables 1 to 3 are a set of estimated variance and covariance parameters, namely \( \hat{\Sigma} \) and \( \hat{\Delta} \). Estimates of these matrices are presented in Table 4 below. In Table 4 the figures in parentheses are estimates of the relevant standard errors. These standard errors are calculated as the square root of the elements of the inverse of the estimated Hessian matrix. With one exception, all covariance parameters are statistically different from zero at the usual levels of significance. The exception is \( \hat{\Delta}_{24} = \hat{\Delta}_{42} \) which is an estimate of the covariance between the error term in the capital share equation and the time and individual varying component of the error term in the cost function.
TABLE 4: Estimated Variance-covariance Parameters

\[ \bar{\Sigma} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0056769 \\ \end{bmatrix} \]

\[ \bar{\Delta} = \begin{bmatrix} .0065172 & .0046438 & -.0012610 & .0132141 \\ (.000381) & (.000377) & (.000211) & (.002646) \\ -.0094489 & -.0018754 & -.0012789 \\ (.000553) & (.000259) & (.0003110) \\ .0037626 & .0046610 \\ (.000220) & (.002068) \\ .5212699 \\ (.031551) \end{bmatrix} \]

\(^a\)Numbers in parentheses are estimates of standard errors

The correlation coefficients between disturbances in the input and cost equations are given by the matrix \( \bar{\rho} \):

\[ \bar{\rho} = \begin{bmatrix} 1 & 0.59 & -0.25 & -0.13 \\ 1 & -0.31 & -0.13 \\ 1 & -0.02 \\ 1 \end{bmatrix} \]

The off-diagonal elements of this matrix are predominantly negative, with only the correlation coefficient between disturbances in the land (i=1) and capital (i=2) share equations positive. This shows that unexpectedly larger shares of capital and land tend to occur together, but for all other inputs unexpectedly higher shares are associated with unexpectedly lower shares for other inputs.

The variance associated with \( \ln(\theta_t) \) is given by the only non-zero element of \( \Gamma \). Thus, the estimate of this variance is \( \sigma^2 = 0.0057 \). This estimate has an asymptotic t-value of 2.58, indicating that \( \sigma^2 \) is significantly different from zero. The variance of \( \theta_t \) is \( \exp(\sigma^2) \) and is therefore estimated to be 1.006. The mean of \( \theta_t \) is unity by assumption.

Some implications of our estimate of \( \sigma^2 \) become apparent when we use it to calculate the range of values for \( \theta_t \) in which 95% of observations will occur. This range is 0.85 to 1.17. Thus, there is a 95% probability that output will be between 85% and 117% of mean production as a result of
variations in climatic conditions. This range does not seem to be inconsistent with the assumption used by Duloy and Woodland (1967) of a 20% reduction in agricultural production in a severe drought.

Another way of evaluating the contribution of the common disturbance $\theta_t$ to the model is to calculate the proportion of the variance of the total disturbance in the cost equation, $u_{it}$, due to the common component $\ln(\theta_t)$. The estimate of the variance of $\ln(\theta_t)$ is 0.0057 and the estimate of the variance of the total disturbance is $0.0057 + 0.5213 = 0.5270$. Thus, the proportion of variance in unit costs of an individual firm due to $\ln(\theta_t)$ is estimated to be only 0.011 or 1.1%. The conclusion is that, while the role of $\theta_t$ is statistically significant, its contribution to the disturbance for cost is swamped in size by the individual sector-specific effects. For a group of $n_{gt}$ firms the variance in the disturbance for unit costs is estimated to be $0.0057 + 0.5213/n_{gt}$. If, for example, there are $n_{gt} = 10$ firms in the group the proportion of variance due to the common component $\theta_t$ is estimated to be 0.0986, so $\theta_t$ contributes just under 10% of the total variance.

The conventional goodness-of-fit statistic, $R^2$, was calculated for each input share equation in each sector, and the results are reported in Table 5. Each $R^2$ value was calculated as one minus the ratio of the sample variance of the input shares to the sample variance of the residuals. Because the residuals do not necessarily sum to zero the range of $R^2$ is $[-\infty, 1]$. Bearing this in mind, the degree of variation in input shares provided by variations in relative input prices appears reasonably high.

<table>
<thead>
<tr>
<th></th>
<th>Merino Woolgrowing</th>
<th>Non-Merino Woolgrowing</th>
<th>Prime Lamb Producing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land</td>
<td>0.72</td>
<td>0.54</td>
<td>0.30</td>
</tr>
<tr>
<td>Capital</td>
<td>0.72</td>
<td>0.52</td>
<td>0.33</td>
</tr>
<tr>
<td>Livestock</td>
<td>0.79</td>
<td>0.65</td>
<td>0.77</td>
</tr>
<tr>
<td>Labour and other inputs</td>
<td>0.77</td>
<td>0.60</td>
<td>0.24</td>
</tr>
</tbody>
</table>

4.4 Estimates of $\theta_t$

Estimates of the random variables $\theta_t$ are presented in Table 6. Recall that the $\theta_t$ are random variables which give rise to production uncertainty, and they have a theoretical mean of one. They measure the effects of climatic and seasonal conditions on levels of production. Estimates of $\theta_t$ which are less than 1 imply that realised outputs are less than planned outputs. For example, $\bar{\theta}_6 = 0.869$ means that
<table>
<thead>
<tr>
<th>Year</th>
<th>t</th>
<th>$\theta_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952-53</td>
<td>1</td>
<td>1.046</td>
</tr>
<tr>
<td>1953-54</td>
<td>2</td>
<td>.934</td>
</tr>
<tr>
<td>1954-55</td>
<td>3</td>
<td>.932</td>
</tr>
<tr>
<td>1955-56</td>
<td>4</td>
<td>1.044</td>
</tr>
<tr>
<td>1956-57</td>
<td>5</td>
<td>1.042</td>
</tr>
<tr>
<td>1957-58</td>
<td>6</td>
<td>.869(a)</td>
</tr>
<tr>
<td>1958-59</td>
<td>7</td>
<td>.999</td>
</tr>
<tr>
<td>1959-60</td>
<td>8</td>
<td>1.056</td>
</tr>
<tr>
<td>1960-61</td>
<td>9</td>
<td>1.058</td>
</tr>
<tr>
<td>1961-62</td>
<td>10</td>
<td>1.061</td>
</tr>
<tr>
<td>1962-63</td>
<td>11</td>
<td>1.053</td>
</tr>
<tr>
<td>1964-65</td>
<td>12</td>
<td>.997</td>
</tr>
<tr>
<td>1965-66</td>
<td>13</td>
<td>.916</td>
</tr>
<tr>
<td>1966-67</td>
<td>14</td>
<td>1.022</td>
</tr>
<tr>
<td>1967-68</td>
<td>15</td>
<td>.877(b)</td>
</tr>
<tr>
<td>1968-69</td>
<td>16</td>
<td>1.054</td>
</tr>
<tr>
<td>1969-70</td>
<td>17</td>
<td>1.036</td>
</tr>
<tr>
<td>1970-71</td>
<td>18</td>
<td>.954</td>
</tr>
<tr>
<td>1971-72</td>
<td>19</td>
<td>1.120</td>
</tr>
<tr>
<td>1972-73</td>
<td>20</td>
<td>.946(c)</td>
</tr>
<tr>
<td>1973-74</td>
<td>21</td>
<td>1.014</td>
</tr>
<tr>
<td>1974-75</td>
<td>22</td>
<td>.961</td>
</tr>
<tr>
<td>1975-76</td>
<td>23</td>
<td>.991</td>
</tr>
</tbody>
</table>

(a) Central Australia and adjacent areas of Queensland, South Australia, Western Australia, New South Wales and Northern Australia were affected by prolonged drought to varying degrees of intensity over the years 1957-66.

(b) Severe drought in Victoria, southern N.S.W., South Australia and Tasmania.

(c) Severe drought in most of Victoria, western and central N.S.W., South Australia and north-eastern Tasmania.
realised outputs are estimated to be only 86.9% of planned outputs in 1957-58. The estimates presented in Table 6 are plausible in the sense that they vary about the theoretical mean of one, they appear not to be consistently increasing or decreasing over time, and very low estimates of $\theta_t$ (such as $\hat{\theta}_e$) coincide with major drought periods in the eastern states. Notice that the estimates of $\theta_t$ lie within the range 0.85 to 1.17 calculated in Section 4.3.

6. Conclusion

Two important characteristics of agriculture in general, and the Australian wool and lamb producing industries in particular, are that producers must choose input levels before output prices are known, and that the level of output associated with a particular level of inputs is uncertain. There are a number of theoretical models of agricultural production response which account for these two types of uncertainty. Until now, however, these theoretical models have not found an empirical application. Our empirical application, to the wool and lamb industries, was partly motivated by the high degree of variability in wool and sheepmeat production and prices revealed in studies such as that by Motha et al (1975).

The theory of producer behaviour under price and output uncertainty is relatively straightforward. Output is specified as a stochastic function of inputs, and since output price is unknown at the time the input decision is made, it is assumed that producers maximise the expected utility of profits. Importantly, if the production technology is multiplicatively separable in its deterministic and stochastic components, $q=f(x)\theta$, this paper has revealed that the expected utility maximisation problem implies cost minimisation of a special type. Specifically, firms must choose input levels in order to minimise the cost of producing a so-called planned output level, $q = q/\theta$.

The implications for empirical work are two-fold. First, the parameters of the production technology can be estimated without making any assumptions concerning the firms' underlying utility function. Second, an estimable form of the firms cost function will involve $\theta$, a random variable which gives rise to discrepancies between planned and realised outputs. Indeed, the presence of this random variable means that the disturbance term in the cost function is of an error components type. In this paper we confirmed that in the case of the wool and lamb industries the error components specification was appropriate.

The empirical results we present include the usual parameter and elasticity estimates which can be used to assess the effects of existing or proposed government input price policies. These estimates allow policy makers to assess, for example, the likely consequences of input specific subsidies on the pattern of factor demands. In addition, we estimate the changes in aggregate production levels which can be attributed to industry-wide climatic effects. To be specific, we estimate that there is a 95% probability
that output will be between 85% and 117% of mean production as a result of variations in climatic conditions. We have also estimated that in 1967-68, for example, output levels fell to 88% of mean production levels as a consequence of drought in the eastern states. Estimates of this kind are useful in determining appropriate levels of government intervention in the form of, for example, adjustment assistance and drought relief.

Some possible extensions of the model include a more detailed component structure for the error term in the cost equation. In this paper the error term comprised a time-varying component (θₜ) and an individual and time-varying component (e₄ₙt). One possibility is to allow the time-varying component to vary by time and sector (i.e. use θₖₜ instead of θₜ) or geographical location. Another is to introduce a separate random variable to represent sector-specific variations, so that the error term in the cost equation is then comprised of three components (e₄ₙt, θₜ, and ηₖ). Other possible extensions include the truncation of these error components to reflect the fact that there is an upper limit on the level of output associated with a particular level of factor inputs. Finally, variations on the model to account for the dynamics of input adjustments and for non-constant returns to scale may be useful.
Appendix A

Expected Utility Maximisation and Cost Minimisation

The producer maximisation problem is given by:

\[(1) \quad \max_{x \geq 0} E[U(pq - w'x)]\]

Equivalently:

\[(A.1) \quad \max_{x \geq 0} E[U(p0f(x) - w'x)]\]

The cost minimisation problem implied by this expected utility maximisation problem is given by:

\[(A.2) \quad \min_{x} \{w'x : f(x) \geq \bar{q}, x \geq 0\}\]

To see this, note that the Lagrangean for (A.2) is:

\[L = -w'x - \lambda(\bar{q} - f(x))\]

with vector of first order conditions:

\[w' + \lambda \frac{\partial f(x)}{\partial x_i} = 0\]

With an appropriate choice of \(\lambda\), this vector of first order conditions is identical to the vector of first order conditions associated with the maximisation problem (A.1). The appropriate \(\lambda\) is:

\[\lambda = \frac{E[U'(pq - w'x)p\theta]}{E[U'(pq - w'x)]]}\]
Appendix B

Data

The ASIS data was used to construct observations on q (output), C (total cost) and w (input prices) as follows:

Output

Firms in the data set are primarily engaged in the production of wool and lambs as joint products. At the same time however, it is not unusual for firms to derive income from other enterprises. To overcome difficulties in allocating inputs to different enterprises it is convenient to convert all enterprise outputs into a common measure, namely kilograms of wool equivalents. Kilograms of wool equivalents are obtained by dividing total wool and non-wool revenue by an estimate of the price of the composite output. It is assumed that wool and lambs are produced in fixed proportions so that a suitable estimate of the price of the composite output is given by:

\[ \hat{p} = \frac{p_{\text{wool}} + \hat{c}p_{\text{lamb}}}{1 + \hat{c}} \]

where \( \hat{p} \) is an estimate of \( p \),

\( \hat{c} \) is an estimate of the factor of proportionality which is assumed to differ from one type of firm to another,

and \( p_{\text{wool}} \) and \( p_{\text{lamb}} \) are the prices of wool and lambs.

For firms of a particular type the estimated factor of proportionality is calculated as the average ratio of wool produced to lambs marked for all firms of that type which participated in the ASIS surveys from 1952-53 to 1975-76. The estimates and their standard errors are presented in Table B.1 below.

<table>
<thead>
<tr>
<th>Type of Firm</th>
<th>Estimate of ( c )</th>
<th>Estimated Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merino woolgrowing</td>
<td>.047903</td>
<td>.00041</td>
</tr>
<tr>
<td>Non-merino woolgrowing</td>
<td>.083243</td>
<td>.00399</td>
</tr>
<tr>
<td>Prime lamb producing</td>
<td>.11428</td>
<td>.00437</td>
</tr>
</tbody>
</table>

Finally, total revenue is calculated as the sum of wool revenue and total non-wool revenue. Total non-wool revenue includes positive livestock trading gains and positive livestock operating gains. Livestock trading gains are simply net livestock sales revenues. Livestock operating gains are defined...
as the difference between opening and closing livestock numbers multiplied by closing prices. Positive livestock operating gains are included in total revenue to represent output which is not sold. This practice has previously been adopted by Lawrence and McKay (1980).

Total Cost

Firms in each sector use inputs which are grouped into one of four categories, namely land, physical capital, livestock and a group of other inputs including labour, equipment, supplies, services and overheads. The cost of each type of input is calculated by imputation or direct measurement or both. The sum of these costs is then used as a measure of total cost.

Land, physical capital (including water, fencing, yards, buildings and plant) and livestock are durable inputs which provide a flow of services into the production process over several years. The cost of these inputs is measured as the cost of this service flow and not as the value of the input stock. Lawrence and McKay (1980) suggest that the cost of the annual service flow from a durable input is comprised of three components, namely depreciation, the cost of maintenance and opportunity cost. This principle is applied to the measurement of land, capital and livestock costs as follows:

Land: Land is essentially an inexhaustible asset and is regarded as having zero depreciation. Regarding the maintenance of land, all expenses associated with maintenance (through the application of fertilisers, sound management and so on) are included in the cost of equipment and supplies. The remaining component of costs, opportunity cost, is in fact the only non-zero cost component associated with the land input. This opportunity cost is measured as the current market value of land multiplied by the yield on 2 year government securities.

Capital: The depreciation associated with items of physical capital is a separate item in the ASIS records. All expenses associated with the maintenance of physical capital are included in the cost of equipment and supplies. Finally, the opportunity cost is measured as the current market value of capital items multiplied by the yield on 2 year government securities.

Livestock: The depreciation of the livestock stock is zero under the assumption that the stock is properly maintained. Some maintenance costs are included in the cost of equipment and supplies, and these costs include the costs of drenches and labour inputs. Other maintenance costs must be attributed to the livestock input, and these costs include the cost of replacing culls and animals which die. These particular costs are measured as the absolute value of negative livestock trading gains (the net cost of livestock purchases) plus the absolute value of negative livestock operating gains (representing a running
down of stocks in order to produce output). Finally, the opportunity cost is again measured as the current market value of livestock multiplied by the yield on 2 year government securities.

Costs associated with the final group of inputs are measured directly. These costs are the costs of labour, equipment, services and overheads. The cost of labour includes wages paid to hired labour as well as the cost of imputed labour (excluding the operator). In addition, the cost of labour includes the costs of shearing, crutching and store classing as well as the cost of stores and rations. The cost of equipment and supplies includes the costs of seed, fodder, fuel, fertiliser, packs, bags, twine, drenches, dips, licks, vermin destruction and maintenance to plant and improvements. Finally, the cost of services and overheads includes rates and land taxes, insurance, rent, droving and agistment, freight, cartage and other miscellaneous service expenses.

*Input Prices*

A Laspeyres index of prices paid for the final group of inputs (labour, equipment and so on) is calculated regularly by ABARE. The list of inputs which are used to construct this index appears to be slightly but not significantly different from the list of inputs described above. To be specific, ABARE appears to include interest in the calculation of its index. However, because the weight attached to this input is relatively small, and because movements in the price of this input are broadly in line with movements in the prices of other included inputs, this small discrepancy is quite safely ignored.

The prices of the remaining inputs, land, capital and livestock, are not so readily available. Instead, an implicit price index for each input is obtained by dividing the cost of the input by a measure of the amount of the input consumed. Lawrence and McKay (1980) assume that the amount of the input consumed in a particular year is a constant proportion of the input stock in that year. Adopting this practice, the amount of the land input consumed is measured as average farm size. The amount of the capital input consumed is measured as the value of the capital stock divided by an ABARE index of prices paid for machinery items. Finally, the amount of the livestock input consumed is measured as opening livestock numbers measured in sheep equivalents. For this purpose, sheep equivalents are calculated as the number of sheep carried plus eight times the number of cattle carried. This is the conversion factor used by Anderson and Griffiths (1981).
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