AN ANALYSIS OF STRESS TESTING FOR ASIAN STOCK PORTFOLIOS

by

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Abstract

While extreme asset price movements are a common feature of the global financial system, recent financial crises have witnessed an increase in the use of serious stress testing in risk management. This paper examines the performance of a bivariate normal distribution model and a bivariate mixture of two normal distributions model in the institutional context of five Asian stock markets, namely Bangkok, Hong Kong, Seoul, Taipei and Tokyo. To assess the performance of the two models, the data from the five stock markets for the period 4 January 1990 to 28 February 1998 are employed. The results show that the bivariate normal distribution model outperforms the bivariate mixture of two normal distributions model. This seems to suggest that the latter model can more precisely capture the fat-tailed property of left and right tails in return distributions.

Key Words: Stress Testing; Bivariate Normal Distribution Model; Bivariate Mixture of Two Normal Distributions Model; Backtesting.

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An Analysis of Stress Testing for Asian Stock Portfolios

1. Introduction

The Asian financial crisis in 1997 has witnessed a renewed interest amongst scholars and practitioners alike in stress testing as an important risk management tool for asset portfolio assessment. As defined by International Organization of Securities Commissions (IOSCO 1995), stress tests apply particular ‘worst-case’ assumptions to a given portfolio to determine the effects of specific and severe adverse market movements on financial institutions, and to identify what potential losses would arise if the envisaged ‘worst-case’ scenario eventuated. The Basle Capital Accords of the Bank for International Settlement (BIS 1995, 1996) stipulate that financial institutions should use Value-at-Risk (VaR) models to calculate the potential risk on capital. It also requires these institutions should also perform stress tests as an additional dimension of risk management. Moreover, RiskMetrics (1999) has indicated that, employed in tandem, VaR and stress tests provide a ‘broader picture of risk’.

The BIS (1996) stresses two important complementary features of stress testing. Thus, while its quantitative characteristics enable analysts to identify plausible stress scenarios to which financial institutions could be exposed, its qualitative characteristics allow managers to evaluate the capacity of a financial institution to absorb capital large capital losses. Managers are thus able to identify those measures a financial institution can take to reduce its risk and to conserve its capital. The 1999
IOSCO report also underlines the unique functions of stress testing. This report argues that stress testing should be performed regardless of whether the institution uses VaR models, since stress tests quantify the extreme risks that may threaten the firm. Accordingly, regulatory advice from international supervisory organizations, as well as potentially serious losses contingent upon financial crises, have induced financial institutions to disclose both the methodology and/or the results of stress testing in their annual reports. Leading examples include Citibank, Chase, United Bank of Switzerland, Deutsche Bank and the Canadian Imperial Bank of Commerce\(^1\).

The need for stress testing is well documented. For example, Best (1998, 1999) argues that the primary purpose of risk management is to prevent a financial institution from suffering catastrophic losses, defined either as total institutional failure or as severe material damage to its competitive position. In procedural terms, the risk management function should report the estimated stress losses to senior management. This information can then be employed to design the long-term risk profile and determine the stress limits. Sound contingency plans should only be developed after such limits have been decided. However, traditional stress testing methods ignore the assessments of probabilities of extreme events, and thus their results may be misleading for risk management in financial firms. Furthermore, the

\(^1\) Please refer to the 1998 annual reports of these banks.
BIS (1999) holds that the performance of stress test systems solely based on historical or hypothetical scenario analysis is not satisfactory.

In this paper, the specifications for stress testing are compared for Kupiec’s (1998) multivariate normal distribution model and Kim and Finger’s (1999) bivariate mixture of two normal distributions model. To assess the comparative performance of these two models, data from the Tokyo, Seoul, Bangkok, Hong Kong and Taipei stock markets are used for the period 4 January 1990 to 28 February 1998.

The paper itself is divided into three main sections. In section II, we specify and compare Kupiec’s (1998) multivariate normal distribution model and Kim and Finger’s (1999) bivariate mixture of two normal distributions models from the perspective of stress testing. Section III discusses the empirical application of these models to data drawn from the five Asian stock markets. The paper ends with some brief concluding comments in section IV.

2 Methods and Model Specifications for Stress Testing

Traditional stress testing methods are intrinsically scenario-orientated: namely, the standard scenario approach, the historical scenario approach and the hypothetical scenario approach. In the first place, the standard scenario approach employs a set of conceivable situations, representing widely accepted specific extremal market
conditions, to evaluate the stress losses of specific portfolios. For example, the nine specific market movements defined by the Derivatives Policy Group (1995) constitute a standard scenario set. Breuer and Krenn (1999) point out that a regulatory authority can easily compare the possible historical extremal loss changes of some given institution, or alternatively, compare the differences of possible stress losses between various institutions at a given point of time, if they are provided with standard scenario stress test results by financial firms. However, since not all of the portfolios held by financial institutions are the same, this method cannot evaluate precisely the maximum losses that firms may actually face. Thus financial institutions should develop portfolio-specific standard scenario stress testing to comprehend the possible maximum losses contingent upon the composition of particular portfolios.

The historical scenario approach represents a second technique that financial institutions can use to conduct stress tests. This method employs historical market extremal changes to assess their effects on portfolios. For instance, a stock dealer may use information from the stock market ‘crisis’ of 1987 to measure the impact on its current portfolios. The data requirements of this technique are straightforward. Moreover, the method is uncontroversial: no senior management can ignore the likelihood of such scenarios. However, it may be not useful for newly developed
financial products for which no past data exists. Furthermore, historical extremal scenarios may be not the possible worst-case scenarios for a given portfolio (see, for example, Dunbar and Irving (1998), Breuer and Krenn (1999), and Blanco (1999)).

The final scenario-orientated approach is to hypothesize the worst-case conditions for particular portfolios (Breuer and Krenn 1999). This involves designing the scenarios by specifying possible extremal changes in risk factors, volatilities, correlations, etc., and then assessing the value changes of portfolios from these hypothetical scenarios. This method emphasizes the quality of risk measures (i.e., whether the scenario in question is possible or probable) and is thus dependent on the value judgments of risk analysts. However, this method may encounter the problem of ‘risk ignorance’ (Kimball 2000): Risk managers may overlook some important risk factors that can have a great influence on portfolios.

While the three basic methods described above all have strengths and weaknesses, they share the common problem of assigning probabilities to specific stress scenarios. Although both Breuer and Krenn (1999) and Best (1998) contend that this problem is essentially trivial, it has been argued elsewhere that the probabilities of particular market conditions provide vital information for risk management (Wang et al. 2001). Indeed, some recent studies have tried to estimate stress losses with associated probabilities, including Zangari (1997), Kupiec (1998), Berkowitz (1999)

2.1 Kupiec’s multivariate normal distribution model

A natural consideration is to extend the univariate normal distribution to a multivariate distribution when the stress tests are conducted for a multi-asset portfolio. However, this may result in overly complicated estimations and computations. Kupiec (1998) has developed a simplified multivariate normal distribution model. In this model, the stress tests can be performed using the characteristics of conditional multivariate normal distribution in the framework of VaR.

Assume that there are \( N \) assets in a portfolio, and that the first \( N - k \) are non-care and the remaining \( k \) are core assets. A partitioned vector will represent the return vector of the assets and is denoted as \( \tilde{R}_t = \begin{pmatrix} \tilde{R}_{1t} \\ \tilde{R}_{2t} \end{pmatrix} \), where \( \tilde{R}_{1t} \) is a \((N-k)\times1\) vector, and \( \tilde{R}_{2t} \) is a \( k \times 1 \) vector. The return vector follows a \( N \)-dimension normal distribution and is denoted as \( \tilde{R}_t \sim \mathcal{N} \left( \begin{pmatrix} \tilde{\mu}_{1t} \\ \tilde{\mu}_{2t} \end{pmatrix}, \begin{pmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} \\ \tilde{\Sigma}_{21} & \tilde{\Sigma}_{22} \end{pmatrix} \right) \). If the stress loss of core assets is defined as \( \tilde{R}_{2t} = R_2 = [r_1, r_2, \ldots, r_k] \), then the expected stress loss of portfolio is shown as:

\[ \mathbb{E} \left[ \tilde{R}_{2t} \right] = R_2 = [r_1, r_2, \ldots, r_k] \]

For details, please refer to Kupiec (1998) and Appendix 1 in this paper.
\[
W_{2i}R_2 + W_{1i}\left[ \mu_1 + \Sigma_{12}^{-1}(R_2 - \mu_2) \right] , \text{ where } W_{i,t} \text{ is the investment weight vector for } N-k \text{ non-care and } k \text{ core assets, } i=1,2. \text{ In the case of a two-asset portfolio, if } \tilde{r}_1 \text{ and } \tilde{r}_2 \text{ are assumed to be the returns of non-core and core assets, and if the stress loss of core assets is defined as } \tilde{r}_2 = r_2, \text{ then the expected stress losses of the portfolio can be shown as:}
\]

\[
W_{2i}r_2 + W_{1i}\left[ \mu_1 + \frac{\sigma_{12}}{\sigma_2^2}(r_2 - \mu_2) \right] = W_{2i}r_2 + W_{1i}\left[ \mu_1 + \frac{\sigma_1}{\sigma_2} \rho_{12} (r_2 - \mu_2) \right], \quad (1)
\]

where \( \rho_{12} \) is the unconditional correlation coefficient of \( \tilde{r}_1 \) and \( \tilde{r}_2 \).

### 2.2 Kim and Finger’s bivariate mixture of two normal distributions model

Kim and Finger (1999) considered the stress testing of a two-asset portfolio. If we assume that the returns on two different assets are \( x \) and \( y \) respectively, \( x \) is core asset and \( y \) is non-core asset, then a bivariate mixture of two normal distributions model can be written as:

\[
\begin{bmatrix} x \\ y \end{bmatrix} \sim N^{2}\left[ \begin{bmatrix} \mu_{x1} \\ \mu_{y1} \end{bmatrix}, \begin{bmatrix} \sigma_{x1}^2 & \frac{\sigma_{x1}\sigma_{y1}\rho_{x1y1}}{\sigma_{y1}} \\ \frac{\sigma_{x1}\sigma_{y1}\rho_{x1y1}}{\sigma_{y1}} & \sigma_{y1}^2 \end{bmatrix} \right], \quad \text{Pr } ob = w \quad \text{(quiet period)}
\]

\[
\sim N^{2}\left[ \begin{bmatrix} \mu_{x2} \\ \mu_{y2} \end{bmatrix}, \begin{bmatrix} \sigma_{x2}^2 & \frac{\sigma_{x2}\sigma_{y2}\rho_{x2y2}}{\sigma_{y2}} \\ \frac{\sigma_{x2}\sigma_{y2}\rho_{x2y2}}{\sigma_{y2}} & \sigma_{y2}^2 \end{bmatrix} \right], \quad \text{Pr } ob = 1 - w \quad \text{(hectic period)}
\]

\]

\[
(2)
\]

where 1 and 2 are denoted as the ‘quiet’ and ‘hectic’ periods respectively.

In a similar vein to the Kupiec (1998) model, if the stress loss of core assets is defined as \( x = \hat{x} \), then the expected stress losses of the portfolio can be
shown as:

\[ W_{x_1} \hat{x} + W_{y_1} [\mu_{y_2} + \frac{\sigma_{y_2}}{\sigma_{x_2}} (\hat{x} - \mu_{x_2})] = W_{x_1} \hat{x} + W_{y_1} [\mu_{x_2} + \frac{\sigma_{x_2}}{\sigma_{y_2}} \rho_{x_2 y_2} (\hat{x} - \mu_{x_2})], \quad (3) \]

However, in contrast to the Kupiec (1998) model, the mean, standard deviation and correlation coefficient parameters are estimated in the ‘hectic’ periods. To avoid complexity in estimating parameters, Kim and Finger (1999) estimate the parameters initially by the whole sample data of \( x \) and then weight the parameters by the conditional probabilities of \( x \) being derived from ‘hectic’ period\(^3\).

### 2.3 Investment strategies specifications and stress losses formula

To represent different investment strategies (such as long and short positions) in different markets, the stress losses of two-market portfolios estimated by these models can be classified into four main categories. The stress loss formulae for four categories of Kupiec’s (1998) model are summarized in Table 1. Moreover, for the sake of expositional clarity, we assume that the stress scenario for the core asset is set at \( \alpha = 0.002 \) (\( \alpha = 0.998 \)) only. Using the same methodology set out in Table 1, we can also derive the stress losses for the four categories for Kim and Finger’s (1999) model. The abbreviated formulae are

\(^3\) For details, please refer to Kim and Finger (1999) and also Appendix 2 to this paper.
shown in the Table 2. The stress scenario for core asset is once again calculated at $\alpha = 0.002$.

**Table 1. Summary for Stress Losses Calculation in Bivariate Kupiec’s (1998) Model**

<table>
<thead>
<tr>
<th>Core asset</th>
<th>Non-core asset</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long</td>
<td>Long</td>
<td>$W_2(\mu_2 + Z_\alpha \sigma_2) + W_1(\mu_1 + \rho_{12} \sigma_1 Z_\alpha)$</td>
</tr>
<tr>
<td>Long</td>
<td>Short</td>
<td>$\min{W_2(\mu_2 + Z_\alpha \sigma_2) +</td>
</tr>
<tr>
<td>Short</td>
<td>Long</td>
<td>$\min{-W_2(\mu_2 + Z_{1-\alpha} \sigma_2) +</td>
</tr>
<tr>
<td>Short</td>
<td>Short</td>
<td>$-W_2(\mu_2 + Z_{1-\alpha} \sigma_2) - W_1(\mu_1 + \rho_{12} \sigma_1 Z_{1-\alpha})$</td>
</tr>
</tbody>
</table>

Note: $\alpha = 0.002; Z_\alpha = -2.88$ and $Z_{1-\alpha} = 2.88$

**Table 2. Summary for Stress Losses Calculation in Bivariate Kim and Finger’s (1999) Model**

<table>
<thead>
<tr>
<th>Core asset</th>
<th>Non-core asset</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long</td>
<td>Long</td>
<td>$W_2(\mu_{x2} + Z_\alpha \sigma_{x2}) + W_1(\mu_{y2} + \rho_{x2,y2} \sigma_{y2} Z_\alpha)$</td>
</tr>
<tr>
<td>Long</td>
<td>Short</td>
<td>$\min{W_2(\mu_{x2} + Z_\alpha \sigma_{x2}) +</td>
</tr>
<tr>
<td>Short</td>
<td>Long</td>
<td>$\min{-W_2(\mu_{x2} + Z_{1-\alpha} \sigma_{x2}) +</td>
</tr>
<tr>
<td>Short</td>
<td>Short</td>
<td>$-W_2(\mu_{x2} + Z_{1-\alpha} \sigma_{x2}) - W_1(\mu_{y2} + \rho_{x2,y2} \sigma_{y2} Z_{1-\alpha})$</td>
</tr>
</tbody>
</table>

Note: $\alpha = 0.002; Z_\alpha = -2.88$ and $Z_{1-\alpha} = 2.88$

### 3 Empirical Evidence

Five Asian stock markets, Bangkok, Hong Kong, Seoul, Taipei and Tokyo, are considered in the empirical study. By way of historical background, all of these stock
exchanges represent are emerging markets, with the sole exception of Tokyo. In the 1990s, the Bangkok, Hong Kong, Seoul and Taipei stock markets all thrived on the basis of their outstanding national economic growth rates. Furthermore, the regulatory authorities governing these emerging markets were eager to deregulate in order to attract the foreign capital inflows and to develop their financial markets as regional financial centers. However, these markets were all severely ‘shocked’ by the financial crisis in 1997 and the subsequent Asian contagion. This emphasizes the crucial need for international financial asset management institutions that specialize in investing in Asian equity markets to take a much more considered view of the risk of extremal events.

3.1 Data and descriptive statistics

The empirical data employed in this paper are the daily closed indices of five markets from 4 January, 1990 to 28 February, 1998. These indices include the SET Index (Bangkok (BK)), Heng Seng Index (Hong Kong (HK)), Seoul Securities Exchange Index (Seoul (SL)), TSEC Index (Taipei (TP)) and Nikkei 225 Index (Tokyo (TK)). Since trading days are different for all markets, the sample sizes of returns are 2024, 2022, 2043, 2323 and 2010 respectively. Moreover, because the empirical estimations will consider only the case of two-asset portfolios, in order to
to get more consistent results, the data were trimmed to keep a common number of trading days for all five markets. This procedure reduces the original sample sizes to 1709 observations for all five markets. The standardized indices of the five markets are shown in Figure 1, where the five indices are all 100 on 4 January, 1990.

**PLEASE INSERT FIGURE 1 HERE**

To understand the data structure primarily, the descriptive statistics of daily returns are summarized in Table 3. Table 3 shows that the means of daily return series are all near zero, and the standard deviations of five market returns are not apparently different and are all in the range of 0.72% and 1.05%. However, the skewness and kurtosis are very different among the five markets. The skewness of Hong Kong is negative, but for the other four markets it is slightly positive. Furthermore, the kurtosis of the five markets is higher than 3.0, the value held by a normal distribution. The information given in Table 3 indicates that most of empirical distributions are centralized at zero, but may have quite different types of peaks and tails in the respective markets.

Table 3 also presents the sample means plus (minus) three times the standard deviations, 1 and 99 percentiles, and historical maxima and minima. If the data structure is judged from the relative positions of means plus (minus) three times
standard deviations, and 1 and 99 percentiles, then it is possible to conclude that the distributions should be symmetrical. However, when these statistics are compared with the historical extremes, then the differences are quite apparent. This implies that the behavior of the tails (i.e., the returns in the extremal market conditions) may be different from the normal distributions.

Table 3. The Descriptive Statistics of Five Asian Stock Markets

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Tokyo</th>
<th>Seoul</th>
<th>Bangkok</th>
<th>Hong Kong</th>
<th>Taipei</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (A)</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>0.0003</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Standard deviation (B)</td>
<td>0.0072</td>
<td>0.0082</td>
<td>0.0093</td>
<td>0.0078</td>
<td>0.0105</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.3450</td>
<td>0.2678</td>
<td>1.5034</td>
<td>-0.0156</td>
<td>0.1117</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.5012</td>
<td>8.2845</td>
<td>19.2339</td>
<td>15.6098</td>
<td>4.8891</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0340</td>
<td>-0.0631</td>
<td>-0.0445</td>
<td>-0.0661</td>
<td>-0.0633</td>
</tr>
<tr>
<td>A-3B</td>
<td>-0.0217</td>
<td>-0.0248</td>
<td>-0.0280</td>
<td>-0.0231</td>
<td>-0.0345</td>
</tr>
<tr>
<td>1% fractile</td>
<td>-0.0186</td>
<td>-0.0231</td>
<td>-0.0259</td>
<td>-0.0230</td>
<td>-0.0299</td>
</tr>
<tr>
<td>99 fractile</td>
<td>0.0206</td>
<td>0.0245</td>
<td>0.0257</td>
<td>0.0200</td>
<td>0.0278</td>
</tr>
<tr>
<td>A+3B</td>
<td>0.0212</td>
<td>0.0248</td>
<td>0.0277</td>
<td>0.0237</td>
<td>0.0313</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0526</td>
<td>0.0510</td>
<td>0.1134</td>
<td>0.0731</td>
<td>0.0647</td>
</tr>
</tbody>
</table>

Note: The sample period is from Jan. 1990 to Feb. 1998.

3.2 Correlation coefficients

The parameters of two-asset portfolios will be estimated on the basis of equation (1) and equation (3), and the stress losses calculated in terms of the formulae in Table 1 and Table 2. Correlation coefficients play a key role in the
calculation of stress losses. The historical correlation coefficients will be used in Kupiec’s (1998) model. They will also be employed by weighting the conditional probabilities of core asset being in ‘hectic’ periods in Kim and Finger’s (1999) model.

The historical correlation matrix calculated by means of the full sample data is shown in Table 4. It is immediately evident that the correlation coefficients between any two markets are not very high. The specific cases of Hong King and Seoul, and Hong Kong and Bangkok are used to illustrate the estimations of aggregated stress losses of two-asset portfolios since these markets were influenced most deeply by the Asian financial crisis in 1997. Furthermore, we shall assume that the Hong Kong market position is the core asset (and thus carries a higher investment weight), and the other two markets positions (Seoul and Bangkok) are the non-core assets (and thus attract lower investment weights).

Table 4. The Linear Correlation Coefficients of Returns among Five Asian Stock Markets

<table>
<thead>
<tr>
<th></th>
<th>Tokyo</th>
<th>Seoul</th>
<th>Bangkok</th>
<th>Hong Kong</th>
<th>Taipei</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tokyo</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seoul</td>
<td>0.1084</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bangkok</td>
<td>0.1650</td>
<td>0.2252</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hong Kong</td>
<td>0.3052</td>
<td>0.1713</td>
<td>0.3737</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Taipei</td>
<td>0.1870</td>
<td>0.1194</td>
<td>0.1785</td>
<td>0.1931</td>
<td>1</td>
</tr>
</tbody>
</table>

3.3 Backtesting
In order to check the accuracy of the models considered in this paper, we performed backtests using the method developed by Kupiec (1995). Let $P$ be the ‘theoretical failure rate’, which is defined as the proportion of the total of estimated stress losses being exceeded in a given sample. And let $N / T$ be the ‘observed failure rate’, which is defined as the number of observations ($N$) in which estimated stress loss is exceeded, over the total observed data ($T$). Kupiec (1995) developed a likelihood ratio test with the test statistic being:

$$LR = -2 \log[(1 - p)^{T-N} p^N] + 2 \log[(1 - (N/T))^{T-N} (N/T)^N].$$

(4)

which is a distributed chi-square with one degree of freedom. If $LR$ is significant, then the accuracy of our model is rejected. Conversely, one can also define the non-rejection regions of $N$ such that the accuracy of our model will not be rejected. Since the sample size of our return data is 1708 ($T=1708$), the non-rejected regions of alternative ‘theoretical failure rates’ at significance levels, can be set at 0.01, 0.05 and 0.10 respectively. These non-rejected regions are summarized in Table 5. For instance, if $N = 0.005$ and significance level=0.05, the non-rejection region is $3 < N < 15$. 

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Table 5. Non-rejection regions for number of failures (N) (Sample = 1708)

<table>
<thead>
<tr>
<th>Critical probabilities</th>
<th>Significant level = 0.01</th>
<th>Significant level = 0.05</th>
<th>Significant level = 0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0100</td>
<td>0.9900</td>
<td>7 &lt; N &lt; 29</td>
<td>9 &lt; N &lt; 26</td>
</tr>
<tr>
<td>0.0075</td>
<td>0.9925</td>
<td>4 &lt; N &lt; 24</td>
<td>6 &lt; N &lt; 21</td>
</tr>
<tr>
<td>0.0050</td>
<td>0.9950</td>
<td>2 &lt; N &lt; 18</td>
<td>3 &lt; N &lt; 15</td>
</tr>
<tr>
<td>0.0025</td>
<td>0.9975</td>
<td>N &lt; 11</td>
<td>N &lt; 9</td>
</tr>
<tr>
<td>0.0020</td>
<td>0.9980</td>
<td>N &lt; 10</td>
<td>N &lt; 8</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.9985</td>
<td>N &lt; 8</td>
<td>N &lt; 7</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.9990</td>
<td>N &lt; 7</td>
<td>N &lt; 5</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.9995</td>
<td>N &lt; 5</td>
<td>N &lt; 4</td>
</tr>
</tbody>
</table>

3.4 Empirical results

The estimated results of Kupiec’s (1998) and Kim and Finger’s (1999) models are presented in Table 6 and Table 7 respectively. The estimates in Table 6 show that the stress losses fall in the range of 0.93% and 2.15%. The absolute numbers of the data sample exceeding estimated stress losses (shown in the corresponding parentheses) indicate that the predictive accuracy of Kupiec’s (1998) model can be rejected, if we use the information provided in the Table 5.

Table 6. Stress Losses Estimations of Two-asset Portfolios with Different Investment Weights in Bivariate Kupiec’s model (α = 0.002)

<table>
<thead>
<tr>
<th>Stock markets</th>
<th>Weights (HK: Others)</th>
<th>60:40</th>
<th>75:25</th>
<th>90:10</th>
<th>Hong Kong</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Long</td>
<td>Short</td>
<td>Long</td>
<td>Short</td>
</tr>
<tr>
<td>Seoul Long</td>
<td></td>
<td>0.0149</td>
<td>0.0120</td>
<td>0.0176</td>
<td>0.0161</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25)</td>
<td>(28)</td>
<td>(22)</td>
<td>(15)</td>
</tr>
<tr>
<td>Seoul Short</td>
<td></td>
<td>0.0117</td>
<td>0.0153</td>
<td>0.0156</td>
<td>0.0181</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(36)</td>
<td>(27)</td>
<td>(22)</td>
<td>(19)</td>
</tr>
</tbody>
</table>
The results in Table 7 demonstrate that the stress losses are in the range of 3.06% and 5.78%. The numbers of sample exceeding estimated stress losses (shown in the corresponding parentheses) suggest that the predictive accuracy of Kim and Finger’s (1999) model should not be rejected, and that all of these numbers are all less than 2. Accordingly, we can conclude that the Kim and Finger (1999) model is much more precise than the Kupiec’s (1998) model. A common characteristic of both models is that the estimated stress losses are symmetrical when the investment strategy is either one-way (i.e., long or short positions in both markets) or mixed (i.e., a long position in one market and a short position in the other market).

However, the estimated stress losses from former model are insufficient to cover the extremal losses. Put differently, the estimated stress losses from the Kim and Finger (1999) model can satisfy the requirements of backtests.

Table 7. Stress Losses Estimations of Two-asset Portfolios with Different Investment Weights in Bivariate Kim and Finger’s model ($\alpha = 0.002$)

<table>
<thead>
<tr>
<th>Stock markets</th>
<th>60:40</th>
<th>75:25</th>
<th>90:10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hong Kong</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Investment strategies</strong></td>
<td>Long</td>
<td>Short</td>
<td>Long</td>
</tr>
<tr>
<td>Seoul</td>
<td>Long</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0402 (1)</td>
<td>0.0344 (1)</td>
<td>0.0485 (1)</td>
</tr>
</tbody>
</table>
Concluding Remarks

While the importance of stress testing is widely recognized in the literature, traditional stress testing methods ignore the assessments of probabilities of extreme events and the results may thus mislead risk management decision-making. Moreover, little empirical work has been done on the performance and accuracy of existing models for stress testing.

This paper compares the Kupiec (1998) and Kim and Finger (1999) models for stress testing under the bivariate normal distribution model, and bivariate mixture of two normal distributions model in two-asset positions. These models appear to be theoretically sound: they can also estimate the stress losses and associated probabilities simultaneously. In principle, the Kupiec (1998) model shares at least some of the operational advantages of the Kim and Finger (1999) model, but it may suffer from some problems associated with fat-tailed return distributions.

To assess the performance of these models, data from the five Asian stock markets were employed. The results of backtesting estimated stress losses show that

<table>
<thead>
<tr>
<th></th>
<th>Short</th>
<th>Long</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bangkok</td>
<td>0.0345 (1)</td>
<td>0.0441 (1)</td>
</tr>
<tr>
<td></td>
<td>0.0403 (2)</td>
<td>0.0367 (1)</td>
</tr>
<tr>
<td></td>
<td>0.0449 (0)</td>
<td>0.0509 (1)</td>
</tr>
<tr>
<td></td>
<td>0.0485 (1)</td>
<td>0.0462 (1)</td>
</tr>
<tr>
<td></td>
<td>0.0553 (1)</td>
<td>0.0577 (1)</td>
</tr>
<tr>
<td></td>
<td>0.0567 (2)</td>
<td>0.0558 (1)</td>
</tr>
<tr>
<td>Short</td>
<td>0.0306 (0)</td>
<td>0.0443 (1)</td>
</tr>
<tr>
<td></td>
<td>0.0425 (1)</td>
<td>0.0510 (2)</td>
</tr>
<tr>
<td></td>
<td>0.0543 (1)</td>
<td>0.0578 (2)</td>
</tr>
</tbody>
</table>

Note: the number of sample exceeding estimated stress losses are in the corresponding parentheses.
the Kim and Finger (1999) model is preferred for performing stress tests. Our results imply that Kim and Finger (1999) model possess superior precision in capturing the fat-tailed nature of emergent market return distributions, and in determining the extremal correlation between assets in stressful market conditions. These empirical results may be useful to risk managers who are interested in Asian stock markets.
References


Appendix 1

Since \( \vec{R}_t \sim N^h \left( \begin{pmatrix} \vec{\mu}_t \\ \vec{\Sigma}_{22} \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \right) \), the return of the portfolio can be calculated as

\[
\Delta \vec{V}_t = W_{2t} \vec{R}_2 + W_{lt} \vec{R}_{lt}.
\]

If the stress scenario of core assets is defined as \( \vec{R}_{2t} = R_2 = [r_1, r_2, ..., r_k] \), then

\[
\Delta \vec{V}_t = W_{2t} R_2 + W_{lt} \vec{R}_{lt}.
\]

The distribution of \( \vec{R}_{lt} \) conditioned on the \( \vec{R}_{2t} = R_2 = [r_1, r_2, ..., r_k] \) can be derived as

\[
\vec{R}_{lt} \mid \vec{R}_{2t} = R_2 \sim N(\mu_c, \Sigma_c),
\]

where \( \mu_c = \mu_1 + \Sigma_{12}^{-1}(R_2 - \mu_2) \), and \( \Sigma_c = \Sigma_{11} - \Sigma_{12}^{-1}\Sigma_{22}\Sigma_{21} \).

Therefore, the return to this portfolio can be written as

\[
\Delta \vec{V}_t \mid \vec{R}_{2t} = R_2 = W_{2t} R_2 + W_{lt} \vec{R}_{lt} \mid \vec{R}_{2t} = R_2,
\]

and their expected value and variance are

\[
E(\Delta \vec{V}_t \mid \vec{R}_{2t} = R_2) = W_{2t} R_2 + W_{lt} \mu_c = W_{2t} R_2 + W_{lt} \left( \mu_1 + \Sigma_{12}^{-1}(R_2 - \mu_2) \right),
\]

and

\[
V(\Delta \vec{V}_t \mid \vec{R}_{2t} = R_2) = W_{lt} (\Sigma_{11} - \Sigma_{12}^{-1}\Sigma_{22}) W_{lt}^T.
\]

The estimated stress loss of the portfolio can be computed like the VaR:

\[
\text{StressVaR}(95\%) = \frac{E(\Delta \vec{V}_t \mid \vec{R}_{2t} = R_2) - Z_{0.95} \sqrt{V(\Delta \vec{V}_t \mid \vec{R}_{2t} = R_2)}}{W_{2t} R_2 + W_{lt} (\Sigma_{12}^{-1}\Sigma_{22} R_2) - 1.65 \sqrt{W_{lt} (\Sigma_{11} - \Sigma_{12}^{-1}\Sigma_{22}) W_{lt}^T}}.
\]

So the expected value of stress loss is

\[
E[\text{StressVaR}(95\%)] = E(\Delta \vec{V}_t \mid \vec{R}_{2t} = R_2) = W_{2t} R_2 + W_{lt} \left( \mu_1 + \Sigma_{12}^{-1}(R_2 - \mu_2) \right).
\]
Appendix 2

Since the marginal distribution $x$ can be shown as $x \sim N(\mu_{x_1}, \sigma_{x_1}) \text{ with prob } w \text{ (quiet period)}$ $\sim N(\mu_{x_2}, \sigma_{x_2}) \text{ with prob } 1 - w \text{ (hectic period)}$ the conditional probabilities of $x$ being from ‘hectic’ periods can be calculated by $\alpha_i$ using the Bayesian theorem:

$$\alpha_i = \frac{(1 - w)f(x_i | \mu_{x_2}, \sigma_{x_2})}{(1 - w)f(x_i | \mu_{x_2}, \sigma_{x_2}) + (w)f(x_i | \mu_{x_1}, \sigma_{x_1})}.$$ 

The $\alpha_i$ can be used as weight factors for estimating the parameters in Kim and Finger’s (1999) model (equation 3). Thus,

$$\mu_{x_2} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i x_i, \quad \mu_{y_2} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i y_i, \quad \sigma^2_{x_2} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i (x_i - \mu_{x_2})^2,$$

$$\sigma^2_{x_2} = \frac{1}{n} \sum_{i=1}^{n} \alpha_i (x_i - \mu_{x_2})^2, \text{ and } \rho_{x_2y_2} = \frac{\sum_{i=1}^{n} \alpha_i (x_i - \mu_{x_2})(y_i - \mu_{y_2})}{\sigma_{x_2} \sigma_{y_2} \sum_{i=1}^{n} \alpha_i}.$$
Figure 1. The Standardized Indices of Five Asian Stock Markets