Towards a New Patent System

Samuel Meng

Abstract

The need to balance the positive and negative effects of patent monopoly is at the heart of design and reform of the patent system. However, by scrutinizing prior studies on optimal patent design, this paper has found that the basis for this balanced approach is flawed. Furthermore, a macroeconomic analysis of the patent system confirms that there is no trade-off in patent protection, unless the patent system generates only marginal negative effects. Based on this analysis, the paper suggests an alternative patent system design which is able to stimulate innovation directly while minimizing deadweight loss.

Key Words: patent law, optimal patent design, patent monopoly, social benefit, deadweight loss

JEL Code: K11, K12, K23

1. Introduction

The patent system can be traced back to medieval guild practices in Europe. To raise revenue, medieval European monarchs frequently sold certain of their privileges, for example monopoly over trade in specified commodities. The first formal patent law was enacted by Venetian Senate in 1474, which stated: ‘we have among us men of great genius, apt to invent and discover ingenious devices (…). Now, if provision were made for the works and devices discovered by such persons, so that other who may see them could not build them and take the inventor’s honour away, more men
would then apply their genius, would discover, and would build devices of great utility to our commonwealth’ (cited in Kaufer, 1989). However, the principle that monopoly should be granted only for innovators was first laid down by Francis Bacon in 1602, and was adopted by the British Parliament as the Statute of Monopolies in 1623. Section Six of the Statute authorized ‘letters patent and grants of privilege for the term of fourteen years or under, hereinafter to be made, of the sole working or making of any manner of new manufactures within this Realm, to the true and first inventor of such manufactures, which others at the time of making such letters patent shall not use.’ The patent system was first internationalized in 1883, when 11 countries ratified the ‘Paris convention of the international union for the protection of industrial property’ (The ‘Paris Convention’). The Paris Convention has subsequently been amended on numerous occasions, and now has approximately 175 member countries. In the mean time, other international conventions or treaties have been established. Notable examples include the Patent Co-operation Treaty of 1970, the European Patent Convention of 1973, and the Community Patent Convention of 1975.

While there has been considerable progress in the internationalization and broadening of the scope of the patent system, relatively little has been achieved with regard to its reform. Nonetheless, of those that have been undertaken in the last three decades, the United States (U.S.) has been at the forefront in accomplishing such reform. Major changes have included prolonging the duration of patents, as well as extending patent rights to new subject areas, such as computer programming, genetically engineered life forms, and business methods. The most recent patent reform in the U.S., the America Invents Act of 2011, has made three notable achievements possible. The first of these has been to change the ‘first-to-invent’ to ‘first-to-file’. Second, a post-grant-review process has been created to allow challenges to a patent within nine
months of its issuance. Third, the U.S. Patent Office has been granted the right to set patent registration fees. Currently there are different versions of the patent laws in different countries. It should also be noted that while many reforms of patent laws have been made worldwide, Section Six of the Statute of Monopolies enacted in Britain in 1623 remains the foundation of patent laws around the world.

Despite many patent law reforms undertaken worldwide, the effectiveness of the modern patent system has been cast in doubt due to many problems associated with the system, including poor patent quality, low usage of patent technology, patent trolls, and most notably, the impediment of incremental innovations. Of these problems, incremental innovation, or innovation based on prior innovations, came to the attention of Scotchmer (1991), and is now a substantial concern in the U.S. Scotchmer (1991) identified the cumulative nature of research as the reason that patent law may hinder innovation. To address this problem, Scotchmer considered it necessary to reward early (junior) innovators fully for the technological foundation they provide to later (senior) innovators, but also to reward later innovators adequately for their improvements to existing innovation, as well as for the new products they develop. Scotchmer considered the ‘nature’ system of property rights unable to solve this problem. He called for prior agreements among research firms, in the form of joint research ventures. Green and Scotchmer (1995) further argued that, due to the difficulties associated with dividing profit, patent lives need to be long enough so that one can assume that the whole sequence of innovations occurs in a single firm. Gallini (2002) suggested that, in the case of sequential research, an enhanced ability to enforce patents may impede rather than promote innovation.

The problems associated with the patent system are largely due to the monopolistic
nature of patent rights, which has caused long standing controversy about patents. Those proposing stronger patent protection believe that such protection is necessary to encourage invention. Others argue that the patent system should be abolished, claiming that the monopoly power provided by the patent system to patent holders not only hinders economic growth but also obstructs incremental innovation. However, the most common argument is in favour of a balanced approach, where a weak patent system is able to strike a balance between the positive and negative effects of patent rights. In other words, there exist optimal patent length and breadth which maximize net social benefit. This balanced approach is based on trade-off (or optimal protection) theory. The purpose of this paper is to show that those studies underpinning trade-off theory are flawed and to present a new approach to patent system reform.

The remainder of this paper is organized as follows. In section 2, the research on optimal patent protection is reviewed, and flaws in this research identified. Using a macroeconomic approach, section 3 proves that, generally speaking, there is no trade-off between stimulating innovation and preventing the abuse of patent monopoly. Section 4 suggests a new patent system capable of providing maximum protection for innovation while limiting the negative impact of patent monopoly on economic growth. In section 5, the paper is summarized and concluding remarks offered.

2. The research on optimal patent protection

In order to mitigate the negative effect (deadweight loss), substantial research has been devoted to identify the optimal design that a patent system might take. Specifically, this involves using appropriate length and breadth of patent rights to maximize net social welfare. Notable studies on optimal patent design include Nordhaus (1967, 1969, 1972), Scherer (1972), Tandon (1982), Gilbert and Shapiro
(1990), Klemperer (1990), Gallini (1992), and Scotchmer (1991, 1996). Due to space limitations, only those studies directly relevant to this paper are reviewed.

Nordhaus (1967) is regarded as the pioneering researcher with regard to optimal patent systems, and his approach has largely been accepted and followed by subsequent researchers. It is therefore important to review his research in depth. Nordhaus extended the model developed by Arrow (1962) to investigate optimal patent duration in the context of small (run-of-the-mill) and drastic innovations. The basic logic of Nordhaus’ approach was as follows: patent royalty \( r \) is tied to the amount of research embodied in the patented innovation \( R \), i.e. the invention possibility function \( r=B(R) \). Extending this function, one can express the innovating firm’s profit maximization condition as a function of patent duration \( T \) as well as the amount of research \( R \). Using this profit maximization condition as a constraint to maximize the net social welfare of a patented innovation, one can obtain the optimal length of patent protection. Assuming a linear demand function, Nordhaus derived a finite optimal patent length for the run-of-the-mill innovation, a slightly complicated solution for the drastic innovation, and a more complicated solution for product innovations. The solution for the run-of-the-mill case is reproduced as follows:

\[
\phi = (dB + 1) / [dB(1 + \sigma/2) + 1]
\]

where

\[
\phi = 1 - e^{-\rho T}, \quad \sigma = -B''B/B^2.
\]

\( T \) – the length of patent protection,
\( \rho \) – the constant rate of discount,
\( d \) – demand elasticity (in absolute value), and
\( B \) – cost reduction as a result of innovation. \( B \) is a function of the amount of research
for the innovation R, i.e. $B=B(R)$.

B’ and B”’ are the first and second derivatives of B, respectively

Since it is assumed that $B>0$, $B'>0$ and $B''<0$, it must be $\sigma>0$. The elasticity d is measured as an absolute value, so $d>0$. As a result, Nordhaus was able to conclude that $0<\phi<1$, necessitating a finite optimal patent length $T=-(1/\rho)\log(1-\phi)$.

Nordhaus’ research appears to provide sound mathematic proof that there exists a finite optimal patent length, at least for the run-of-the-mill case. However, his model has three major defects. First, the firm’s profit maximization condition in his model implied that the firm’s decision on investment in innovation research (sR) is dependent on the return to innovation ($B(R)$). This is hardly true. The return on innovation is unknown at the time of planning innovation research so the firm cannot make a profit maximization decision according to an unknown return. One may argue that the firm can make a decision based on the ‘projected’ return to innovation, but the projections on return to innovation are frequently proved to be remarkably wrong. For successful innovations, the projections have been vastly below the actual value, e.g. Facebook, Google, Microsoft and Yahoo. For other innovations, the estimation tends to be too high. For example, the Concorde overestimated the demand for superfast air flight and thus markedly overestimated the return to supersonic jets. So the firm’s profit maximization equation in Nordhaus’ model is not true in reality. On the other hand, the important effect of return to innovation on inspiring or stimulating future innovation research is not reflected in Nordhaus’ model since his model is a closed system – only one innovation is considered in this system.

Second, Nordhaus set royalty equal to the return to innovation $B(R)$ (or ‘invention possibility function’ in Nordhaus’ words) and assume $B(R)$ increases with the amount
of research activity R at a decreasing rate. There is no theoretical base that the size of royalty is related to the amount of research activity. In reality, the size of royalty is more likely to be related to the monopoly power of an innovation or the elasticity of demand for an innovation, thus setting \( r = B(R) \) is problematic in the first place. On the return to innovation, although the assumption of ‘decreasing return to innovation research’ is reasonable at the macro level, it is not true in the micro setting of Nordaus’ model. A macro-level measurement of return to innovation includes all kinds of innovation research – failed and successful. For failed research, the return is zero, so it is obviously not related to the amount of research done at all. If the failed research is excluded, the return to the rest of innovation research could be positively related to the amount of research at either a constant or even an increasing rate, given the high failure rate in innovation research. Moreover, it is possible that an innovation with lower cost has higher return and vice verse, so at the micro level the return to innovation could be positively or negatively related to the amount of research.

Finally, even if all the assumptions in Nordhaus’ model are correct, his conclusion that ‘Since \( \sigma \) is positive, we know that the optimal life (of a patent) is a finite, positive period’ (Nordhaus, 1967, p9) is not correct since he ignored ‘the horrible second-order condition’ (Nordhaus, 1967, p7) for the constrained maximization problem. Using Nordhaus’ social welfare function, we can derive the following second-order condition (see Appendix for detailed derivation).

\[
\varphi < 1 + \frac{BB''}{(4B'^2 + 5BB'^2 + B^2B''d)}, \text{ or }
\]

\[
T < \frac{-BB''}{(4B'^2 + 5BB'^2 + B^2B''d)}.
\]

This second order condition means that the limited duration of patent protection is a prerequisite of the true optimal length of patent protection provided by Nordhaus. It is
therefore not surprising that Nordhaus found a finite optimal length of patent. If assigning some numbers in the second order condition and in Nordhaus’ solution, one can grasp this issue more readily. Following Nordhaus (1967), the author assumes an innovation response function \( B = R^\alpha \), and use the numbers \( \alpha = 0.1 \), or \( \sigma = 9 \), suggested by Nordhaus. For convenience of calculation, the author further assign \( d = 0.5 \), \( \rho = 0.1 \), and \( R = 1 \). The second order condition requires \( 1 - \phi > \frac{(-0.09)}{0.02} = 4.50 \), or \( T < -10 \ln 4.5 = -15.04 < 0 \). This suggests a requirement of negative length of patent protection for an optimal solution. However, the Nordhaus solution suggests an optimal length of patent protection being \( 1 - \phi = 1 - 1.5/3.75 = 0.6 \), or \( T = -10 \ln (1 - \phi) = -10 \ln 0.6 = 5.12 \). Apparently, this solution is not a true optimal patent duration because it does not satisfy the second order condition. Believing that his results were valid, Nordhaus was in great difficulty in explaining the counterintuitive implication of his results – the optimal length of a patent for more important innovations should be shorter.¹ There were other implausible implications arising from the Nordhaus model. Some of them were identified by Domar and Stigliz (1969). Since the optimal length derived by Nordhaus is false, the belief that there is a trade-off point in the length of patent protection cannot be supported.

Patent protection has two dimensions, length and breadth, so there may be a tradeoff between these dimensions. From the early 1990s a number of researchers have set out to identify this tradeoff. Gilbert and Shapiro (1990) defined patent breadth as the flow rate of profit available to the patentee, or the ability of the patentee to raise price. Using this concept they maximized social welfare subject to the optimal reward

¹ Nordhaus admitted that it was almost universally agreed that more important inventions should have longer patent lives because their development poses a higher risk to the inventor and those funding their work, and requires longer periods of development. To explain his implausible results he overlooks the greater social welfare of more important inventions, and exaggerates its deadweight loss by saying ‘in general more important inventions involve larger second order effects and thus should have shorter lives’.
available to the patentee. They concluded that the optimal patent was of an infinite length, with patent breadth adjusted to provide the required reward for innovation. In other words, a long and narrow patent is preferred. Klemperer (1990) defined patent breadth as the distance between the patented product and the products in a product space that competing firms can sell without infringing the patent. By minimizing the social cost per dollar of profit to the patentee (or the ratio of social cost to the patentee’s profit), Klemperer concluded that when all consumers have identical transport costs, infinite length narrow patents are optimal. However, when all consumers have identical reservation prices for the most-preferred product variety, a short-lived patent with infinite width is optimal. Gallini (1992) links patent length and breadth to imitation costs. She defined patent breadth as the flow of profits earned by the innovator during patent life, which was similar to Klemperer (1990). Imitation cost is assumed to be positively related to patent breadth, while incentive to imitate is determined by the return to the imitator during the length of patent protection. Minimizing the discounted dead-weight loss plus profits lost to imitation, while letting the return to the innovator equal imitation cost, Gallini claimed that the optimal patent policy consists of broad patents with patent lengths adjusted to achieve the desired reward to the patentee. Denicolo (1996) studied optimal patent length and breadth in the context of many firms racing to establish a patent. He defined patent breadth as the fraction of the cost reduction that does not spill out as freely available technology to non-innovating firms. Assuming that the losers of the patent race can profit from the post-innovation equilibrium, Denicolo maximized the total social welfare subject to the condition that the returns to all firms in the patent race equal the equilibrium R&D investment level. He derived a number of optimal patent policies under different circumstances. However, he concluded that generally, the less efficient
is the competition in the product market, the more likely it is that broad and short patents are socially optimal.

The aforementioned studies used a method very similar to Nordhaus (1967). That is, maximizing social welfare (or minimizing social cost) subject to the optimal return to the patentee (or to all firms in a patent race). In the analysis undertaken in these studies, the behavior of deadweight loss with respect to the breadth of a patent is crucial. Gilbert and Shapiro (1990) assume social welfare (W) becomes increasingly costly as patent breadth increases (W’<0 and W’’<0), so it is not surprising that their conclusion is in favour of long but narrow patents. For Klemperer (1990), when the ratio of the social cost to the patentee’s profit r(w) is minimized at zero breadth of a patent (w=0), the social cost will increase faster than the profit to the patentee as the breadth of the patent increases. Thus it is natural that a narrow patent is preferred in this case and a wider patent preferred otherwise. Due to the assumptions of Gallini (1992) regarding imitation cost and incentive to imitate, the long patent duration encourages imitation and thus is not desirable. On the other hand, a wide patent breadth will increase imitation costs and thus deter imitation. So, it is understandable that her study is in favour of broad and short-lived patent protection. Denicolo (1996) made different assumptions about the behavior of social cost of patents under different scenarios, and reached different conclusions accordingly.

The diverse or even contradictory results arising from the above research may be not a shortcoming of the research. However, there is a common defect in the studies on the trade-off between patent length and patent breadth. That is, total social welfare is assumed unchanged when the patent breadth increases. In reality, when a patented technology has a greater breadth, which means the technology can be applied more
widely to products or processes, the patent is said to be of greater importance in its ability to generate greater social welfare. As such, as the patent breadth increases, net social welfare (total social welfare less deadweight loss) may increase, even if deadweight loss increases with patent breadth. Omission of the positive impact of patent breadth on social welfare invalidates the optimization equations and thus the conclusions in these studies.

To sum up, although these previous studies on optimal patent protection make a worthwhile contribution to this issue, their conclusions differ greatly and even contradict each other. More importantly, many such studies are flawed mainly due to the microeconomic (firm) level approach they adopt. Such an approach includes details of the firm, such as the firm’s innovation possibility function, its profit and associated deadweight loss. However, this makes it is easy to lose sight of the impact of patent policy on the economy as a whole, such as the impact on total social welfare, or to overlook or exaggerate the importance of other part of the whole picture. As such, there is a serious shortcoming in exploring optimal patent protection from a microeconomic perspective alone. Since the impact of patent policy is a macro issue, a macroeconomic approach is required to address optimal patent protection more adequately and efficiently.

3. Patent protection in a macroeconomic context

From a macro perspective, a real world case will include multiple innovations with different breadths, and different quantities of social benefits and deadweight loss. However, before we consider the general and complex cases of the real world, it is useful to begin with a simple case, where there is only one innovation that is applicable to only one product. In this case, the degree of patent protection is reflected
by patent duration.

The welfare effect of this case is illustrated in Figure 1. As the patentee will monopolize production of the innovative product, they will establish the output and price so as to maximize their profit. The monopoly patentee sets the output at $Q_M$, so that marginal cost equals marginal revenue. Compared with production under a perfect competition scenario at point C, where marginal cost equals market demand, the monopoly produces less ($Q_M < Q_C$) and charges a higher price ($P_C < P_M$). As a result, the monopoly achieves a super profit $P_MBEPC$ but leads to a deadweight loss $ABC$.

If we assume that the above super profit and deadweight loss occur for each year where patent protection exists, and that all future values are discounted to the present value by the same discount rate, the present value of these two parts (super profit and deadweight loss) will increase with the length of patent protection. However, since the present values of these two parts will change by the same degree over time\(^2\), the ratio between them will be unchanged. Thus there is a constant trade-off between patentee’s profit and social cost. This constant trade-off is used by many researchers (e.g. Tandon, 1982, Klemperer, 1990, Gallini 1992, and Denicolo, 1996) in determining the optimal patent duration, but nobody has proved so far that it is appropriate to ignore other parts related to the social cost and benefit of an innovation, e.g. the consumer surplus, producer surplus, and the cost of research. Next, the author provides a formal proof and consequently, an illustration that a positive and finite optimal patent duration does not exist.

\(^2\) This is because the discount rates are the same for these two parts and because there is a constant ratio between these two parts for each year within the duration of patent protection, when the patent expires both parts becomes zero
Assuming that the marginal cost of production before the innovation is $MC_0$, the total cost of research for the innovation is $SR$, and the inter-temporal discount rate is $\rho$, the social benefit of the innovation is the area $C_0AB$ during the patent protection and the area $C_0CB$ after the patent is expired. However, an important benefit of patent protection is still unaccounted – the future innovation research induced by the super profit obtained by the innovator, the area $PMBEP$ in Figure 1. Let $q$ be the impact factor of super profit on the future innovation research, the net social benefit (NSB) of a patented innovation can be expressed as follows:

\[
NSB = \int_0^T C_0AB e^{-\rho t} dt + \int_T^\infty C_0CB e^{-\rho t} dt + q\int_0^T PMBEP e^{-\rho t} dt - SR
\]

To find the optimal patent duration, we find the derivative of NSB with respect to $T$:

\[
\frac{\partial NSB}{\partial T} = (q*PMBEP_C - ACB) * e^{-\rho T}
\]

Only two areas in Figure 1 appear in the above equation: $PMBEP_C$ and $C_0AB$ apparently because other areas are not related to patent duration. The first order condition for optimal patent duration requires $\frac{\partial NSB}{\partial T} = 0$. However, the sign of $\frac{\partial NSB}{\partial T}$ is not related to $T$. If $q*PMBEP_C > ACB$, $\frac{\partial NSB}{\partial T} > 0$. This means that the positive effect of patent protection (the incentive to innovate) always outweighs the negative effect (the deadweight loss) and thus the longer patent protection generates higher net social benefit. In this case, the optimal solution is to extend the duration of patent protection infinitely. On the other hand, If $q*PMBEP_C < ACB$, $\frac{\partial NSB}{\partial T} < 0$. This means that patent protection has negative net social effect all the time and thus,
the optimal solution is no patent protection. In the case $q^* \quad P_{M\text{BEP}_C} = ACB$, $\partial \text{NSB}/\partial T = 0$. This means that positive and negative effects of patent protection are cancelled out. In this case, any length of patent protection has the same result. Out of these three scenarios, it is widely believed that the patent system has a positive impact overall\(^3\). Therefore, an infinite patent length should be desirable for society.

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**Figure 1: Deadweight loss in monopoly production**

It is more complicated in the case of patent breadth. The first complication arises from the definition of patent breadth. As mentioned in the previous section, there are many definitions\(^4\), of which Denicolo(1996) provides a summary. In this paper, a general and intuitive definition taken from Matutes et al. (1996) is used: the breadth of a patent is the number of products or procedures to which an innovation can apply and these applications are reserved for the patentee. Assuming a patent innovation has

\(^3\) Survey data show that, without the patent system, about 65% of innovation will not occur in these industries while it is 8% for all industries (Tay and Siberston, 1973; Mansfield, 1986).

\(^4\) The different definitions of patent breadth are largely due to their convenience to explain the models used by different researchers.
breadth of \( n \), it can be applied to \( n \) products and the innovator is able to monopolize production of \( n \) products. For each product, there will be profit for the inventor and deadweight loss, similar to those shown in Figure 1. Since there is no reason that the ratios of super profit to deadweight loss are correlated to the size of patent breadth, for simplicity it is assumed that these ratios are the same for each product. As such, a variation of patent breadth will affect the total amounts of the inventor’s profit, and of the deadweight loss, but it will not affect the ratios. Consequently, we reach a conclusion similar to the patent length: there is no trade-off point in patent breadth.

A second complication arises due to the difference between the patent breadth claimed and the true breadth of a patent technology. A true patent breadth represents the importance of an invention: a widely applicable invention will have a large positive impact on the economy. The claimed patent breadth, on the other hand, is generally positively related to the amount of super profit to the patentee and thus has a positive impact of stimulating innovation but, in the mean time, it is also positively related to the amount of deadweight loss. Ideally, the claimed patent breadth and the true patent breadth should be equal. If the breadth claimed is less than the true breadth of the patent (that is, some applications of the invention are not claimed in the patent), there will be a smaller deadweight loss than when the breadth of invention is fully claimed; but this level of patent protection will deliver less income to the inventor and thus has less power to stimulate innovation in the future. If the breadth claimed is greater than the true breadth of the patent (that is, some claims in the patent are not associated with the innovation in practice), there will be a greater deadweight loss to the society and greater unjustifiable super profit to the patentee – the part of super profit arising from the over-claimed patent protection. Since the over-claimed part of patent breadth is not a true innovation, patent protection of this part does not reward innovation and
thus plays no role in stimulating further innovation. By this reasoning, the desirable maximum breath of patent protection should be the true breadth of a patented innovation.

Although the effect of patent protection of an individual innovation can be explained intuitively by using a graph (Figure 1), in order to analyze patent protection rigorously, it is necessary to apply math in the context of a macro economy. For all innovations that occurred each year in an economy, it is arguable that some innovation may have occurred naturally, without the patent system. And even for innovations induced by the patent system, it may be argued that they may occur sooner or later without the patent system in place. So it is arguable that the effect of the patent system is to accelerate the timing of innovations. This is the starting point of the analysis next. Moreover, although the features of innovations will vary in practice, for ease of analysis and discussion it is assumed initially that all innovations induced by the patent system have the same characteristics, i.e. the number of years advanced, and the amounts of social benefit and deadweight loss. Initially, only the impact of the patent length is considered, the patent breadth for each patented innovation is set as 1.

Let $N_0$ be the number of innovations occurring naturally, and $N$ be the number of innovations induced by the patent system. For each innovation (either naturally occurring or induced by the patent system), $i$ is the yearly discount rate, $SS$ is the social surplus (the area below the demand curve and above the marginal cost curve), and $R = r * SS$ is the cost of innovation, where $r$ is a positive parameter (in the case of diminishing return to innovation research, $r$ increases as $R$ increases). For each innovation induced by the patent system, $n$ is the number of years advanced, $w$ is the percentage of deadweight loss in the total social surplus generated by each innovation,
\( t \) is the duration of patent right, which is assumed longer than the number of years advanced \((t>n)\), and \( N \) is a positive function of \( t \) (the longer patent duration leads to more profit to patentees and thus will stimulate more innovations), namely \( N=N(t)>0 \).

With this setting, the total net social benefit without a patent system \( B_{NP} \) and that with a patent system \( B_P \) can be expressed as follows:

\[
B_{NP} = N(t)[(1-i)^0 + (1-i)^{n+1} + \ldots] + N_0[SS-r*SS][1+(1-i)+(1-i)^2+\ldots];
\]
\[
B_P = N(t)[(1-i)^0 + (1-i)^{t+1} + \ldots] + N_0[SS-r*SS][1+(1-i)+(1-i)^2+\ldots];
\]

So the overall effect of patent protection \( EP \) can be calculated as:

\[
EP = B_P - B_{NP} = N(t)[(1-i)^0 + (1-i)^{t+1} + \ldots - w][1+(1-i)+(1-i)^2+\ldots + (1-i)^{t-1}]
\]

\[
= N(t)[1-(1-i)^n]/\bar{i} - w[1-(1-i)^t]/\bar{i}, \text{ or}
\]
\[
EP = N(t)* SS* (1-r)*[1-(1-i)^0-w+w(1-i)^t]/\bar{i}.
\]

The number of innovations induced by patent system \( N \) and the social surplus should be positive. Likewise, \((1-r)\), \( i \) and \((1-i)\) should also be positive, because \( 0<r<1 \) and \( 0<i<1 \). Thus, whether the economic impact of the patent system is positive or negative depends on the sign of \([1-(1-i)^0-w+w(1-i)^t]/\bar{i}\).

Since both \( w \) and \((1-i)\) have a value between 0 and 1, and the number of innovations induced by the patent system is large, it is clear that the size of negative impact of patent duration \( t \) on \( w(1-i)^t \) is much less than the size of its positive impact on \( N(t) \). As a result, if \([1-(1-i)^t-w]/\bar{i}\) is positive, longer patent duration will have a greater positive contribution to the economy. Consequently, the best outcome in this case would be an infinite patent right duration. On the other hand, if \([1-(1-i)^t-w]/\bar{i}\) is negative, then the
whole term \([1-(1-i)^n-w+w(1-i')i)]\) will most likely be negative. Thus, the patent system will have negative impact on the economy, and a longer patent duration would magnify this negative impact. The best outcome in this case would be a zero patent right duration; in other words, no patent protection. There is a small chance that, when \([1-(1-i)^n-w]\) is negative and in the vicinity of zero, a small \(t\) value may lead to a greater value of \(w(1-i')i\) and thus a positive \([1-(1-i)^n-w+w(1-i')i]\). In this case, a weak patent protection can avoid the marginal negative effect and achieve a marginal positive result and thus is the most desirable option.

Next, we consider patent breadth. Let \(b\) be the patent breadth claimed, and \(m\) the true breadth of the patented technology. Since the true patent breadth \(m\) represents the importance of an invention, a greater \(m\) indicates a widely applicable invention that has a larger positive impact on economy. So, \(SS\) (social surplus) should be a positive function of \(m\): namely \(SS=SS(m)\). On the other hand, the claimed patent breadth \(b\) may have a positive impact in stimulating innovation, so \(N\) (the number of innovations induced by the patent system) should be a positive function of \(b\). Since zero patent protection will be a result of either zero duration of patent right or zero patent breadth claimed by patentee, we assume \(N=N(t^*b)^5\). The size of claimed patent breadth relative to the size of true breadth of patent innovation is positively related to the size of deadweight loss relative to total social welfare, so we assume deadweight loss is a positive function of \(b/m\), namely, \(w=w(b/m)\). As such, the overall effect of patent protection \(EP\) can be expressed as:

\[
EP = N(t^*b)^*SS(m)^*(1-r)^*[1-(1-i)^n-w(b/m)+ w(b/m)*(1-i')i]/i.
\]

\(^5\) More accurately, only the part of \(b\) which is less than or equal to \(m\) stimulates innovation, so the function for innovation stimulation should be something like \(N=N(t^*(2mb-b^2)^{1/2})\). For simplicity, a simple function is used in the paper, but this will not affect the discussion.
As is the case with patent duration, the breadth of patent protection should have a larger impact on $N(t^*b)$ than on $w(b/m)$, but it may change the size of $w(b/m)$, that may in turn alter the sign of $EP$. As stated previously, over-claimed patent breadth has no effect on stimulating innovation but will cause deadweight loss, so the maximum desirable patent breadth should be equal to the true breadth of the patented technology. If $[1-(1-i)^n]-w(b/m)+ w(b/m) *(1-i^j))]$ is positive, a maximum breadth of patent protection $(b=m)$ will increase $N$ to the greatest degree and thus generate maximum net benefit for the economy. However, if $[1-(1-i)^n]-w(b/m)+ w(b/m) *(1-i^j))]$ is negative, the best solution will be zero patent protection (zero breadth or zero duration). When $[1-(1-i)^n]$ is small, that is when a low discount rate $i$ is used and when the number of years advanced is low, the increase in breadth and/or duration of patent protection may reverse the sign of $EP$. In this case, there may be an optimal breadth and length of patent protection to maximize $EP$.

Finally, we can relax the assumption that each innovation has the same characteristics. For innovation $j$, let the cost-benefit ratio of the innovation be $r_j$, the number of years advanced $n_j$, the true breadth of innovation $m_j$, the claimed patent breadth $b_j$, and the patent duration $t_j$. The overall effect of patent protection $EP$ can be written as:

$$EP = [N(\bar{t}*\bar{b})/i]*\sum_{j=1}^{N} SS(m_j)*(1-r_j)*[1-(1-i)^n_j]-w(b_j/m_j)(1-(1-i^j))]/N$$

In the above equation, we assume that the number of innovations induced by the patent system depends on average patent length and average patent breadth ($\bar{t}$ and $\bar{b}$). Since the overall effect of the patent system is the sum of the net social benefits of all innovations induced by the system, both the number of innovations and the average contribution of an innovation are important. It is useful to make a general assumption
that some innovations have positive net social benefits and some have negative net social benefits. If the positive net social benefits outweigh the negative ones, the average contribution of an innovation $j$ is positive, and so is the overall effect of the patent system. On the other hand, if the net social benefit of an average innovation $j$ is negative, the effect of the patent system will be negative. The size of the average contribution of an innovation has important implications for the design of a patent system. If the average contribution of an innovation $j$ is significant, where $[1-(1-i)^n]$ is significantly greater or smaller than $[w(b_j/m_j)(1-(1-i)^t)]$, then the change in $t$ and $b$ will not affect the sign of the average contribution. Since $t$ and $b$ have a large positive effect on $N(t*b)$, thus magnifying the effect of the average contribution of an innovation, the best solution is maximum $t$ and $b$ if $[1-(1-i)^n]-[w(b_j/m_j)(1-(1-i)^t)] > 0$ and zero $t$ and/or zero $b$ if $[1-(1-i)^n]-[w(b_j/m_j)(1-(1-i)^t)] < 0$. In a scenario where the positive net benefit of some innovations is largely cancelled out by the negative net welfare of the others, the impact of the patent system will be insignificant. Only in this case, can we fine-tune $t$ and $b$ to achieve a marginally positive result.

In short, there is generally neither an optimal patent duration nor an optimal breadth of patent protection. If the economic impact of a patent system is positive, both patent duration and patent breadth should be maximized in order to maximize the contribution of the patent system to the economy. However, if the impact of a patent system is negative, the system should be abolished. When a patent system has only a marginal economic impact (either positive or negative), a combined optimal patent duration and breadth of patent protection may exist, and thus weak patent protection is preferred. However, the likelihood of the patent system having only a marginal
economic impact is not high. If this were the case, the impact of the patent system would not be significant, and thus finding an optimal combination of patent duration and breadth would be of little importance.

4. The new patent system

The traditional approach to patent system reform attempts to determine the trade-off point which allows a balance of the positive and negative effects of patent monopoly. However, the preceding analysis demonstrates that, generally speaking, there is no optimal duration and breadth of patent protection. As such, this traditional approach is without a logical foundation. A new approach is therefore required to improve the patent system.

The new approach proposed here comes from a scrutiny of the purpose and mechanism of a patent system. The system allows inventors to profit from their inventions by prohibiting others from doing so. Traditionally, inventors will profit by producing and selling the products they have invented. So the obvious solution is to grant inventors the monopoly right for production of the products they have invented. In this way, inventors become producers and they are rewarded through their engagement in production activity. As a result, the ‘traditional’ patent system encourages innovators to become producers and, in the mean time, necessitates deadweight loss from monopoly production. Ideally, if a patent system was able to separate the role of innovator and the role of producer, and reward innovators directly for their innovative behaviour, deadweight loss would be avoided and the system would become much more efficient. The answer to improving the patent system in this way is licensing. If patent right is defined as the exclusive right to grant non-exclusive production licenses, the innovator will be rewarded directly from
licenses fees. Production monopoly will not be a necessary outcome because the production licenses are non-exclusive.

It should be acknowledged however that such an approach may have potential difficulties. The patentee may ask an excessive price for a patent license, so that nobody is willing to purchase the license. As a result, the patentee will become a monopoly producer. If the intention of the patentee is to produce the patented product themselves, monopoly production is indeed possible. However, this probability is not high because licensing will become the better way to obtain profit under the new patent system. Producing patent goods may take a long time and require substantial capital and effort and managing the production process may not be the strength of the patentee, whose strength is more likely to be the invention of new products. The exclusive right of granting production licenses provides the patentee with the opportunity to obtain their desired profit quickly and easily, so it is more likely that they will take the easy way. Furthermore, a non-exclusive license will give the patentee unlimited opportunity to grant licenses later, and the prices of these licenses can be adjusted when necessary, according to market demand. This gives the patentee countless chances to harvest the commercial potential of the patent and thus the patentee will not hesitate to grant licenses. As a result, granting non-exclusive licenses to licensees will become the most likely practice.

Patent licensing has already become an adopted practice but it is not popularly used. The main reason may be because the practice is too complex and too difficult to implement. The patent licensing practice is currently in the form of unregulated agreements and suffers from the inappropriate and unclear definition of patent rights, as well as restrictive rules within the current patent system. Patent reform is required
to redefine patent rights, and simplify and standardize patent licensing practice, aiming at strengthening monopoly in patent licensing and removing monopoly for the production of invented goods. One way to reinforce patent licensing monopoly is to remove restrictions on this practice such as the duration of a patent and the compulsory rule. To remove the monopoly on production, the practice of granting exclusive patent licenses must stop. Moreover, most non-commercial uses of patent technology (e.g. for research or further invention purposes) has a large positive externality and will not affect the profit of inventors significantly. This type of use can and should be exempted from the requirements of a patent license. In short, the principle of patent reform is to enhance the patentee’s exclusive right to grant non-exclusive licenses to commercial producers. To fully protect the benefits of inventors and thus encourage innovation activity, several changes to current patent laws are considered necessary. These are now discussed in turn.

(1) clarifying the patent right.

Current patent laws grant patentees the exclusive right to implement their innovations. For example, in the US, patent rights is defined as the rights granted to inventors by the federal government, pursuant to its power under Article I, Section 8, Clause 8, of the U.S. Constitution, that permit them to exclude others from making, using, or selling an invention for a definite, or restricted, period of time. This right is broad and vague and this makes it difficult to separate the role of inventor from the role of producer. Under this patent rights definition, patentees are able to grant all kinds of licenses and sign all sorts of agreements, or even refuse to grant license without any reason. This broad and vague definition is subject to interpretation and can lead to abuse of patent monopoly. Moreover, by rewarding innovation through the production
and sale of patented products, it is the implementation of innovative technology that is actually being rewarded and stimulated. As a result, although it is widely accepted that innovation is vital to the economy, those who generally receive the highest salary are the CEOs of companies rather than the inventors. Some inventors do end up being rewarded very highly, such as the creators of Microsoft, Apple, and Google, but their rewards come from their role in producing new products rather than in creating them.

Defining the patent right as the exclusive right of patentees to grant non-exclusive licenses has two fundamental implications. One such implication is that, as stated earlier, the approach will separate the role of innovator from the role of producer. Under the new definition of patent rights proposed here, innovation is rewarded directly, allowing the innovation/research department to be an independent, self-financed entity. Separating the two activities of invention and production will lead to the more efficient use of specialized human capital. For example, Bill Gates would not have become the previous CEO of Microsoft and would have been available to work on more innovations. The separation of innovation and production into distinct companies would also solve the problem of sequential innovation now concerning many economists and practitioners alike. Since the innovation companies would not commercially produce patented products, they do not need to obtain a license from any patentee. Instead, it is the production firms that need to obtain production licenses from all inventors involved in the new products.

The concept of separating innovation activity from production activity is a contentious one. It may be argued that the two activities are closed linked together. Indeed it is true that some innovations are closed associate with production (e.g. process innovation) while some are not (e.g. product innovation). The function and
structure of companies vary greatly, so it is not necessary that all innovation departments split away from production departments under the modified patent system. The key point here is that, as long as innovation activities can be rewarded directly and independently through licensing income, the value of the innovation will be determined by the market and realized by company management or investors. Consequently, innovation funds will be efficiently channeled by the market according to the importance of the innovations.

A second and most fundamental implication of the new definition of patent right is that it will lay the foundation for an efficient patent market. The current definition of patent right is broad and production-focused, and thus patent rights are not marketable. This leads to complexity and hinders the formation of a patent market. Without an efficient market to channel research funds, innovation activities are under invested and, consequently, the scarcity of new products imposes a tight constraint on economic growth. The new definition provides a clearly identified product that has a distinct value and is transferrable, thus helping to form an efficient patent market. An efficient patent market will in turn standardize the practice of patent licensing and automatically stimulate innovation.

(2) standardizing patent licensing.

Under current patent laws, patent licensing is implemented by an unregulated patent license contract. Given the open-ended nature of these contracts, many inappropriate contents may be included, leading to various abuses of patent monopoly. In order to avoid these problems, the new patent law needs to forbid any exclusive license, and to simplify and standardize patent licensing.

It may be argued that patent laws should not ban the granting of exclusive licenses,
because this imposes constraints on the patentee’s profit maximization decision. However, this is a common misperception. Normally it is the producer that desires an exclusive license because the producer likes monopoly power of setting price for the patented product. An exclusive license actually imposes great constraint on the patentee. After granting an exclusive license, the value of the patent will decrease substantially because the patentee is no longer able to grant another license in the same region. If exclusive licensing is banned, licensees are no longer able to seek an exclusive license. This means the patentee has unlimited opportunities to grant licenses later. The other benefits of banning exclusive licenses include reducing the patent license price (a non-exclusive license will be much cheaper than an exclusive license) and thus the prices of patented products, and fostering an efficient patent market. The reduced patent license price will not affect the income of the patentee negatively because they retain unlimited rights to grant licenses later. On the contrary, it can increase the patentee’s income because the patentee may adjust the patent license price according to market demand so as to maximize their profit. The reduced prices of new products, thanks to the reduced license fee and increased competition in production, will also benefit consumers greatly. The most important impact of forbidding exclusive licenses is that it will lead to a thicker patent market – there will be more buyers of patent licenses. Buyers’ willingness to pay will reveal the true market value of the patent license, which in turn reveals the value of the patent. Through changes in the price of the patent license and thus in the price of the patent, the market will reward the inventor adequately for their work, and encourage further innovation at a degree which is most beneficial to the economy.

Under this new patent system, patent licensing would be very simple. Even a statement ‘I grant xxx the right to use the method/technology in patent No. xxx’
would be sufficient. The simplicity of patent licensing would improve market efficiency and avoid all sorts of problems in current patent practice, for example, tie-ins. Tie-in refers to the practice where the patent holder inserts some contents in a licensing contract so as to extend their monopoly power, for example requiring the licensee to purchase his nominee goods. This problem can be avoided by standardizing the license agreement, allowing only contents such as permission of use of patent technology in return for a royalty or annual fee. In reality, in order to implement the patented technology, the licensee may require technical assistance from the patentee. However, this kind of cooperation does not involve the right to use the patented technology, so it should and can be included in a separate agreement. The advantage of a separate agreement regarding production cooperation is that any attempt to extend patent monopolistic power will be delinked from patent licensing and thus will eventually fail.

(3) prolonging the duration of patent rights infinitely and abolishing the compulsory license rule.

Due to concern regarding the negative effects of patent monopoly, current patent laws grant only temporary patent rights. The duration of patent right varies from country to country and depends on different kinds of inventions. Currently, the maximum patent duration is generally 20 years, with medical patents being 5 years longer. This time limit is unnecessary and harmful.

As demonstrated in previous section, a finite optimal duration of patent protection exists only in some rare cases when the effect of the patent system is marginal. Since empirical evidence suggests that the patent system is beneficial overall, the macroeconomic analysis suggests that an infinite length of patent protection will
maximize the social benefit of the patent system. Moreover, With the exclusive license banned under the new patent system, the social cost of patent protection would be reduced substantially, as the deadweight loss in production would be eliminated. Consequently, the concern about the negative effects of the patent system is unwarranted. One point is worth making in this regard: although the patent law may provide an infinite patent duration, the effective life of a patent is limited. As the patented technology is outdated or the patented product has been phased out of the market, the patent loses its value and disappears naturally.

Limit in patent right duration should be removed because it is a major obstacle to a functioning patent market, which may have a tremendous positive influence on efficient resource allocation and robust economic growth. Limited duration means that the property right of a patent is temporary only. This temporary property right significantly reduces incentives to innovate, and imposes a great distortion on the patent market. Without a functioning market, resource allocation is inefficient, and patented innovation is no exception in this regard.

An extreme case of limited patent duration is the compulsory license rule. To address abuses of patent rights, many countries have adopted a compulsory license rule. For example, while Article 48(3) of Patents Act 1977 in UK lists different kinds of abuse of patent monopoly, Article 48(1) stipulates:

At any time after the expiry of 3 years, or of such other period as may be prescribed, from the date of the grant of the patent, any person may apply to the comptroller on one or more of the grounds specified in subsection (3) below: (a) For a license under the patent, (b) For an entry to be made in the register to the effect that licenses under the patent are to be
available as of right, or (c) Where the applicant is a government department, for the grant to any person specified in the application of a license under the patent.

If the government is able to provide an optimal price for patent licenses, as assumed by Tandon (1982), this rule is of no harm. However, nobody can detect the optimal price except the market. If the ‘optimal price’ designated by the government is only a fraction of the market price for the patent, this compulsory license rule effectively reduces the duration of patent to three years. This rule is unnecessary because abuse of patent rights can be avoided without the rule. For example, the tie-in problem can be fixed by standardizing patent licensing, while the excess prices of patent rights and patent burying can be solved by a functioning patent market. Even so, some may argue that the compulsory license rule is still required to safeguard the implementation of some very important patents such as generic invention patents and patents on pharmaceutical and pollution control equipment. This caution is understandable, but the problem should be solved using methods that will not hinder the patent market, such as government involvement in production of certain goods after purchasing the relevant patent licenses. If less protection is provided for these extremely important innovations by employing the compulsory license rule, the public may gain some benefit in the short run, by having cheaper patent licenses and thus cheaper prices for new products. However, the public will be worse off in the long run, as fewer innovations are made.

The compulsory license rule is not only unnecessary but also very harmful. It wrongly imposes the task of implementing and diffusing patent technology on inventors. This distracts the energy of inventors from continuing invention activity to implementing
the invention, a job more appropriately suited to the entrepreneur. More importantly, the extremely short period of time stipulated by the compulsory license rule greatly disadvantages inventors in the negotiation of patent license prices. As Reid (1993) realized, ‘[t]here have been relatively few compulsory license applications in recent decades… nevertheless they have probably wielded an influence wider than might have been expected from the paucity of the case law. The background threat of a compulsory license application is a potent lever in the hands of a person applying to a patentee for a voluntary license on reasonable terms.’ If a patent right does not command a fair price due to the pressure of limited time, further innovation activity becomes unattractive. As a result, inventors or potential inventors will try to find a more attractive career. In considering its damaging effects on the patent system, the compulsory license rule should therefore be abolished.

(4) improving the quality of patent and widening the patent protection scope

As illustrated in section 3 of this paper, the optimal breadth of patent protection is one that matches the true breadth of the patent innovation. It is the responsibility of the patentee to make sure that the claimed breadth of their patent in their application is not less than the true breadth of their innovation. However, there is a tendency for patent applicants to exaggerate the patent breadth of an innovation so that it may be larger than the true breadth, meaning that some of the claimed breadth will not be part of the innovation. An extreme case of this is where a non-innovative technology (or procedure) is approved for a patent protection. These kinds of patents are of low quality, or as some researchers term them, as low height, e.g. Foster and Breitwieser (2011). This has a tremendous negative impact because it imposes a huge social cost without any social benefit. Recent events in the US provide a worrisome example of
this. Due to a large increase in patent application numbers and lack of resources in the patent office, many patents were approved even if they were not essentially innovative, such as Amazon’s ‘one click’ patent. With the patent office obtaining budget increases in the recent patent reform in the U.S., this situation is expected to change. However, in order to prevent granting low quality patents, the decision of the patent office needs to be challengeable in the court system.

For historical and practical reasons, current patent laws only protect practical inventions. Neither patent laws nor copyright laws protect scientific discoveries and theoretical breakthroughs, which are the foundation of practical inventions. As a result, theoretical and fundamental research is severely underfunded: it heavily depends on government funding because few private companies are interested in it. Since the function of a fundamental discovery is mainly to contribute to humanity’s knowledge or ideas base, it is impractical to grant a patent right. Legal scholars unanimously claim that knowledge and ideas are not patentable (Pretnar, 2003). However, these kinds of discovery will have widespread although indirect influence on the economy. It is possible and desirable to impose a fundamental-research tax on patent applications or on patent trading transactions. This could be one of many ways to protect the source and base of invention, and thus to guarantee adequate innovation in the economy.

(5) enhancing international coordination in patent protection.

Due to globalization, a patent innovation can easily cross national boundaries, but the patent law of one country can protect the benefit of inventors only within that country and become powerless beyond its boundaries. It is arguable that the country thoroughly revising its patent law first will enjoy an enriched pool of innovations and
thus stimulate its economic growth the most. However, producers in this country will initially be unfairly disadvantaged, because countries without a strict patent law will be able to utilize these innovations at reduced cost, or even for free. To protect the benefit of inventors on a global scale, and to give all companies an equal footing in producing patented products, it is highly desirable to enhance international coordination in revision and implementation of patent laws. The development of the patent system provides an example of good international coordination. For example, the agreement on trade-related aspects of intellectual property rights (TRIPS) was incorporated into the charter of the World Trade Organization (WTO) in 1994 and this resulted in uniform patent standards throughout the membership of the WTO. The globalization of the patent law is necessary to stimulate global innovation and thus boost the world economy.

6. Conclusion

This paper has reviewed the research on optimal patent design, and revealed that the belief that a trade-off in patent protection exists is based on flawed microeconomic research. To avoid the shortcomings of this micro-level approach, a macroeconomic analysis was undertaken. This analysis has confirmed that, except for some rare cases, there is no trade-off in the length and/or breadth of patent protection. Empirical studies show that the patent system is effective overall. Consequently, to maximize the benefit of the patent system, the duration of patent right should be infinite, and the breadth of patent protection should be set equal to the true breadth of patent innovation.

Rejecting the concept of a trade-off point in patent protection puts the current rationale behind the design and reform of the patent system on an imperfect footing,
and suggests that a new approach is required. Through a scrutiny of the purpose and mechanism of the patent system, the author suggests a new design for this system. By redefining patent rights as an exclusive right to grant non-exclusive patent licenses for commercial production, the patent system can separate the role of inventor from the role of producer, thus rewarding the inventor directly and avoiding the deadweight loss from monopoly production. A new patent system can be formed by forbidding the granting of exclusive patent licenses, abolishing the compulsory license rule, standardizing patent license practice, prolonging the duration of patent rights indefinitely, improving the quality of patents, widening the scope of patent protection, and enhancing international coordination. This new patent system can be expected to facilitate the formation of an effective patent market, which can solve the problems plaguing the current patent system, encourage innovation in a more efficient manner, and automatically channel more appropriate funds into innovation activity.
References:


Appendix: the derivation of the second order condition for Nordhaus solution

Social welfare function in his study (Nordhaus, 1967, P7) is as follows:
\[ W = rY_0/\rho + r^2d[1-\phi]/(2\rho) - sR. \]
Since he defined \( r = B = B(R) \), the welfare function becomes:
\[ W = BY_0/\rho + B^2[1-\phi]d/(2\rho) - sR. \]
The relevant derivatives to obtain the second order condition are:
\[ f_1 = \partial W/\partial \phi = -B^2d/(2\rho), \]
\[ f_2 = \partial W/\partial R = B'Y_0/\rho + B'B(1-\phi)d/\rho - s, \]
\[ f_{11} = \partial f_1/\partial \phi = 0, \]
\[ f_{12} = \partial f_1/\partial R = -BB'd/\rho, \]
\[ f_{22} = \partial f_2/\partial R = B''Y_0/\rho + (B'^2+BB'') (1-\phi)d/\rho. \]
The second order condition for a true optimization requires:
\[ f_{11}f_{22} - 2f_{12}f_{21} + f_{22}f_{11} < 0. \]
Substituting the first and second derivatives into the above condition we have,
\[ f_{11}f_{22} - 2f_{12}f_{21} + f_{22}f_{11} = -2\left[ -BB'd/\rho \right] \left[ B''Y_0/\rho + BB'(1-\phi)d/\rho - s \right] + \]
\[ [B''Y_0/\rho + d(B'^2+BB'') (1-\phi)/\rho] \left[ -B^2d/(2\rho) \right] = -(B^3B'd^2/\rho^3) \left[ B'Y_0 + BB'(1-\phi)d - sp \right] - \]
\[ (0.25B^3d^2/\rho^3) \left[ B''Y_0 + (B'^2+BB'') (1-\phi)d \right]. \]
Plugging into above equation Nordhaus’ results (1967, p6) from the firm’s profit maximization condition, \( sp = B'\phi Y_0 \), we have:
\[ f_{11}f_{22} - 2f_{12}f_{21} + f_{22}f_{11} = -0.25B^3d^2/\rho^3 \left[ 4B^2Y_0 (1-\phi) + 4BB'^2 (1-\phi)d \right] - \]
\[ (0.25B^3d^2/\rho^3) \left[ B''Y_0 + (B'^2+BB'') (1-\phi)d \right] \]
\[ = -(0.25B^3d^2/\rho^3)((1-\phi)(4B^2Y_0+5BB'^2d+B^2B''d)+BB''Y_0) < 0. \]
By definition \( B, d, \rho \) are all positive, so the above inequality requires the content in the square bracket to be positive. Following Nordhaus’ convention to let \( Y_0 = 1 \) so that \( d \) represents demand elasticity in absolute value. We have the following second order condition:
\[ 1-\phi > -BB''/(4B'^2+5BB'^2d+B^2B''d), \]
\[ \phi < 1 + BB''/(4B'^2+5BB'^2d+B^2B''d) \]
Given that \( \phi = 1 - e^{-\rho T} \), the above inequality is equivalent to
\[ T < -(1/\rho) \ln[-BB''/(4B'^2+5BB'^2d+B^2B''d)]. \]