IDENTIFICATION AND ESTIMATION OF ELASTICITIES OF SUBSTITUTION FOR FIRM-LEVEL PRODUCTION FUNCTIONS USING AGGREGATIVE DATA

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Identification and Estimation of Elasticities of Substitution for Firm-Level Production Functions Using Aggregative Data

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Abstract

Firm-level stochastic CES and VES production functions are defined and conditions are specified such that elasticities of substitution can be identified and efficiently estimated from aggregative data on firms within different asset-size categories. Attention is given to the estimation of production functions using cross-sectional and time-series data. The sufficient conditions specify that levels of factors of production be the same for firms within designated categories.

1. Introduction

Empirical studies, in which elasticities of substitution are estimated using official published statistics, consider production functions defined in terms of aggregate input quantities for firms in the given asset-size categories for which the data are reported.

A review of literature on the estimation of elasticities of substitution reveals a somewhat confusing array of results. Nerlove (1967, p.58) concludes that "the major finding of this study is the diversity of results; even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity." Zarembka (1970, p.47), however, challenged Nerlove's contention and claimed that a correction of the labour and wage-rate variables for quality variations and the use of seemingly unrelated regression led to results such that "the use of different time periods does not produce different estimates of the elasticity". Griliches (1967, p.296), on the other hand, concluded that labour-quality variables contributed little in the
estimation of the elasticity. Berndt (1976, p. 59) listed a variety of hypotheses that have been advanced to explain the diversity of results, but concluded that, in general, empirical studies attempting to take account of these deficiencies have produced unsatisfactory results. Morawetz (1976) came to the same conclusions after examining several studies for a variety of developing countries. He noted that it was impossible to find industries with consistently high or low elasticities in either developing countries or advanced economies, such as the United States of America. Comprehensive reviews of empirical studies are given by Walters (1963), Nerlove (1967) and Gaude (1975).

It is generally observed that the elasticity estimates obtained from time-series data are significantly lower than those obtained from cross-sectional data. Boddy (1967) suggested that this is so because the estimates based upon time-series data are obtained from regressions involving aggregate factor proportions or average practice coefficients. He stated that the changes in factor costs, and the expectations of future costs based upon them, are only reflected in the factor proportions of the newest or best practice techniques. He claimed that the elasticity estimates based upon time-series data are unbiased if, and only if, the ratio of average practice to best practice factor proportions is constant for successive cross-sectional observations. This implies Hicks neutral technological change.

The above argument is related to the so-called "impossibility theorem" of Diamond and McFadden (1965) which states that the elasticity of substitution and the biases of technical change cannot be identified from smooth time-series data [see also Sato (1977, p. 361)]. Gaude (1975) claimed that the
lower estimates obtained from time-series data are due to the simultaneity between inputs and their prices, misspecification of adjustment lags between inputs and outputs and the dominance of cyclical conditions in time-series data.

In this paper, we define production functions in terms of data on individual firms and present conditions under which elasticities of substitution can be identified and efficiently estimated on the basis of aggregative data which is generally available in official statistical publications.

2. Econometric Modelling

We initially define models for stochastic production functions in terms of individual firms, in particular industries, for a given year and location. Subsequently, we consider the estimation of the elasticity of substitution on the basis of aggregative data for firms in different asset-size categories and different time periods. We consider both constant-elasticity-of-substitution (CES) production functions and variable-elasticity-of-substitution (VES) production functions.

(a) CES Production Functions

For a given industry, the value-added, capital and labour for the j-th reporting firm in the i-th asset-size category in a given location are denoted by $Y_{ij}$, $K_{ij}$ and $L_{ij}$, respectively. Suppose that the stochastic constant-returns-to-scale CES production function [cf. Arrow, et al. (1961)] applies:

$$Y_{ij} = \gamma (\delta K_{ij}^{-\rho} + (1-\delta)L_{ij}^{-\rho})^{-1/\rho} e^{U_{ij}},$$

$$j=1,2,\ldots,r_i, \ i=1,2,\ldots,n,$$

where the efficiency parameter, $\gamma$, is a positive constant; the distribution parameter, $\delta$, is a positive constant between zero and one; the substitution
parameter, \( p \), is a constant greater than minus one; the random errors, 
\[ u_{ij}, \ j=1,2,...,r_i, \ i=1,2,...,n, \]
are independently and identically distributed as normal random variables with means zero and variances, \( \sigma^2 \). \( r_i \) represents the number of reporting firms in the \( i \)-th asset-size category; and \( n \) represents the number of asset-size categories for the industry.

As is well known, the so-called indirect form of the CES production function (1) is defined by
\[
\log(Y_{ij}/L_{ij}) = \beta_0 + \beta_1 \log w_{ij} + u_{ij},
\]
(2)
\[ j=1,2,...,r_i, \ i=1,2,...,n, \]
where \( w_{ij} \) represents the wage rate for labour in the \( j \)-th firm within the \( i \)-th asset-size category. Under the assumption of perfect competition in the factor and product markets, the parameter, \( \beta_1 \), in (2) is identically equal to the elasticity of substitution, \( \sigma \), for the CES production function (1), i.e.,
\[
\beta_1 = \sigma \equiv (1 + p)^{-1}.
\]
It thus follows that the estimator for \( \beta_1 \), obtained by the ordinary least-squares regression of the logarithms of value-added per unit of labour on the corresponding logarithms of wages for individual firms, is the minimum-variance unbiased estimator for the elasticity of substitution, given that the wage data are known values, measured without error.

The above firm-level model and consequent efficient method of estimation of the elasticity of substitution is generally unfeasible when only aggregative data are available. We now, however, introduce simplifying conditions by which efficient estimation of the elasticity of substitution, as defined above, is possible on the basis of aggregative data for the asset-size categories. Suppose that the wages are the same for all labourers employed by firms in a given asset-size category, i.e., \( w_{ij} = w_i \) for all \( j=1,2,...,r_i, \ i=1,2,...,n \). This implies that the indirect form of the CES production function (2) is expressed by
This model is equivalently expressed by

\[ \log(\frac{Y_{ij}}{L_{ij}}) = D_0 + \beta_1 \log w_i + U_{ij}, \quad j=1,2,\ldots,r_i, \quad i=1,2,\ldots,n. \tag{3} \]

where \( Y_{ij} \) and \( L_{ij} \) denote the geometric means of the value-added and labour quantities for firms in the \( i \)-th asset-size category; and \( U_{ij} \) are heteroscedastic arithmetic means of the random errors associated with firms in the different asset-size categories.

It is well known that the least-squares estimator for \( \beta_1 \) in the model (3) is equivalent to the generalized least-squares estimator for \( \beta_1 \) in the model (4) which is obtained by ordinary least-squares regression after the observations in model (4) are multiplied by the square root of the corresponding numbers of reporting firms in the asset-size categories.

However, it is generally the case that when firm-level data are not available, the geometric means of value-added and employment levels for firms in different asset-size categories are also unavailable. In general, totals of value-added, employment and wages are available for reporting firms within different asset-size categories. A sufficient condition, by which the elasticity of substitution can be efficiently estimated by regression analysis involving sample (arithmetic) means for different asset-size categories, is as follows:

**Condition A:** Wages and factor inputs are the same for all firms in a given asset-size category, i.e., \( w_{ij} = w_i, L_{ij} = L_i \) and \( K_{ij} = K_i \) for all \( j=1,2,\ldots,r_i, \quad i=1,2,\ldots,n. \)
Given Condition A, the indirect form of the CES production function (2) is expressed by

\[ \log(Y_{ij}/L_i) = \beta_0 + \beta_1 \log w_i + U_{ij}, \tag{5} \]

\[ j=1,2,...,r_i, \ i=1,2,...,n. \]

From model (5) it is evident that value-added, \( Y_{ij} \), is expressed by

\[ Y_{ij} = e^{\beta_0 + \beta_1 \log w_i + U_{ij}}, \tag{6} \]

\[ j=1,2,...,r_i, \ i=1,2,...,n. \]

Thus the sample mean of value-added for reporting firms in the i-th asset-size category, \( \bar{Y}_i \), is expressed by

\[ \bar{Y}_i = \frac{1}{r_i} \sum_{j=1}^{r_i} Y_{ij}, \tag{7} \]

where \( \bar{V}_i = \frac{1}{r_i} \sum_{j=1}^{r_i} V_{ij} \) and \( V_{ij} \) has lognormal distribution. The logarithmic form of model (7) is expressed by

\[ \log(\bar{Y}_i/L_i) = \beta_0 + \beta_1 \log w_i + \log \bar{V}_i, \ i=1,2,...,n. \tag{8} \]

It readily follows from standard asymptotic methods that the random variable, \( r_i (\log \bar{V}_i - \frac{1}{2} \sigma^2_U) \), converges in distribution, as \( r_i \) approaches infinity, to a normal random variable with mean zero and variance, \( (e^{\sigma^2_U} - 1) \).

Thus, if \( r_i \) is sufficiently large, then the distribution of \( \log \bar{V}_i \) is approximately normal with mean \( \frac{1}{2} \sigma^2_U \), and variance, \( (e^{\sigma^2_U} - 1)/r_i \). Given this result, and appropriate regularity conditions, it follows that a consistent and asymptotically efficient estimator for the elasticity of substitution, \( \sigma \equiv \beta_1 \), is obtained by applying weighted least-squares regression to model (8).
Suppose that the stochastic variable-returns-to-scale CES production function [cf. Brown and de Cani (1962)] applies:

\[ U_{ij} = \gamma(\delta K_{ij}^{-\rho} + (1-\delta) L_{ij}^{-\rho})^{\gamma-\rho} \rho e^{U_{ij}} \]  

\[ j=1,2,\ldots,r_i, \quad i=1,2,\ldots,n, \]

where, in addition to the parameters and random errors defined for (1), the homogeneity parameter, \( \gamma \), is a positive constant. For this CES production function, the elasticity of substitution is the same as for the constant-returns-to-scale CES model (1).

A possible indirect form for the model (9), based on the assumption of perfect competition in the factor and product markets, is defined by

\[ \log(Y_{ij}/L_{ij}) = \beta_0 + \beta_1 \log w_{ij} + \beta_2 \log L_{ij} + U_{ij}, \]

\[ j=1,2,\ldots,r_i, \quad i=1,2,\ldots,n, \]

where the parameters \( \beta_1 \) and \( \beta_2 \) are defined by \( \beta_1 = \gamma(\gamma+\rho)^{-1} \) and \( \beta_2 = \rho(\gamma-1)(\gamma+\rho)^{-1} \) [cf. Behrman (1982, p.161)]. This indirect form differs from that given by Dryhmes (1965) and Katz (1969, pp.70-72) in which the logarithm of value-added is involved on the right-hand side of their equation.

Given the specifications of Condition A, it follows that the estimable indirect form of the variable-returns-to-scale CES production function is given by

\[ \log(Y_{i}/L_{i}) = \beta_0 + \beta_1 \log w_i + \beta_2 \log L_i + \log \bar{V}_i, \]

\[ i=1,2,\ldots,n, \]

where \( \log \bar{V}_i \) has the distributional properties as in (8).

It is evident that the indirect form (8) of the constant-returns-to-scale CES production function (1) is obtained from model (11) if \( \beta_2 = 0 \). Thus, it follows that the t-statistic associated with the weighted least-
8.

squares estimator for $\beta_2$ in model (11) has approximately $t_{n-3}$ distribution of the constant-returns-to-scale CES production function (1) applies.

It is readily verified that the parameters, $\beta_1$ and $\beta_2$, of the indirect form (11) are expressed in terms of the elasticity of substitution, $\sigma$, by

$$\beta_1 = (1 + \beta_2)\sigma.$$  \hspace{1cm} (12)

From this result, it follows that

(i) if $\beta_2 = 0$, then $\beta_1 = \sigma$;

(ii) if $\beta_2 \neq -1$, then $\sigma = \beta_1(1 + \beta_2)^{-1}$; and

(iii) if $\sigma = \sigma_0$, where $\sigma_0$ is a known constant, then $\beta_1 = (1 + \beta_2)\sigma_0$ and, conversely, if $\beta_1 = (1 + \beta_2)\sigma_0$, then $\sigma = \sigma_0$, provided $\beta_2 \neq -1$.

These results imply that the weighted least-squares estimator for the coefficient of the logarithm of wages, $\beta_1$, in model (11), is not an unbiased estimator for the elasticity of substitution, unless the constant-returns-to-scale CES production function (1) applies. If the coefficient of the logarithm of labour, $\beta_2$, in model (11), is not minus one, then the elasticity of substitution is defined and, given appropriate regularity conditions for the indirect form (11), a consistent estimator for the elasticity of substitution is defined by

$$\hat{\sigma} = \hat{\beta}_1(1 + \hat{\beta}_2)^{-1}.$$  \hspace{1cm} (13)

where $\hat{\beta}_1$ and $\hat{\beta}_2$ represent the weighted least-squares estimators for $\beta_1$ and $\beta_2$ in the indirect form (11). Although this estimator for the elasticity of substitution does not have a finite mean or variance, the variance of the asymptotic distribution associated with it can be estimated by standard
methods involving a first-order Taylor expansion of the estimator.

Further the result of (iii) above is significant for obtaining hypothesis tests on the elasticity of substitution. That is, provided the coefficient of the logarithm of labour, $\beta_2$, in the indirect form (11) is not equal to minus one, the null hypothesis, $H_0: \sigma = \sigma_0$, that the elasticity of substitution is equal to some given value, $\sigma_0$ (say, zero or one), is equivalent to the null hypothesis, $H_0: \beta_1 = (1 + \beta_2)\sigma_0$, which can be tested by standard regression methods.

(b) VES Production Functions

Suppose that the stochastic constant-returns-to-scale VES production function [cf. Lu and Fletcher (1968)] applies:

$$ Y_{ij} = \gamma \left( \delta K_{ij} - \rho \right) + (1 - \delta) \eta L_{ij} K_{ij} / L_{ij} \gamma^{-c} (1 + \rho)^{-1} / \rho \epsilon U_{ij}, $$

$$ j=1,2,\ldots,r, \quad i=1,2,\ldots,n, $$

(14)

where, in addition to the parameters and random errors defined for (1), $\eta$ is a positive parameter. Given the assumption of perfect competition in the factor and product markets, the indirect form of this VES production function is defined by

$$ \log(Y_{ij}/L_{ij}) = \beta_0 + \beta_1 \log w_{ij} + \beta_2 \log(K_{ij}/L_{ij}) + U_{ij}, $$

$$ j=1,2,\ldots,r, \quad i=1,2,\ldots,n, $$

(15)

where $\beta_1 \equiv (1 + \rho)^{-1}$ and $\beta_2 \equiv c$.

The elasticity of substitution for this VES production function is expressed in terms of the parameters of its indirect form (15) by

$$ \sigma = \beta_1 (1 - \epsilon \beta_2)^{-1} $$

(16)

where $r$ is the interest rate on capital and $\epsilon \equiv (wL + rK)/rK$ is the ratio of
the total factor costs to the cost of capital.

If the specifications of Condition A are satisfied, then the indirect form (15) is expressed by

\[ \log(Y_{ij}/L_{ij}) = \beta_0 + \beta_1 \log w_{ij} + \beta_2 \log(K_{ij}/L_{ij}) + U_{ij}, \]

\[ j=1,2,...,r_i, \quad i=1,2,...,n. \]

From this the following estimable model is obtained:

\[ \log(Y_{i}/L_{i}) = \beta_0 + \beta_1 \log w_{i} + \beta_2 \log(K_{i}/L_{i}) + \log \bar{V}_i, \]

\[ i=1,2,...,n, \]

where \( \log \bar{V}_i \) has the same statistical properties as in model (8). Given that \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) denote the weighted least-squares estimators for \( \beta_1 \) and \( \beta_2 \) in the indirect form (18) and necessary regularity conditions are satisfied, then a consistent estimator for the elasticity of substitution (16) is defined by

\[ \hat{\sigma} = \hat{\beta}_1 (1 - \hat{\beta}_2)^{-1}. \]

Suppose that the stochastic variable-returns-to-scale VES production function [cf. Yeung and Tsang (1972)] applies:

\[ Y_{ij} = \gamma \left\{ \delta K_{ij}^{\rho} + (1-\delta)\gamma L_{ij}^{\rho}(K_{ij}/L_{ij})^{-c(1+\rho)} \right\}^{-\rho/-\rho \cdot e^{-U_{ij}},} \]

\[ j=1,2,...,r_i, \quad i=1,2,...,n. \]

Given the assumption of perfect competition in the factor and product markets, the indirect form of this VES production function is defined by

\[ \log(Y_{ij}/L_{ij}) = \beta_0 + \beta_1 \log w_{ij} + \beta_2 \log(K_{ij}/L_{ij}) \]

\[ + \beta_3 \log L_{ij} + U_{ij}, \]

\[ j=1,2,...,r_i, \quad i=1,2,...,n, \]
11.

where $\beta_1 \equiv \nu(\nu + \rho)^{-1}$, $\beta_2 \equiv c$ and $\beta_3 = (\nu - 1)(1 - \beta_1)$.

If the specifications of Condition A are satisfied, then the estimable indirect form, associated with aggregate data for asset-size categories, is given by

$$\log(\bar{y}_i/L_i) = \beta_0 + \beta_1 \log w_i + \beta_2 \log(K_i/L_i) + \beta_3 \log L_i$$

$$+ \log \bar{v}_i$$

$$i = 1, 2, \ldots, n.$$ (22)

It is evident that the indirect form (8) of the stochastic constant-returns-to-scale CES production function is obtained from the VES production functions if $\beta_2 = 0$ in (18) or if $\beta_2 = 0$ and $\beta_3 = 0$ in (22). Further, if $\beta_3 = 0$ in model (22), then the VES production function has constant returns to scale.

Given that the random variables, $\log \bar{v}_i$, $i = 1, 2, \ldots, n$, in models (18) and (22) have approximately normal distribution with mean, $\mu_0$, and variance, $(\nu \delta U - 1)/\nu_1$, it follows that

(a) the t-statistic associated with the weighted least-squares estimator for $\beta_2$ in model (18) has approximately $t_{n-3}$ distribution, if the constant-returns-to-scale CES production function (1) applies;

(b) the F-statistic associated with the weighted least-squares estimators for $\beta_2$ and $\beta_3$ in model (22) has approximately $F_{2,n-4}$ distribution, if the constant-returns-to-scale CES production function applies; and

(c) the t-statistic associated with the weighted least-squares estimator for $\beta_3$ in model (22) has approximately $t_{n-4}$ distribution, if the constant-returns-to-scale VES production function (14) applies.
(c) Modelling for Time-Series Data

Often in empirical studies, elasticities of substitution are estimated on the basis of time-series data. We now briefly consider the specification of the above stochastic production functions for more than one time period. In the most general situation, the parameters and variables of the production functions are indexed by the time periods involved.

Suppose that, for a given industry and location, the constant-returns-to-scale CES production function (1) applies for T time periods, such that the production functions are defined by

\[
Y_{tij} = \gamma \left( \delta_t K_{tij} + (1 - \delta_t) L_{tij} \right)^{-1/\rho_t} e^U_{tij},
\]

\[ j=1,2,...,r_{ti}, \quad i=1,2,...,n_t, \quad t=1,2,...,T, \]

where \( Y_{tij} \), \( K_{tij} \) and \( L_{tij} \) denote the value-added, capital and labour, respectively, for the \( j \)-th firm in the \( i \)-th asset-size category at the \( t \)-th time period; \( \gamma_t, \delta_t \) and \( \rho_t \) denote the parameters of the CES production function for the \( t \)-th time period; the random errors, \( U_{tij} \), \( j=1,2,...,r_{ti}, \quad i=1,2,...,n_t, \quad t=1,2,...,T, \) are independently distributed as normal random variables with means zero and variances, \( \sigma^2_U; \quad r_{ti} \) denotes the number of reporting firms in the \( i \)-th asset-size category in the \( t \)-th time period; and \( n_t \) denotes the number of asset-size categories in the \( t \)-th time period.

As outlined above, given the assumptions of perfect competition in the factor and product markets in each time period, the indirect forms for the CES production functions (23) are defined by

\[
\log(Y_{tij}/L_{tij}) = \beta_{t0} + \beta_{tl} \log w_{tij} + U_{tij},
\]

\[ j=1,2,...,r_{ti}, \quad i=1,2,...,n_t, \quad t=1,2,...,T, \]

where \( \beta_{t0} \) and \( \beta_{tl} \) are expressible in terms of the parameters of the CES production
function for the t-th time period. In particular, $\beta_{t1} \equiv (1+p_t)^{-1}$ is the elasticity of substitution for the t-th time period. It is obvious that, unless these elasticities are the same for all T-time periods, it is not meaningful to discuss estimation of the elasticity of substitution, on the basis of time-series data.

The hypothesis that the elasticity of substitution is constant over time for the CES production functions (23) can be tested if cross-sectional data are available for the time periods involved. If the specifications of Condition A are satisfied for the aggregative cross-sectional data for each time period, then the appropriate indirect forms are defined by

$$\log(\bar{Y}_{ti}/L_{ti}) = \beta_{t0} + \beta_{t1} \log w_{ti} + \log \bar{V}_{ti},$$

where $\bar{Y}_{ti} \equiv \frac{1}{r_{ti}} \sum_{j=1}^{r_{ti}} Y_{tij}$ is the sample mean of value-added for the reporting firms in the i-th asset-size category in the t-th time period; and $\log \bar{V}_{ti}$ has approximately normal distribution with mean, $\mu^{2}$, and variance, $(e^{U-1})/r_{ti}$, if the number of reporting firms, $r_{ti}$, in the i-th asset-size category in the t-th time period is sufficiently large.

Given the indirect form (25), the hypothesis that the slope parameters are equal (i.e., $\beta_{t1} = \beta_{1}$ for all $t=1,2,...,T$) can be tested by traditional regression methods. Further, it is possible to test the hypothesis of Hicksian neutral technological change (i.e., $\beta_{t0} = \beta_{0} + \delta_{t}$ and $\beta_{t1} = \beta_{1}$ for all $t=1,2,...,T$).

Suppose, however, that neither firm-level data nor aggregative data for asset-size categories are available. If only totals of value-added and input quantities are available for each time period, then the elasticity of substitution is estimable under the following restrictive condition:
Condition B:

(a) Wages and factor inputs are the same for all firms at any given time period, i.e., \( w_{tij} = w_t, L_{tij} = L_t \) and \( K_{tij} = K_t \) for all \( j=1,2,...,r_t, \)
\( i=1,2,...,n_t, t=1,2,...,T; \)

(b) The substitution parameters, \( \rho_t, t=1,2,...,T, \) are the same (and hence the elasticity parameters, \( (1+\rho_t)^{-1}, t=1,2,...,T, \) are the same over time); and

(c) The efficiency and distribution parameters, \( \gamma_t \) and \( \delta_t, t=1,2,...,T, \) are expressible as known functions of time, involving a reduced number of parameters.

If, for example, the case of Hicksian neutral technological changes exists (i.e., the distribution parameters are constant over time, \( \delta_t = \delta, t=1,2,...,T, \) and the efficiency parameters are an exponential function of time, \( \gamma_t = e^{\lambda t}, t=1,2,...,T, \) then, given the specifications of Condition B, the indirect forms (25) are expressed by

\[
\log(\frac{Y_{tij}}{L_t}) = \beta_0 + \delta_0 t + \beta_1 \log w_t + U_{tij},
\]
\( j=1,2,...,r_t, \)
\( i=1,2,...,n_t, t=1,2,...,T, \)

where \( \beta_0 = -\sigma \log(1-\delta), \delta_0 = \rho \sigma \) and \( \beta_1 = \sigma = (1+\rho)^{-1}. \)

Given that only total value-added is available for each time period, it follows that the estimable model associated with (26) is defined by

\[
\log(\tilde{Y}_t/L_t) = \beta_0 + \delta_0 t + \beta_1 \log w_t + \log \tilde{Y}_t,
\]
\( t=1,2,...,T, \)

where \( \tilde{Y}_t = \frac{1}{n_t} \sum_{i=1}^{n_t} \frac{1}{r_t} \sum_{j=1}^{r_t} Y_{tij} / \sum_{i=1}^{n_t} r_{ti} \) is the sample mean of value-added for the firms
reporting in the \( t \)-th time period; and 
\[
\tilde{V}_t = \sum_{i=1}^{n_t} \sum_{j=1}^{r_{ti}} \frac{V_{tij}}{r_{ti}} \sum_{i=1}^{n_t} r_{ti}
\]
where \( V_{tij} = e_{tij} \).

The elasticity of substitution, \( \beta_1 = \sigma \), is thus estimated by a weighted least-squares regression analysis associated with the indirect form (27).

However, it is evident that the specifications of Condition B, upon which the elasticity is identified and estimable using time-series data, are highly restrictive and unlikely to be satisfied in practice.

Similar difficulties to those indicated above apply for the estimation of elasticities when related products are aggregated to obtain a composite product, such as electrical machinery or metal products. This emphasises the desirability of obtaining data at the firm level, for well-defined products, at the time periods of interest.

3. Conclusions

The specification of stochastic production functions in terms of aggregate value-added and input quantities, as in most previous empirical studies, have two basic problems:

(i) It is unlikely that the stochastic errors involved are homoscedastic, as generally assumed; and

(ii) The parameters involved in such aggregative models may not be identified in terms of those for corresponding models, defined for firm-level data.

The first problem implies that the traditional estimates of the parameters of aggregative models are likely to be inefficient and precision of estimators be incorrectly estimated. The second problem is more significant because it has implications for the interpretation of the estimated elasticity of substitution. This is the focus of aggregation in production function analysis [see, e.g., Green (1964), Fisher (1965, 1968, 1969), Sato (1975) and Theil (1971)].
If the required specifications are not true in a given empirical situation, then it may be impossible to identify the elasticity of substitution or obtain a consistent estimator by weighted least-squares regression involving aggregative data on value-added and input quantities. This result follows by considering the arithmetic average of the values of model (2) for the i-th asset-size category:

\[ \log \left( \frac{Q_i}{L_i} \right) = \beta_0 + \beta_1 \log \hat{w}_i + \mathcal{U}_i, \]  

(28)

where \( \hat{w}_i \) denotes the geometric mean of the wages for firms in the i-th asset-size category. If the values of wages are not the same for all firms within given asset-size categories, then \( \hat{w}_i \neq \bar{w}_i \), so the estimator for \( \beta_1 \) obtained by the weighted least-squares regression of the logarithm of the ratio of the average value-added to average labour inputs on the logarithm of the average wage rates will be biased and inconsistent. However, provided the specifications of Condition A are not grossly violated, then the biases incurred may not be too serious.

An important shortcoming of our analyses and the traditional methods of estimation of the elasticity of substitution is the assumption that there are only two factors of production, homogeneous labour and homogeneous capital, used in the production of a single homogeneous output. This gives no consideration to the role of land, education, entrepreneurship, labour mix and capital mix in production. The malleability of factors is also an abstraction from reality. The possible effect of capacity utilization also needs more attention. This is especially relevant for manufacturing industries in developing countries, in which capital-intensive techniques of production have been installed, but often remain grossly underutilized due to lack of skilled labour, raw materials or access to markets.

The assumption of perfectly competitive markets is likely to be a serious
misrepresentation of the reality of the situation, particularly in developing countries. Thus the use of the indirect forms associated with the CES and VES production functions may result in biased estimation of elasticities of substitution. Further, the traditional production-function approach to the estimation of elasticities of substitution assumes that the data points lie on a common function. In practice, technical and allocative inefficiencies exist, which imply that observed data are likely to be below the defined production frontier.

Despite these shortcomings of approaches to estimate elasticities of substitution, the methodology outlined above is applied in the analysis of data for manufacturing industries in Pakistan in Battese and Malik (1986).
Footnote

1. The number of asset-size categories, n, as well as the numbers of firms within asset-size categories, must approach infinity. The matrix of independent variables for the model (8) must be regular [see, e.g., Theil (1971, p.363)].
References


21.


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