UNE Business School Occasional Papers

Number: 2014 - 01

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ABSTRACT

Forecasting volatility is fundamental to forecasting parametric models of Value-at-Risk. The exponentially weighted moving average (EWMA) volatility model is the recommended model for forecasting volatility by the Riskmetrics group. For monthly data, the lambda parameter of the EWMA model is recommended to be set to 0.97. In this study we empirically investigate if this is the optimal value of lambda in terms of forecasting volatility. Employing monthly realized volatility as the benchmark for testing the value of lambda, it is found that a value of lambda of 0.97 is far from optimal. The tests are robust to a variety of test statistics. It is further found that the optimal value of lambda is time varying and should be based upon recent historical data. This paper offers a practical method to increase the reliability and accuracy of Value-at-Risk forecasts that can be easily implemented within an Excel spreadsheet.

JEL Classification Codes: C5, G17

Keywords: EWMA, volatility, lambda, Value-at-Risk

1. INTRODUCTION

In 1994 leading financial services firm J. P. Morgan first developed and documented a set of procedures to quantify risk known collectively as Riskmetrics. One key metric of risk outlined in this document was the notion of Value-at-Risk (VaR). Value-at-Risk measures the most that a portfolio can lose in a given time horizon with a given probability. Basel II, the second of the Basel accords published in June 2004 had the explicit goal of monitoring and quantifying credit risk, operational risk and market risk. Market risk in particular was to be monitored and quantified using Value-at-Risk as the standard metric.

Both parametric and non-parametric methodologies to quantify Value-at-Risk have been widely adopted. This study focuses on one of the parametric methods outlined in the 1996 Rickmetrics technical document. VaR_α measures the most that a portfolio can lose (R) in a given time horizon with probability $1-\alpha$. It is defined as:

$$P(R < -VaR_\alpha) = \alpha \tag{1}$$

where typically α = 1% or 5%. Moosa and Bollen (2002) demonstrate that a forecast of VaR_α for a financial asset at time t can be written as:

$$\widehat{VaR}_{\alpha,t+1} = P_t \times z_\alpha \times \hat{\sigma}_{t+1} \tag{2}$$

where P_t is the price of the asset at time t, z_α is the one sided critical value taken from the normal distribution for a given α. (Note that the use of normal critical values does not rest on the assumption that returns are normally distributed but does assume that scaled returns (r_t/σ_t) are normally distributed. The literature supports this assumption, see for example Andersen *et al* (2000)). Variable $\hat{\sigma}_{t+1}$ is an estimate of the asset's volatility. It is clear from equation (2) that a robust parametric estimate of VaR_α is critically dependent upon being able to forecast asset volatility. To this end the 1996 Rickmetrics technical document recommends the use of the Exponentially Weighted Moving Average (EWMA) volatility model. Mina and Xiao (2001) recommend that the lambda (λ) decay parameter in the EWMA volatility model be set to 0.97 when using monthly data. This study is an empirical investigation as to what the value of λ should be when working with monthly data.

The paper proceeds as follows. In section (2) the EWMA volatility model is formally defined and its properties evaluated. In section (3) the nature of the testing procedures in this study are outlined with particular reference to monthly realized volatility as the benchmark when testing volatility forecasts. In section (4) the methodology and empirical results are reported. Both in-sample estimates and out-of-sample forecasts are reported to evaluate the optimal value of lambda. Section (5) offers some concluding comments.

2. THE EXPONENTIALLY WEIGHTED MOVING AVERAGE MODEL

Despite considerable academic interest in the Auto Regressive Conditional Heteroscedastic (ARCH) family of volatility models over the last 30 years, finance practitioners have tended to employ far simpler volatility models that can be easily implemented within an Excel spreadsheet. The simplest of these volatility models is the historical volatility estimator (see Figlewski (1997)). If portfolio volatility were constant over time it can be estimated efficiently with the historical volatility estimator. The historical volatility estimator is the sample variance of historical returns defined as:

$$\hat{\sigma}_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_{t-i} - \bar{r})^2 \tag{3}$$

where r_t is the portfolio return in month *t*, n is the length of the sample period, $\bar{r}$ is the mean return over the sample period and $\hat{\sigma}_t^2$ is the estimated portfolio volatility in month *t.*

If, however, portfolio volatility clusters in high and low periods, a more robust way to estimate volatility is to implement a model where more weight to given to recent returns and less weight to more distant returns. One such model is the Exponentially Weighted Moving Average (EWMA) model which is defined as:

$$\hat{\sigma}_{t+1}^2 = \lambda\hat{\sigma}_t^2 + (1-\lambda)r_{t-1}^2 \tag{4}$$

where λ is the decay factor $(0 < \lambda < 1)$ and all other variables and parameters are as previously defined. The lower the decay factor, the lower the influence of more distant squared returns. Equation (4) can alternatively be expressed as a geometrically declining lag process in squared historical returns as:

$$\hat{\sigma}_{t+1}^2 = (1-\lambda)\sum_{i=0}^{n}\lambda^i r_{t-i}^2 \tag{5}$$

where in theory $n = \infty$ but in practice n is set to a finite number such that the influence of more distant squared returns becomes negligible. Harris and Shen (2003) state in regard to the EWMA model, "… it is often found to generate short-run forecasts of the variance-covariance matrix that are as good as those of more sophisticated volatility models …" (page 805). The EWMA model is a special case of the IGARCH(1,1) model where volatility innovations have infinite persistence. The assumption that volatility innovations infinitely persist through time may appear theoretically tenuous, however it appears to be a reasonable assumption for short term volatility forecasting. Moosa and Bollen (2002) find that the EWMA volatility estimator outperforms a range of ARCH volatility models when forecasting *VaR* over short time horizons.

3. DATA AND REALIZED VOLATILITY

The analysis is conducted using both daily and monthly data on the S&P 500 Index from January 1957 to September 2013 (680 trading months). The data was sourced from the Federal Reserve Bank of St Louis[1]. Ideally testing of the EWMA volatility model should be conducted by comparing volatility forecasts from this model with the 'true' volatility for a particular month. The problem with this approach, of course, is that volatility is a latent unobserved variable and we do not have access to the 'true' volatility of any trading month.

However, it is possible to develop an efficient and consistent estimate of the 'true' volatility of any trading month by estimating the realized volatility of each trading month. This is done by calculating the daily return of each trading day and then summing the square of daily returns. Monthly realized volatility in month *t* is thus given as:

$$\sigma_t^2 = \sum_{i=1}^{n_t} r_{it}^2 \tag{6}$$

[1] See http://research.stlouisfed.org/fred2/series/SP500/downloaddata

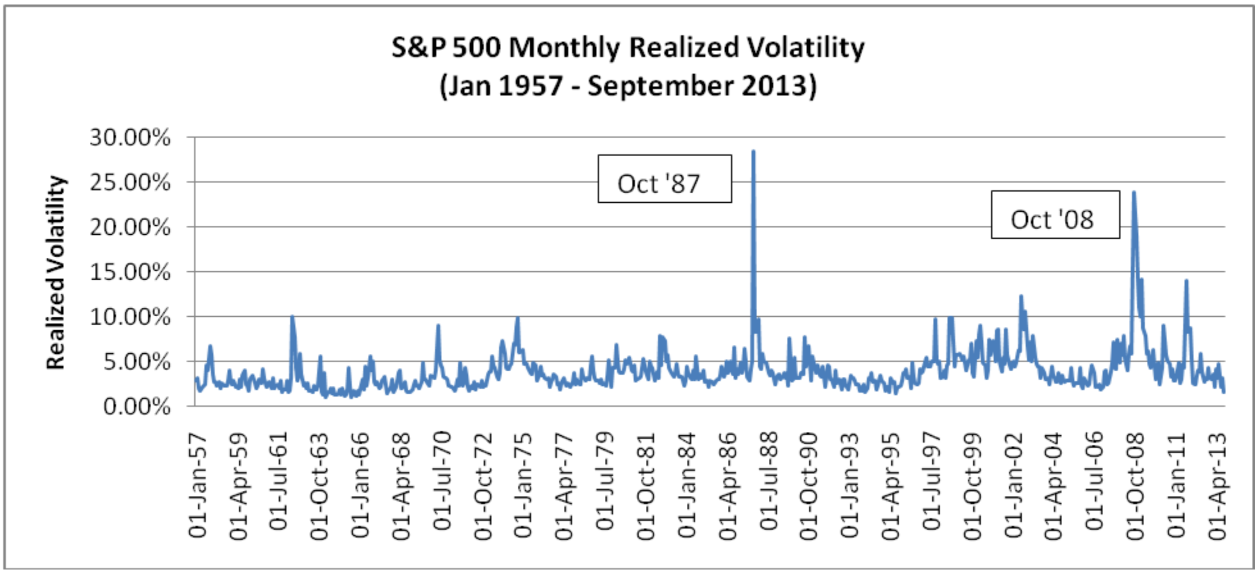
where r_{it} is the i^{th} trading day return in month t and n_t is the number of trading days in month t. This is unconditional, *ex post* estimator of monthly volatility does not require the daily return series to be homoskedastic, but it does require daily returns to be uncorrelated. In this study the monthly realized volatility is employed as the benchmark to test the EWMA volatility forecasts. Andersen *et al* (1999) argues that realized volatility can be regarded as the actual and observed volatility measure if there are a sufficient number of return observations. This statement is justified by noting that:

$$plim_{n_t \to \infty}\left(\sum_{i=1}^{n_t} r_{it}^2\right) = \sigma_t^2 \tag{7}$$

Consequently the monthly realized volatility of the S&P500 is calculated by summing the square of daily returns for each trading month over the period January 1957 to September 2013. The level of monthly realized volatility over this trading period is displayed in Figure (1).

Figure (1) – Monthly realized volatility of the S&P500 index.

This figure displays the monthly realized volatility of the S&P500 index over the trading period January 1957 to September 2013. Monthly realized volatility is defined as $\sigma_t^2 = \sum_{i=1}^{n_t} r_{it}^2$ where r_{it} is the i^{th} trading day return in month t and n_t is the number of trading days in month t.



From an inspection of Figure (1) it can be seen that volatility does indeed cluster in low and high periods as modeled by the ARCH family of the volatility models. The two prominent volatility spikes correspond to the financial market crash of October 1987 and the collapse of Lehman Brothers in September-October 2008 in the midst of the Global Financial Crisis.

4. METHODOLOGY

The method to evaluate the optimal value of λ is in terms of one month ahead volatility forecasts requires a metric to evaluate by how much a EWMA volatility forecast varies from the benchmark realized volatility. Four statistics that compare realized volatility with the forecasts from an EWMA model are reported, the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), the heteroskedasticity adjusted RMSE (HRMSE) and the heteroskedasticity adjusted MAE (HMAE). These four statistics are formally defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\sigma_t^2 - \hat{\sigma}_t^2)^2} \tag{8a}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|\sigma_t^2 - \hat{\sigma}_t^2| \tag{8b}$$

$$HRMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right)^2} \tag{8c}$$

$$HMAE = \frac{1}{n}\sum_{i=1}^{n}\left|1 - \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right| \tag{8d}$$

where σ_t^2 is the realized volatility and $\hat{\sigma}_t^2$ is a volatility forecast from the EWMA model in month *t*. These four statistics should not be treated equally. As Anderson *et al* (1999) points out, "… the fact that the highly non-linear and heteroskedastic environment may render the usual measures based on root-mean squared errors [RMSE] unreliable, so we also report the more robust mean absolute errors [MAE]. … To better accommodate the heteroskedasticity in the forecast errors, we also calculate the corresponding heteroskedasticity adjusted statistics [HRMSE and HMAE]".

The analysis is conducted on both in-sample estimates and out-of-sample forecasts. The in-sample analysis proceeds as follows.

i. Calculate the historical volatility of monthly returns over the period $t = 2$ to $t = 36$ (Feb 1957 to Dec 1959). This is the starting volatility that seeds the in-sample EWMA volatility estimates.
ii. Iteratively calculate the EWMA volatility estimates for each month from $t = 37$ to $t = 680$ for a given λ.
iii. Calculate the 'error' between the realized volatility and EWMA volatility estimate in each month for each of the four statistics. For RMSE the error is given as $(\sigma_t^2 - \hat{\sigma}_t^2)^2$, for MAE the error is given as $|\sigma_t^2 - \hat{\sigma}_t^2|$, for HRMSE the error is given as $\left(1 - \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right)^2$ and for HMAE the error is given as $\left|1 - \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right|$.
iv. For the RMSE and HRMSE statistics calculate the square root of the average of their corresponding errors to derive the final RMSE and HRMSE statistics. For

the MAE and HMAE statistics calculate the average of their corresponding errors to derive the final MAE and HMAE statistics.

v. Minimize each of these four statistics by varying λ using a constrained optimization algorithm where λ is restricted to the range $0 \leq \lambda \leq 1$.

The values of λ that minimize the four statistics and the minimum of the four statistics are reported in table (1).

Table (1) – λ estimates which minimize the in-sample RMSE, MAE, HRMSE and HMAE statistics.

This table displays the minimum RMSE, MAE, HRMSE and HMAE statistics calculated from the realized monthly volatility of the S&P500 index and the in-sample estimates from an EWMA model for a given λ. The table displays the minimum of each statistic and the value of λ that minimizes each statistic. The source data is the S&P500 index over the period January 1957 to September 2013.

Statistic	λ	Minimum of the statistic
RMSE	0.7044	0.004492
MAE	0.7292	0.001420
HRMSE	0.8788	2.200232
HMAE	0.8749	0.790978

From Table (1) it can be seen that the value of λ which minimizes the four statistics is highly sensitive to adjustments for heteroskedasticity. Whilst the value of lambda which minimizes the RMSE and MAE statistics are about equal (0.7044 & 0.7292), the value of lambda which minimizes the HRMSE and HMAE statistics are also about equal (0.8788 & 0.8749) but of a significantly higher value than those given by the the RMSE and MAE statistics. Given that financial data is highly heteroskedastic in nature it would seem prudent to give more weight to the results given by the HRMSE and HMAE statistics.

The procedure to empirically investigate the optimal value of λ in terms of one month ahead volatility forecasting (out-of-sample) is based on a rolling 36 month window of EWMA estimates. These estimates are seeded by taking the variance of returns over the 12 months before the rolling window begins. The optimal value of lambda is calculated based upon the four statistics. A one step ahead volatility forecast is then made using this optimal value of lambda for each of the four statistics. Formally, the method proceeds as follows. For each time period t, starting at $t = 50$ (Feb 1961):

i. Calculate the historical volatility of monthly returns over a one year period from time $t - 37$ to time $t - 48$. This is the starting volatility that seeds the out-of-sample EWMA volatility forecasts.

ii. Iteratively calculate the EWMA volatility from $t = -1$ to $t = -36$ for a given λ.

iii. Calculate the 'error' between the realized volatility and EWMA volatility estimate in each month over the period from $t = -1$ to $t = -36$ for a given λ for each of the four statistics.

iv. Calculate the RMSE, MAE, HRMSE and HMAE from their corresponding errors statistics over the period from $t = -1$ to $t = -36$ for a given λ.

v. Minimize each of these four statistics by varying λ using a constrained optimization algorithm where λ is restricted to the range $0 \leq \lambda \leq 1$.

vi. Use this value of λ to construct a forecast of the EWMA volatility at time t for each of the four statistics.

vii. Repeat steps i to vi for all time periods $t = 50$ to $t = 680$.

viii. For all of the 631 one step ahead forecasts, calculate the RMSE of the forecast errors if the selection criteria for the minimum λ was the RMSE of the errors over the preceding 36 months. This is similarly done for the MAE, HRMSE and HMAE statistics.

The average value of λ that minimizes the four statistics and the four statistics for forecast errors are reported in Table (2).

<u>Table (2) – λ estimates which minimize the out-of sample RMSE, MAE, HRMSE and HMAE forecast statistics.</u>

This table displays the average RMSE, MAE, HRMSE and HMAE statistics calculated from the realized monthly volatility of the S&P500 index for one step ahead forecasts from an EWMA model. The table displays each statistic and the average value of λ for each statistic. The source data is the S&P500 index over the period January 1957 to September 2013.

Statistic	Average λ	Value
RMSE	0.7125	0.004425
MAE	0.7201	0.001388
HRMSE	0.7769	2.036870
HMAE	0.7753	0.818455

From Table (2) it can be seen that the average value of λ which minimizes the four statistics is again highly sensitive to adjustments for heteroskedasticity. The average value of lambda which minimizes the RMSE and MAE statistics are about equal (0.7125 & 0.7201) and of similar values to those values found with in-sample testing. The average value of lambda which minimizes the HRMSE and HMAE statistics are also about equal (0.7769 & 0.7753) but are considerably lower than the lambdas found with in-sample testing. The value of the RMSE, MAE and HRMSE of the forecast errors are all reduced relative to the values found with in-sample testing. The value of the HMAE statistic of the forecast errors increased slightly.

It is also found that the optimal value of lambda varies considerably when calculating out-of-sample forecasts. In table (3) the frequency of different values of lambda when conducting the out-of-sample volatility forecasts for the four statistics are displayed.

<u>Table (3) – The frequency of the values of λ for the four statistics for the out-of-sample tests.</u>

This table displays the frequency of the various values of the optimal value λ for the RMSE, MAE, HRMSE and HMAE statistics when conducting out-of-sample testing. The source data is the S&P500 index over the period January 1957 to September 2013.

λ	RMSE	MAE	HRMSE	HMAE
$\lambda = 0$	3	0	0	0
$0 < \lambda < 0.1$	34	1	1	0
$0.1 \leq \lambda < 0.2$	5	6	2	0
$0.2 \leq \lambda < 0.3$	15	41	1	2
$0.3 \leq \lambda < 0.4$	45	74	8	6
$0.4 \leq \lambda < 0.5$	30	31	50	43
$0.5 \leq \lambda < 0.6$	38	30	45	38
$0.6 \leq \lambda < 0.7$	56	54	91	85
$0.7 \leq \lambda < 0.8$	90	72	95	140
$0.8 \leq \lambda < 0.9$	115	108	147	178
$0.9 \leq \lambda < 1$	184	188	156	110
$\lambda = 1$	16	26	35	29

From Table (3) it can be seen that the optimal value of lambda for the 36 month rolling window of volatility forecasts varies considerably. The majority of optimal lambdas are in the range 0.4 to 1, particularly when employing the HRMSE and HMAE statistics.

The final test involves comparing the value of the four statistics when using the value of lambda $(\lambda = 0.97)$ recommended by Mina and Xiao (2001) from the Riskmetrics group with those values obtained from both in-sample and out-of-sample testing. The value of the four statistics when $\lambda = 0.97$ are displayed in table (4).

Table (4) – The value of the RMSE, MAE, HRMSE and HMAE statistics when $\lambda = 0.97$.

This table displays the value of the RMSE, MAE, HRMSE and HMAE statistics when $\lambda = 0.97$. The source data is the S&P500 index over the period January 1957 to September 2013.

Statistic	Value
RMSE	0.004729
MAE	0.001587
HRMSE	2.636429
HMAE	0.866197

A direct comparison of the RMSE, MAE, HRMSE and HMAE statistics in Table (4) with those displayed in tables (1) and (2) clearly show an increase in all four of the statistics. This result must call into question the recommendation of the Riskmetrics group that lambda should equal 0.97 when using monthly data.

5. SUMMARY OF MAIN FINDINGS

This study has endeavored to empirically verify or reject the Riskmetrics group's recommendation that the lambda parameter in the EWMA model should be set to 0.97. This study has offered evidence to reject the Riskmetrics group's recommendation. In the light of this conclusion some recommendations as to what the value of lambda should be set to when using monthly data are offered.

Both in-sample and out-of-sample testing point to a value of lambda of about 0.72 when relying on the RMSE and MAE criteria. But as Anderson *et al* (1999) point out, neither of these two statistics make any adjustment for heteroskedasticity. This is a significant problem when using financial data characterized almost universally by heteroskedasticity. The two heteroskedasticity adjusted statistics (HRMSE and HMAE) however offer differing results. When using in-sample testing both of these statistics point to a value of lambda of about 0.88. In contrast, when using out-of-sample testing both of these statistics point to a value of lambda of about 0.78. This is not a trivial difference and some sort of resolution is required.

The first point to be made in this regard is that the object of the current study is to optimize the forecasting ability of the EWMA model. Thus some weight must be given to the results given by out-of-sample testing. An inspection of the lambdas when using out-of-sample testing shows considerable variation in the value of lambda. Thus it needs to be asked if when implementing the EWMA, if any fixed unconditional value of lambda should be used at all. If it is accepted that lambda is indeed time varying then it made be prudent to only employ recent historical data to find the optimal value of lambda. This can be done by using the optimisation methodology employed in this study to find a value of lambda. Importantly for finance practitioners, this can be easily implemented within an Excel spreadsheet using the Solver function.

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