TOWARD A MATHEMATICAL MODEL OF MALTHUS

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There have been several interesting and ingenious attempts to derive mathematical models of Malthus. Three such attempts - by Eagly 1974, Eltis 1980, and Costabile and Rowthorn 1985 (hereafter abbreviated as CR 1985) - are considered in this paper. The authors describe their models as elementary and unsophisticated, and admit they are based on simplified assumptions. Nevertheless, each of the models succeeds in identifying some significant features of Malthus' system.

But while fully acknowledging the achievements of these three models, this paper argues that they have not succeeded in capturing some important, and even essential, features of Malthus' system; and, by drawing attention to these omitted features, attempts to make some positive contributions toward the formulation of a Malthus model.

The method used in this paper to determine the essentiality of the various elements of Malthus' system is textual analysis. When the paper claims that, in my view, certain elements are essential in Malthus' system, these claims are derived from a detailed study of Malthus' texts and are supported by quotations and/or references which indicate, in Malthus' own words, their essentiality to his system. The earlier models also appear to have adopted textual evidence as their essentiality criterion. They are not criticised therefore on methodological grounds. I would argue, however, that, although their analyses of the texts have identified some elements that are essential to Malthus' system, there remain other elements, which, in my view, are also essential to Malthus' system, and which should therefore be incorporated in a Malthus model. There would be no grounds for criticising the three earlier models if they were explicitly put forward as models of selected parts of Malthus, or as variations on a theme by Malthus, or as deliberate distortions of Malthus' thought for a particular purpose. But any model which claims to be a model of Malthus should surely not exclude any elements which, on the basis of textual evidence, can be shown to have been essential elements in Malthus' system.

The argument of this paper is restricted to features considered to be essential or important. No attempt is made to identify each and every causative factor mentioned by Malthus. An attempt at such comprehensiveness would probably result in a model of great complexity and little utility. The (hopefully constructive) criticisms made of the three models are therefore directed, not at their lack of comprehensiveness, but at the essential or important aspects of Malthus' system which they appear to have omitted.

Doctrine of Proportions

Perhaps the most serious and the most untextual defect of the models being considered here is, in my view, the insufficient attention¹ they give to Malthus' "doctrine of proportions". This doctrine - which was an application to political economy of the traditional ethical notion of the just mean or middle way - was a prominent and essential feature of Malthus' analytical framework. He stated that his aim in writing the *Principles of Political Economy* was to show "how frequently the doctrine of proportions meets us at every turn, and how much the wealth of nations depends upon the relation of parts"²; and he applied this doctrine of proportions to many of the problems he considered, including saving, population, the distribution of property, the balance between productive and unproductive labour, etc. In modern terminology, his aim was to show how wealth and welfare depend upon the attainment of an optimum combination of the determining variables; and it is difficult to understand how any attempt to formulate a mathematical model of Malthus can hope to be successful if it does not give adequate emphasis to this essential aspect of his methodology.³

The implication of Malthus' doctrine of proportions for a Malthus model is that many of its key equations cannot be linear or monotonic, but must express a relationship between income and its determining variables such that up to the optimum point an increase in the determining variable will cause an increase in income, but beyond that optimum point further increases in the determining variable will cause income to decline.

The Eltis model and the CR model do not incorporate this rise-and-fall feature. In their defence it might be argued that they aim to be only highly simplified models in which first-order linear relationships are a justifiable approximation. But in reply to such a defence I would argue that models which are so simplified that they either omit or abstract from the rise-and-fall nature of Malthus' key equations fail to capture an *essential* feature of his system, and result not in procedural simplification but in textual misinterpretation.

Fixed Capital

An assumption of the CR model is that there is no fixed capital, that no raw materials are required for production, and that the only capital is the wages fund. This assumption of no fixed capital leads the authors to a conclusion which they regard as an "essential point" in Malthus' theory, viz. "variations in money wages will not in themselves influence the real wage ... " (CR 1985, p. 426).

The CR model (p.418) gives two reasons for assuming no fixed capital. The first is "lack of space". The second, which is merely stated, not argued, is that the issues covered in the model "can be considered in isolation from the rest of [Malthus'] ideas". Although the no-fixed-capital assumption is quite unrealistic - in the double sense of departing from the real world and departing from Malthus' text - it might at first sight be regarded as a legitimate and helpful procedural simplification, because it reduces the number of concepts and symbols to be handled in the model, and because it can be easily relaxed by incorporating appropriate functional relationships between the wages fund and other forms of capital. However, the authors of the CR model do not subsequently relax the assumption, and do not therefore note that when the assumption is relaxed the conclusion which they regard as an "essential point" in Malthus' theory ceases to be valid.

(1)

Thus, adapting the notation of the CR model,⁴

if K = total capital = wages + fixed capital = L_t M_{t+1}+F_t

F = fixed capital

L = amount of labour employed

M = money wage rate paid to workers

t = year t

then the price per unit of output is given by

$$P_{t} = \frac{L_{t} M_{t+1} + \pi_{t} (L_{t} M_{t+1} + F_{t})}{O_{t}}$$
(2)

where

π = rate of profit = physical output

and real wages are:

0

$$w_{t+1} = \frac{M_{t+1}}{P_t}$$
(3)

$$= \frac{M_{t+1}O_t}{L_t M_{t+1} (1 + \pi_t) + F_t \pi_t}$$
(4)

By assuming there is no fixed capital (i.e. $F_t = 0$), the CR model derived real wages as

$$w_{t+1} = \frac{O_t}{(1 + \pi_t) L_t}$$
 (5)

and concluded that real wages are independent of money wages.

But when the no-fixed-capital assumption is removed, the money wages term (M_{t+1}) cannot be eliminated from the right-hand side of equation (4), and hence remains as a determinant of real wages.⁵ Thus, although the CR conclusion that "variations in money wages will not in themselves influence the real wage ..." is an essential deduction from the limited assumptions of the CR model, it ceases to be valid when the "no fixed capital" assumption is relaxed, and therefore, contrary to the CR claim, it is not an essential aspect of Malthus' theory.⁶

Population Supply Function

Both the Eltis and the CR model establish (in slightly different forms) positive relations between population and income. In both relations the parameter α - the elasticity of population with respect to income or wages - is said to be positive.⁷ But neither model addresses the question of whether this positive coefficient is greater than, less than, or equal to unity. This is a crucial question for any Malthus model. Given a situation in which the population is at subsistence level, any increase in income will result in either more misery, or less misery, or an unchanged amount of misery for each person, depending on whether α is greater than, less than, or equal to unity. It is essential therefore that a Malthus model should specify the size, and not merely the sign of α .

In attempting to specify the size of α it would be necessary to make a distinction between, on the one hand, Malthus' descriptive or pessimistic model which outlines the reasons for the current distress and which warns of what will continue to happen if the world does not heed his message; and, on the other hand, his prescriptive or optimistic model which portrays the beneficial effects of the remedies he proposes. A central feature of his prescriptive or preferred⁸ model is that population should *not* continue to increase indefinitely as wages increase, but that through the practice of prudential restraint and with the spread throughout all social classes of a taste for conveniences, luxuries and leisure, increases in wages would be directed at least in part to increasing living standards, and not directed entirely towards supporting an ever-increasing population at subsistence level. This feature of Malthus' population supply function is evident, for example, in his statement:

From high wages, or the power of commanding a large proportion of the necessaries of life, two very different results may follow; one, that of a rapid increase of population, in which case the high wages are chiefly spent in the maintenance of large and frequent families: and the other, that of a decided improvement in the modes of subsistence, and the conveniences and comforts enjoyed, without a proportionate acceleration in the rate of increase.⁹

The population supply functions put forward in the CR model and the Eltis model do not explicitly signal the population-income relationship which Malthus preferred, and therefore cannot be taken to represent fully Malthus' position on this issue.¹⁰

Saving

(i) The Optimum Level of Saving

The importance which Malthus attached to the "doctrine of proportions" is clearly evident in his discussion of the relation between saving and growth. Whereas Adam Smith said that "every frugal man is a public benefactor" and Lauderdale said that saving is harmful, Malthus using his "doctrine of proportions" tried to occupy the middle ground, arguing that saving would be beneficial up to a point, but harmful if pushed beyond that point (Malthus 1989b, pp. 8, 352).

The Eltis model (1984, p. 175) specifies in formal algebraic terms the condition for a rising trend of investment, and acknowledges Malthus' concept of an optimum saving ratio, stating that "in a more sophisticated model a tangency condition would need to be found" to express that ratio (pp. 176-7). The Eltis model appears to presume that, although it has been content with a simplified rising-trend-of-investment specification, and has not ventured into a more sophisticated optimum-level-of-investment specification, it has nonetheless provided a valid model of Malthus. This presumption could be questioned. It could be argued that the concept of an optimum level of savings and investment is so central to Malthus' thinking, that a simplification which fails to specify this condition is in effect a misinterpretation of Malthus, not a simplification. A Malthus growth model which includes the condition for a rising trend of investment but omits the condition for an optimum level of investment, fails to recognise that in Malthus' system a rising trend of saving and investment is not necessarily a cause of growth; Malthus clearly stated that if saving and investment are pushed too far, they will become a cause of decline, not a cause of growth. The concept of an optimum level of saving played such an important part in Malthus' system that it is doubtful whether any model can pretend to be a Malthus model without it.¹¹

The CR model recognises the importance given by Malthus to the optimum level of saving, and claims to have provided a formal algebraic solution to the problem of determining that optimum level - a problem which Malthus himself had been unable to solve. But this claim is not well founded. The CR model adopts as its equilibrium condition the equality of savings and investment, but, although it recognises the difference between equilibrium conditions and optimality conditions, it does not specify the conditions under which S and I achieve their optimum level, and does not incorporate the optimality conditions in the model's equations.

In Malthus' system it would seem that the equilibrium condition (S = I) is a necessary condition of optimality but not a sufficient condition.¹² In other words, the optimum position for saving is the equilibrium (S = I) position, but the equilibrium (S = I) position is not necessarily the optimum position. Hence, in specifying the condition under which S = I, the CR model provides only a partial solution to the problem of optimum saving. It is conceivable that saving could be equal to investment even though saving and investment are not at their optimum levels. There is, to my knowledge, no textual evidence that Malthus identified equilibrium and optimality.

The Malthus model developed by Eagly (1974) also concentrates on equilibrium conditions rather than optimum conditions. It adopts as its equilibrium condition the equality of the demand for labour (N^D) and the supply of labour (N^S). This equilibrium condition will also be a necessary condition for optimality,¹³ but it is not a sufficient condition for optimality. It is conceivable that the equality of N^D and N^s could be achieved when they are at a level which is either too low or too high for an optimal outcome.

The term "equilibrium" and the concept of equilibrium can be found in Malthus, but his concept of oscillations (or "alternations" or "fluctuations" or "vibrations") played a far greater role. A model of Malthus which specifies equilibrium conditions but does not specify functions capable of producing oscillations, would, I suggest, be omitting a major component of his system. Malthus was clearly more concerned with fluctuation and disequilibrium than equilibrium.

By concentrating on the specification of equilibrium conditions, and not proceeding to the specification of optimal conditions and oscillation conditions, the Eagly, Eltis and CR models seem to be pushing Malthus' thought into an anachronistic Marshallian equilibrium framework.

(ii) Saving by Workers

The CR and Eltis models both incorporate the notion that Malthus either assumed or implied that workers do not save.

Let us follow Malthus in assuming that workers spend their entire income on consumption goods for themselves and their families ... workers do not save in Malthus's model (CR 1985, pp. 423, 424).

It can be assumed as is fully implicit in Malthus's work, and indeed in that of Smith and Ricardo, that workers spend their entire incomes on commodities. Hence, they don't save and they purchase no services. (Eltis 1984, p. 156).

These assumptions, which the authors apparently regard as common knowledge not requiring textual support, can be questioned. In a number of places Malthus referred to the existence and the desirability of saving by labourers. This was particularly emphasised in the case of unmarried labourers, who should "save a sum for common contingencies of the married state".¹⁴ In the 1803 edition of the Essay on Population he admitted that the amount of labourers' savings was not great¹⁵, but that statement was omitted from the 1817 and 1826 editions which contained new material advocating "saving banks" or "provident banks". Patricia James has noted that Malthus himself was listed as a "manager" of one such bank in London in 1816. Obviously, he would not have bothered to support the savings banks movement if he believed that workers do not or cannot save. He argued in 1817 that the success of the savings banks in the current unfavourable period indicated that "in a period of prosperity and good wages, combined with a prospect of diminished parochial assistance, they might spread very extensively" (Malthus 1989a, vol. II, pp. 182-3). He argued that the poor laws discourage saving,¹⁶ but that if the poor law system were reformed along the lines he proposed, workers would save.¹⁷ As a further example of his belief in the institutional determinants of saving, there is his view that the French revolution had encouraged saving by the labouring classes.¹⁸

The idea that workers can and do save appears also in his *Principles of Political Economy.* "Workers and mechanics" are said to have "the means of saving" and "the means of accumulation", even if they receive only "the common wage".¹⁹ He also referred to the savings of menial servants and others engaged in personal services (Malthus 1989b, vol. I, p. 32), and in discussing the possibility of an increase of population from five million to seven million he stated that some of the additional two million "would unquestionably have a part of their wages disposable". He even managed to persuade Ricardo to add a

footnote to the third edition of the latter's *Principles* acknowledging that some part of the wages of labourers will be disposable. Furthermore, Malthus recognised that capital might be furnished by the labourers themselves, thus implying not only that labourers can save, but also that their saving could conceivably be undertaken with a view to investment, not just for postponed or lumpy consumption (Malthus 1989b, vol. I, p. 341). If his expression "the great body of the people" was intended to include labourers, then the following quotation is also relevant:

The prosperity of manufactures and commerce in any state ... shews that the great body of the people ... have both the power and the will to save. (Malthus 1989a, vol. II, p. 40).

It is therefore textually incorrect to say that in Malthus' system labourers do not save, especially if by "Malthus' system" we mean his preferred system. He clearly believed that, insofar as workers do not save, their failure to save was attributable to the current pathological system he was attempting to overturn, and that in his preferred system workers would save. A full model of his preferred system must therefore include a saving-by-workers variable. If, for the sake of methodological simplicity, model makers begin their attempt to model Malthus by assuming that workers do not save, then this simplification should be clearly signalled as an assumption which has been introduced not by Malthus but by the model makers themselves, and one which in fact is directly opposed to what Malthus assumed and hoped for in his preferred system.²⁰

Redundant Capital

Throughout his *Principles* Malthus frequently referred to "redundant" capital,²¹ by which he appears to have meant capital which is actually in existence but which cannot find profitable investment outlets. For example:

That the slow progress of New Spain in wealth and population, compared with its prodigious resources, has been more owing to want of demand than want of capital, may fairly be inferred from the actual state of its capital, which according to Humboldt, is rather redundant than deficient.²²

If a model of Malthus is to capture this essential feature of his theory of growth, it would need to distinguish between (a) capital which is currently invested, and (b) capital which is intended for investment but which cannot find investment outlets affording an acceptable rate of profit.

Eltis (1984, p. 180) appears to interpret Malthus as saying that there is an inequality between *ex post* saving and *ex post investment*.

Ex post saving and *ex post* capital accumulation are therefore distinct, both practically and theoretically.

But it would seem that, in discussing redundant capital, Malthus was thinking of stocks rather than flows, and was intending to distinguish, not between realised saving and realised investment, but between uninvestible capital and invested capital, or between potential investment and actual investment.

Consumption and Investment as Causes of Growth

Throughout his *Principles* Malthus made frequent references to the role of investment in promoting wealth.²³ The Eltis and the CR models contain investment functions, but the Eagly model appears to be deficient in not giving sufficient recognition to the role of investment and in exaggerating the role of consumption. Eagly does not neglect investment altogether; he states that in Malthus' model "equilibrium in the intersectoral market requires the 'proper' level of investment expenditures by capitalists", "the correct level of investment is not likely to be achieved at all times", and "investment demand must increase every time period". (Eagly 1974, p. 97). But Eagly does not discuss the determinants of investment in Malthus' system, and does not stress the interdependence between consumption and investment. He thus reaches a conclusion which, in my view, exaggerates the role of consumption in Malthus' system:

The policy implications of the Malthusian model were centered upon the means to stimulate increases in consumption expenditures. Investment demand was not regarded to be a policy parameter. Increases in investment, by augmenting production capacity, would have a perverse effect upon commodity gluts. At best, any increase in investment would be a very short-run expedient. The basic problem would be only postponed - to emerge again later on. Accordingly, Malthus argued that policy measures must be taken to increase consumption. ... The glut or overproduction of commodities is thus eliminated by the increased consumption demand. (Eagly 1974, pp. 99-100).

Eagly does not give any textual support for his interpretation. Malthus' frequent references to the problem of redundant capital, and to the folly of encouraging more accumulation of capital when the existing capital cannot find profitable employment, might conceivably be interpreted to mean that he was opposed to investment *per se*, or that he thought investment was not an important cause of progress. But these anti-investment comments by Malthus referred only to situations of redundant capital; he did not argue that, at other times, investment was unimportant, or less important than consumption. Eagly has correctly identified the policy prescribed by Malthus for a situation of capital redundancy, but has incorrectly interpreted it as a general policy recommended for all situations.

Demand-Side and Supply-Side Causes of Growth

The CR model argues that in Malthus' system "effective demand, through its effect on income distribution, is the *main* determinant of growth" (p. 427; italics added). There are (at least) two reasons why this statement is unacceptable. The first is that no textual evidence is adduced to support it. The second is that Malthus formally listed seven causes of growth - four on the production or supply side, and three on the distribution or demand side - linking the seven causes with an argument for the "Necessity of a Union of the Powers of Production with the Means of Distribution, in order to ensure a continued Increase of Wealth".²⁴

To say that effective demand through its effect on income distribution is the *main* determinant of growth in Malthus' system is to say that the distribution-side causes were for Malthus more important than the productionside causes. In the absence of supporting textual evidence, that would seem to be an exaggeration of the role of distribution-side factors in Malthus' system. Keynes is sometimes accused of over-emphasising demand and distribution as causes of growth, and under-emphasising supply-side causes. If such an accusation is justified, the unbalanced presentation might have been due partly to Keynes' understandable desire to redress the supply-side bias of conventional economics, and partly to his habitual tendency to adopt a deliberately provocative position in order "to raise a dust" (Kahn 1984, p. 161). If Keynes deliberately intended to over-emphasise demand in his own economics, he might have done the same in his interpretation of Malthus' economics.

The CR thesis that, in Malthus' system, effective demand is the "main" determinant of growth, might possibly reflect a tendency of some Keynesians, but it upsets the balance advocated by Malthus between demand-side and supply-side determinants of growth.²⁵ Such an unbalanced presentation would be uncharacteristic of "Malthus the moderate" (Winch 1987, p. 76).

Although Malthus emphasised the role of consumption as a stimulus to growth, he cannot rightly be classified as an "under-consumptionist" if by that term we mean one who stresses the role of consumption at the expense of investment. His intention appears to have been to advocate, in accordance with his "doctrine of proportions", and optimum level for both consumption and investment.

Interest

Neither the CR nor the Eltis model considers the role of interest in Malthus' system. In the CR model no explicit reason is given for the omission; presumably it was considered to be a minor aspect of Malthus' system which could be ignored in a simplified model. In the Eltis model the omission of interest is justified as follows:

> There has in fact been no reference to the rate of interest because Malthus himself made none when he was considering these vital matters. ... An investment function which attributed investment to profits was the one on which Malthus focused attention in the early nineteenth century, precisely because this may well have been the appropriate assumption at a time when there were not financial markets in the private sector sufficiently sophisticated to allow *a high fraction*

of investment to be financed by borrowing at a competitively determined interest rate.²⁶

It is certainly true that Malthus did not discuss either systematically or in any depth the role of interest rates in the determination of investment. However, there are several reasons why it is justifiable and indeed necessary to include interest as a determining variable in a Malthus model. Firstly, Malthus argued that "those who live upon the interest of a national debt" play an important role in stimulating economic growth.

> they contribute powerfully to distribution and demand; they frequently occasion a division of property more favourable to the progress of wealth than would otherwise have taken place; they ensure that consumption which is necessary to give the proper stimulus to production;²⁷

Secondly, profits cannot be used as a proxy or correlative for interest in Malthus' system. He admitted that in some circumstances a fall in the rate of interest would be evidence of a decline in profits; and he noted that in some periods, e.g. 1797-1817, profits and interest had moved in the same proportion.²⁸ He also stated that interest rates are almost the only criterion for comparing rates of profit in different countries, although that criterion would be "subject to the greatest uncertainty" in countries such as India and China and "in most of the eastern and southern regions of the globe" (Malthus 1989b, vol. I, p. 158). But, in Malthus' view, high interest rates can be accompanied by low profits, and therefore high interest rates do not justify the conclusion that profits are high.²⁹ He recognised that the divergence between interest rates and profit rates can be explained by the different motives involved.³⁰

Thirdly, Malthus noted that, although interest and profits can move in different directions, they are nevertheless interdependent. Profits will be affected by the level of interest, because profits "consist of the difference between the value of the advances necessary to produce a commodity, and the value of the commodity when produced" (Malthus 1989b, vol. I, p. 293), and interest is one of those advances.³¹ But as well as profits being a function of the rate of interest, interest is said to be a function of profits; only high levels of profit will "warrant the borrowing at a very high rate of interest" (Malthus 1989b, vol. I, p. 159).

As far as I am aware, Malthus did not explicitly state that the rate of interest either is or is not an *essential* element of his system, but, on the basis of the above textual evidence, it would seem justifiable and indeed necessary to include interest as a significant determining variable in a Malthus model.

Since interest is one of the costs which has to be met before net profit is determined, one can accept Eltis' statement that Malthus' investment function "attributed investment to profits" without denying that for Malthus one of the determinants of profits and hence of investment is interest. Also, the insertion of interest as an additional determining variable would not be inconsistent with the equations of the Eltis model, and would in fact appear to be a necessary consequence of them. In the Eltis model, capitalists "are responsible for all investment decisions" (1984, p. 175), but the capitalists account for only part of total savings, the other part arising from the savings of the landlords. Eltis thus argued that if planned investment is to equal planned savings, the capitalists must plan to invest not only their own savings but also an amount equal to the savings of the landlords. However, if capitalists are to invest the savings of the landlords (either directly or through financial markets), and pay interest to the landlords.

Eltis argued that growth will occur only if the capitalist class is large enough in relation to the landlords to allow most savings to come from profits and not from rents. But if the landlords' savings are borrowed by the capitalists, it will not be necessary for most saving to *originate* with the capitalists. It is true, as Eltis argued, that the capitalist class would have to be large, but their largeness could be the combined effect of their own savings and the savings they borrow from landlords; their own savings need not by themselves be large.

Wages and Profits

Both the Eltis model and the CR model contain an equation showing a negative or inverse relation between the wage rate and the profit rate. There is no doubt that the notion of a negative relation between wages and profits forms *part* of Malthus' system,³² and that the Eltis and CR models faithfully reflect *that part* of his system. But in a number of passages, Malthus (in opposition to Ricardo) clearly enunciated that wages and profits do not necessarily have a negative relationship, but can also under certain circumstances have a positive relationship, increasing or decreasing together.³³

The Eltis and CR models implicity incorporate this positive relationship in the case where the direction of causation is from output to wages and profits. The two models recognise that when output rises (or falls), both wages and profits can rise (or fall) together, even though when output is constant they have a negative relationship.

Also, the CR model incorporates the positive relationship between wages and profits in the case where the direction of causation is from profits to output to wages; an increase in the rate of profit will cause an increase in employment and hence an increase in total wages paid, assuming the wage rate is constant (CR 1985, p. 423).

But neither the Eltis nor the CR model appears to incorporate the positive relationship that exists between wages and profits in the case where the direction of causation is from wages to output to profits. Although Malthus warned that excessive wages can lead to a decline in output and profit, he also argued that if workers "produce an excess of value above what they consume ... then certainly the power of consumption possessed by the workmen will greatly add to the whole national demand, and make room for the employment of a much greater capital." (Malthus 1989b, p. 472).

Without recognising the existence in Malthus' system of this positive wages-output-profits relationship, as well as the negative relationship between wages and profits, it is not possible to explain a paradox in Malthus' discussion of distribution. In a passage quoted in the CR model (pp. 427-8), Malthus said that an increase in real wages will lead to a decrease in profits and hence to a decrease in investment and employment. But, as the CR model also states (p. 428), Malthus' remedy for the stagnation following the Napoleonic Wars was to increase consumption. These two statements appear at first sight to be inconsistent; the most obvious way to increase consumption would be to increase real wages.

The inconsistency disappears if we recognise that in the relationship between wages and profits in Malthus' system two counteracting tendencies are simultaneously at work, and can produce either a positive or a negative result. Up to a certain point an increase in wages will tend to promote demand, production and profits; but beyond that point the extra costs resulting from higher wages will diminish profits. A model of Malthus should therefore incorporate equations reflecting the double-sided relationship between wages and profits, and should show that a proper proportion between wages and profits is one of the conditions of optimality.

Unproductive Labour and Personal Services³⁴

Both the Eltis and the CR model exclude unproductive labour as a determinant of growth. The Eltis model restricts its attention to "output ... in the commodity-producing sector of the economy" (Eltis 1984, p. 167). The CR model justifies its omission of unproductive labour by alleging, without textual support, that Malthus "regards the incomes of unproductive workers as mere deductions, under various headings, from the distributive shares of these major classes [landlords, capitalists and productive workers]" (CR 1985, p. 419).

But there can be no doubt that Malthus regarded unproductive labourers as a major stimulant to economic growth. They would provide a market for the sale of material goods; and the desire to make use of the services provided by the unproductive labourers would stimulate the productive efforts of the productive labourers. This argument was forcefully presented in the *Principles*, Chapter VII, Section IX, "Of the Distribution occasioned by unproductive Consumers, considered as the Means of increasing the exchangeable Value of the whole Produce".³⁵ Malthus argued that "the employment of a certain number of persons in menial service is in every respect desirable",³⁶ and that "the maintenance of a certain body of consumers who are not themselves engaged in the immediate production of material objects" is one of the causes that will most encourage the progress of wealth.³⁷

In other places Malthus appears to have contradicted himself when he argued that personal servants can be detrimental to the progress of wealth: for example, "there could hardly be a taste more unfavourable to the progress of wealth than a strong preference of menial service to material products".³⁸ Critics might take these apparently contradictory statements on unproductive labourers as a sign of Malthus' muddle-headedness, but a more sympathetic interpretation - and the only interpretation which seems to be consistent with a close reading of the text - is that he was attempting to resolve the question of unproductive labour by once again applying the "doctrine of proportions", i.e. by emphasising the need for a right proportion between the number of productive and

unproductive labourers. This middle-way solution is obvious in his statement that the progress of wealth will be encouraged by the "maintenance of such a proportion of unproductive consumers as is best adapted to the powers of production"³⁹. Given the importance attached by Malthus to this interdependence of the material-goods sector and the personal-services sector in his theory of economic growth, it is quite extraordinary that the Eltis model and the CR model should both have omitted unproductive labour as a determining variable.

Personal services were given an essential role in the Eagly model. Eagly (1974, pp. 93-4) recognised that Malthus' views on personal services represent a significant departure from the classical model, and argued that Malthus was the only post-Quesnaysian who incorporated the distinction between unproductive and productive labour in his model. However, although Eagly noted the importance attached by Malthus to the concept of an optimum balance (e.g. Eagly 1974, p. 92), he did not incorporate optimising conditions into his treatment of unproductive labour. His Malthus model was an equilibrium model defining the conditions under which productive labour and unproductive labour would be in equilibrium - rather than an optimising model defining the conditions under which the material goods sector and the services sector would be at their optimum levels.

Manufacturing, Commerce and Inter-Sectoral Balance

The CR model is purely agricultural. There is only one output, "corn". As the authors state, this neatly removes the problem of relative prices. But it also removes the significant causative roles which Malthus attributed to manufacturing and commerce in the progress of wealth, and prevents any consideration of the importance of an optimum balance between agriculture, manufacturing and commerce. The Eltis model, by contrast, regards commerce as an important determinant.

The CR model does not provide any textual justification for its omission of industry and commerce. As with the omission of fixed capital, it seeks to justify its omission of industry partly on the ground of "lack or space" and partly by the argument that the omission "is not a serious problem" because the issues dealt with in the CR model "can be considered in isolation from the rest of [Malthus'] ideas" (CR, p. 418).

But the overwhelming textual evidence suggests that the omission of industry from a Malthus model is a serious problem; that a consideration of the interrelationships between the three sectors is an essential aspect of Malthus' system; that a purely agricultural model cannot do justice to his system; and that any attempt to consider agriculture in isolation from industry and commerce results in an incomplete and seriously inaccurate model of Malthus.

The importance Malthus attached to commerce as one of the causes of the progress of wealth is evident in Chapter VII, Section VIII of the *Principles* (1989b, vol. I, pp. 440-62): "Of the Distribution occasioned by Commerce, internal and external, considered as the Means of increasing the exchangeable Value of Produce". In addition, there are statements elsewhere in Malthus' writings which stress that commerce can cause an increase in the demand for labour, an increase of population, an increase in the value of the general produce, a wider distribution of land, and in general a more rapid progress of wealth and happiness.⁴⁰ Malthus would surely be surprised to see commerce omitted from a model purporting to be a model of his system.

Malthus' remarks on the importance of manufactures were just as forceful as those on commerce. He argued that the progress of manufactures would create a greater demand for labour, would encourage population growth, and would in general promote the progress of wealth and happiness.⁴¹

In addition to arguing that commerce and manufacturing are in themselves causes of progress, Malthus also stressed the interdependence of commerce, manufacturing and agriculture. Commerce and manufacturing will increase agricultural rents and agricultural prosperity,⁴² but at the same time manufacturing cannot thrive unless the land yields a surplus produce (Malthus 1989b, vol. I, pp. 34-6, 211, 215, 230, 264, 282, 409, 435, 581). The growth of manufacturing will also be dependent upon commerce, because foreign commerce by stimulating a taste for luxuries and extending the market for sales will encourage domestic manufacturing (Malthus 1989b, vol. I, pp. 409, 577).

Although Malthus described commerce and manufactures as only the "apartments" or ornaments or embellishments of the political structure, whereas agriculture is its "foundations", and although he warned of the danger of the commercial sector becoming excessive, an agriculture-only model fails to grasp two essential features of Malthus' system, viz. the causative importance of each of the three sectors, and their interdependence. A purely agricultural model of Malthus might be justified as a model of his early writings, which showed an agricultural or Physiocratic bias, but it must be very inadequate as a model of his mature and total system.

Measure of Value

The CR model seeks to give prominence to Malthus' measure of value (i.e. units of labour commanded), stating that in Malthus' theory of value

labour commanded is the unit of account upon which capitalists' decision-making is based. Whereas most other interpretations regard Malthus's theory of value as of marginal importance, we regard it as essential to a full understanding of his wider theory of demand and growth. (CR 1985, p. 435).

Admirers of Malthus would like to be convinced that there is a tight logical link between his microeconomics and his macroeconomics, and, specifically in this context, between his measure of value and his theory of growth. But regrettably, the evidence so far adduced is not convincing. This is not to deny that for certain purposes labour-commanded is very useful as a measure of value - for example, in comparing values at different times and in different countries - and that Malthus must be given full credit for recognising its usefulness. The CR model shows that the use of units of labour-commanded as a measure of value produces some neat and tidy economies in the use of symbols. L, for example, can represent not only labour employed, but also the wages fund measured in units of labour commanded; and π/L can represent either profits per unit of labour employed, or the rate of profit measured as a proportion of the wages fund. But there seems to be no obvious reason why this particular measure of value is "essential to a full understanding" of Malthus' theory of growth. In fact, only 11 of the 34 equations of the CR model, viz. equations (5) - (15), are expressed in units of labour commanded, and the integrity of the CR model would be in no way impaired if all its equations were expressed in either physical terms or money terms.

The question here is not whether Malthus advocated labour-commanded as a measure of value. He obviously did, at great length; and would have been delighted with the emphasis given to it in the CR model. Nor is the question whether labour-commanded as a measure of value is a useful theoretical tool in its own right. Rather, the question is whether that particular measure of value is, as the CR model claims, "essential to a full understanding of his wider theory of demand and growth" and "the unit of account upon which capitalists' decision-making is based."

In attempting to support this latter claim, the CR model quotes Malthus' statement:

[the] power of commanding labour [is] of the utmost importance in an estimate of the exchangeable value of commodites.

But, in my view, this quotation does not warrant the CR interpretation. It says nothing about the decision-making processes of capitalists.

Moreover, the CR interpretation appears to be contradicted by Malthus in statements such as:

It is the nominal value of goods, or their prices only, which the merchant has occasion to consider.⁴³

Such statements appear to provide convincing evidence that, for Malthus, labour-commanded is *not* "the unit of account upon which capitalists' decision-making is based".

Conclusion

The Eagly, Eltis and CR models can claim to be ingenious and commendable variations on a theme by Malthus, and therefore worthwhile compositions in their own right. But each fails to incorporate some features which appear to be *essential* to Malthus' system, as well as some features which, if not essential, appear to have been given considerable emphasis by Malthus. It is doubtful therefore whether the models, as they currently stand, can rightly claim to be adequate models of Malthus.

It is hoped that the suggestions put forward in this paper will complement the efforts of Eagly, Eltis, Costabile and Rowthorn, and make a positive and useful contribution to the work of formulating a model which will capture the essential complexity and richness of Malthus' theory of growth.⁴⁴ ² Malthus 1989b, vol. II, p. 278. In the first ed. of the *Principles* this reads: "how much the wealth of nations depends upon the proportion of parts".

³ For a more detailed discussion of the doctrine of proportions, see Pullen 1982.

- The CR model uses the subscript "t+1" for wages because it interprets Malthus as saying that " the value of a commodity depends not on the wage rate paid to the workers who actually produced this commodity in the *past*, but on the wage rate which is paid to workers who will be engaged in *future* production" (CR, p. 421). The CR model also uses the symbol "Z" as an index of diminishing returns, but as this concept was not clearly specified and does not appear to be essential to the argument of the CR model, it has been omitted from equations (1) - (5) above. The reader is referred to the CR model for a more detailed explanation of its notation and equations.
- ⁵ If fixed capital were made by labour alone, and if the value of that labour were expressed in money wages, then the money wages term could be eliminated from equation (5). But there is no textual support for the assumption that fixed capital is made by labour alone.
- ⁶ The CR model supports its claim that variations in money wages do not affect real wages by quoting (pp. 427-8) a passage in which Malthus stated that changes in the money-prices of commodities cause changes in the real wages of labour because the money wages of labour are sticky. However, this statement does not appear to justify the interpretation given to it in the CR model. It does not mean that in Malthus' system money wages are not a determinant of real wages. It simply means that one blade of the pair of scissors is held fairly steady while the two blades are cutting.
- ⁷ See Eltis 1984, p. 167 and CR 1985, p. 430.
- 8 The term "preferred" is used here in the sense of the outcome Malthus advocated and hoped for in a welfare sense. He did not see it as an historically inevitable outcome.
- ⁹ Malthus 1989b, vol. I, p. 250; with slight variations in the 2nd edition of the *Principles*.

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It is not argued that the Eltis and CR models ignore the doctrine of proportions entirely. They each make explicit reference to the notion of an optimum level of savings. Also, the Eltis model emphasises the notion of a balanced growth of labour supply and demand, and a balanced growth of capital supply and demand.

- ¹⁰ On Malthus' production and population functions, see Pullen and Baldry 1988.
- Attainment of the optimum level of saving will depend on the distribution of income. On the importance of distribution in Malthus' system, see below, "Demand-Side and Supply-Side Causes of Growth", "Wages and Profits", and "Unproductive Labour and Personal Services".
- ¹² If savings exceed investment, i.e. if capital is redundant, capital resources which could be used to promote growth are lying idle - a situation which is obviously not optimal and which was frequently deplored by Malthus. Also, in Malthus' system, savings cannot be less than investment, because investment occurs out of savings. Therefore, since the optimum level of savings cannot occur if savings exceed investment, and since by definition savings cannot be less than investment, then, at the optimum level of savings, savings must equal investment.
- ¹³ If $N^{D} > N^{S}$, growth will be held back because of an insufficient supply of labour. If $N^{S} > N^{D}$, labour will be redundant and the economy's growth potential will not be realised.
- ¹⁴ Malthus 1989a, vol. II, p. 111; see also pp. 97, 195, 228.
- ¹⁵ "At present the few labourers who save a little money are often greatly at a loss to know what to do with it ..." (Malthus 1989a, vol. II, p. 191).
- "The poor laws ... diminish both the power and the will to save among the common people".(Malthus 1989a, vol. I, p. 359).
- ¹⁷ "it is difficult to conceive that these men [workmen] would not save a part of their high wages for the future support of their families ... if they did not rely on parish assistance for support in case of accidents". (Malthus 1989a, vol. I, p. 359).
- ¹⁸ "The effect of the revolution in France has been to make every person depend more upon himself and less upon others. The labouring classes are therefore become more industrious, more saving and more prudent in marriage than formerly..." (Malthus 1989a, vol. I, p. 378).
- ¹⁹ Malthus 1989b, vol. II, p. 30; not in the 1st ed. of the *Principles*.

- ²⁰ There are two question at issue: Did Malthus believe that workers would save? What would be the amount (if any) of their savings? It is no criticism of Malthus to note that he did not attempt to quantify workers' savings. Given the state of statistics and statistical theory, it would be unreasonable to expect him to have estimated the consumption function of workers, and their marginal propensity to save out of wages. But if this MPS were very low, then even though the CR and Eltis models are incorrect in stating that Malthus assumed that workers do not save, they would be justified, for the purpose of model building, in excluding workers' savings. But is it likely that Malthus would have made frequent references to workers' savings and given support to the savings banks movement if he believed the amount of such savings in his preferred system would be trivial? A model of his preferred system should surely incorporate a saving-by-workers variable, unless there is textual evidence to indicate that it is an insignificant quantity.
- Other terms used synonymously were "excessive capital", "floating capital", "idle capital", "spare capital", "unemployed capital", "vacant capital", and "premature supply of capital".
- Malthus 1989b, vol. I, p. 392-3. Further references to the concept of redundant capital can be found at Malthus 1989b, vol. I, pp. xix-xx, 161, 356, 357, 400, 404, 407, 410, 469, 478, 499, 547, 565, 570, 572, 576, vol. II, pp. 224, 260, 281, 425.
- In particular, Ch. VII, Sect. III, "Of Accumulation, or the Saving from Revenue to add to Capital, considered as a Stimulus to the Increase of Wealth" (Malthus 1989b, vol. I, pp 351-74). The term "investment" was used a few times in the *Essay on Population* but was not used in the *Principles*. The "employment of capital" was used frequently in the *Principles*, and appears to have been equivalent to the modern concept of investment; for example, in his *Definitions in Political Economy* 1827, Malthus defined the "Accumulation of capital" as "The employment of a portion of revenue as capital. Capital may therefore increase without an increase of stock or wealth" (p. 238); and "Capital" was defined as "That portion of the stock of a country which is kept or employed with a view to profit in the production and distribution of wealth" (p. 237).
- See Malthus 1989b, Ch. VII, "On the Immediate Causes of the Progress of Wealth", pp. 345 522.
- It is not suggested that the CR model ignores either demand-side determinants or supplyside determinants. The criticism is directed at the CR model's claim that the "main" determinant of growth is demand.

- 26 Eltis 1984, pp. 177-8. It may or may not be empirically true that, as Eltis suggests, in the early nineteenth century a high fraction of private sector investment was not financed by borrowing, but Senior's *Letters on the Factory Act* provides evidence that, in one part of the private sector, interest on borrowed capital was not an insignificant part of a firm's costs.
- 27 Malthus 1989b, vol. I, pp. 479-80; in the 2nd ed. of the *Principles* "consumption" was altered to "effective consumption". Similar statements about the stimulating role of interest can be found in Malthus 1990b, vol. I, pp. 43, 483-4. These latter statements were omitted from the 2nd ed. of the *Principles*, but it is not clear whether Malthus or the editor of the posthumous second edition of the *Principles* was responsible for the omissions. The retention in the second edition of the above quotation from pp. 479-80 of the first edition indicates that Malthus did not alter his views on this matter.
- 28 Malthus 1989a, vol. II, p. 58; 1990b, vol I, p. 319.
- 29 See Malthus 1989b, vol. I, pp. 160, 373.
- Malthus 1989b, vol. I, p. 159; with some changes in the 2nd ed. of the *Principles*.
- Malthus 1989b, vol. II, p. 218; in the 1st ed. of the *Principles*, interest was not included in the list of advances. It is significant that, in the 2nd ed., interest was said to be a "necessary" advance.
- For example, "a fall of wages raises the profits of stock" (Malthus 1989b, vol. I, p. 161; in the 2nd ed. of the *Principles* "raises" was changed to "tends to raise"); and "a great increase of consumption among the working classes ... must lower profits" (Malthus 1989b, vol. I, p. 402). The following statement in Malthus, *The Measure of Value*, 1823, p. 33, very clearly states that there is an inverse relationship between wages and profits when the level of output is given: "If the increased reward of the labourer takes place without an increase of produce, this cannot happen without a fall of profit, as it is a self-evident truth, that given the quantity of the produce to be divided between labour and profits, the greater the portion of it which goes to labour the less will be left for profits."
- 33 See, for example, Malthus 1989b, vol. I, pp. 154, 162, 302, 323; vol. II, pp. 160, 227. Unfortunately for would-be builders of Malthus models, Malthus did not always bother to distinguish explicitly between the wage rate and the wages bill. The reader is therefore faced with the difficult task of deciding which of the two concepts Malthus was using in the

various contexts. There is no guarantee that his usage remained constant. It is possible that in some contexts "wages" meant "wage rate", and in other contexts "wages" meant "wages bill". A similar problem of interpretation occurs with his use of the term "profits". Although in some circumstances he used the expression "rate of profits", there are other instances in which it is not immediately clear whether "profits" means "rate of profits" or "total profits".

- ³⁴ Malthus' concept of productive labour included labour which produces material goods, and labour which provides services that are directly included in the value of material goods. His concept of unproductive labour included labour which provides services that are not directly related to the production of material goods. The term "unproductive labour" was altered to "personal services" in the second edition of the *Principles*.
- 35 Malthus 1989b, vol. I, pp. 463-90. In the 2nd ed. of the *Principles*, "unproductive Consumers" was altered to "personal services and unproductive Consumers".
- ³⁶ Malthus 1989b, vol. I, p. 479. In the 2nd ed. of the *Principles*, "varying with the neat [sic] revenue of the society" was added after "desirable".
- ³⁷ Malthus 1989b, vol. I, p. 489. In the 2nd ed. of the *Principles*, " consumers objects" was replaced by "unproductive consumers".
- ³⁸ Malthus 1989b, vol. I, p. 479. In the 2nd ed. of the *Principles*, "and ill accommodated followers" was added after "menial service".
- ³⁹ Malthus 1989b, vol. II, p. 284. This statement occurred in the 2nd ed. of the *Principles*, but not in the 1st ed. See also Malthus 1989b, vol. I, p. 463; vol II, pp. 31, 278.
- See, for example, Malthus 1989a, vol. I, p. 431; 1989b, vol. I, pp. 211, 264, 282, 409, 435, 461-2, 581, 585.
- 41 See Malthus 1989a, vol. I, p. 431; 1989b, vol. I, pp. 34-6, 211, 215, 230, 264, 282, 409, 435.
- Malthus 1989a, vol. I, p. 431. See also Malthus 1989a, vol. II, p. 46; and Malthus 1989b, vol. I, pp. 205, 209-11, where a "prosperous commerce" is identified as one of the causes of increased rent and extended cultivation. A clear statement of the interdependence between the three sectors occurred in Malthus 1989b, vol. I, p. 41, where he said that the three

sectors have a "mutual dependence on each other" and act "as stimulants to each other's increase". These statements were part of a 7-page omission from the second edition of the *Principles*, but the recurrence of similar ideas in his later publications indicates that the omission of these interdependence statements did not represent a change of mind on Malthus' part. The theme of a proper balance or proportion between the three sectors was treated at length in the *Essay*, Book III, Chapter C, "Of Systems of Agriculture and Commerce combined". (Malthus 1989a, vol. II, pp. 40-8).

- ⁴³ Malthus 1989b, vol. I, p. 530. See also Malthus 1989b, vol. I, pp. 58, 59, 362, 444-7; vol. II, p. 39.
- It should perhaps be pointed out that the aim of this paper has been to make a contribution to the debate on the identification of Malthus' system, but not to evaluate his system. No attempt has been make in this paper to pass judgement on whether Malthus was a competent modeller, or whether his system is valid in any objective, real-world sense, or whether it correctly explains the events of his time, or whether it has any practical use today.

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