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proprietary "of"**

by

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Abstract

The Marginal Productivity Theory of Distribution has been seen by some writers, notably J.B. Clark, as a rule for both distributive justice and economic efficiency. Its normative implications have been generally rejected, but as a criterion for economic efficiency and profit maximisation it retains a prominent place in modern textbooks and as an equilibrium condition in economic models. The aim of this paper is to show how interpretations of the MPTD have developed over the years, to question its status as a criteria of economic efficiency, and to suggest that, despite explicit rejections of its normative claims persist in many modern presentations.

Key Words: marginal productivity; distribution; J.B. Clark.

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Introduction

Previous linguistic studies of the Marginal Productivity Theory of Distribution (MPTD) have attempted to clarify the meaning of terms, such as marginal, productivity, labour, capital, marginal value product, net marginal product, etc. This paper attempts to extend linguistic analysis of the MPTD by directing attention to the preposition “of” in the phrase “the marginal product of a factor”.

The word “of” is one of the shortest and most frequently used in the English language, but is one of the most difficult to define. The *Oxford English Dictionary* gives no fewer than 63 senses in 16 categories. This paper will be concerned with the 14th category:

In the sense of *belonging or pertaining to*; expressing possession and its converse: ‘the owner of the house’, ‘the house of the owner’,

and in particular with sense no. 49a conveying the notion of proprietorship:

Belonging to a person (etc.) as something that he (etc.) has or possesses.

Discussion of the proprietorial “of” is also extended to other words connoting a proprietorial relationship, such as possessive pronouns (his, hers, its, their, ...); nouns

in the genitive case (labour's, capital's); and words such as "from", "by" and "to" (as used in the phrases "arising from", "created by", "due to", etc.) that suggest a causative and hence a proprietorial connection. It is a study of little words, whose importance in economic discourse is often overlooked and which are often submerged under the weight of the big words, like "marginal" and "productivity", that command attention.

The paper arrives at several conclusions. The fact that "of" and these associated words appear to be used in the MPTD in different senses that are not clearly distinguished raises doubts about the usefulness and indeed the validity of the MPTD as an explanation or justification of the pattern of income distribution. It casts doubt on the validity of the equality between a factor's reward and its marginal product as a criterion for profit maximisation and equilibrium. And it suggests that the use of the first partial derivative of output with respect to a factor ($\partial Q/\partial L$, $\partial Q/\partial K$, ...) is inappropriate as a measure of the marginal product of the factor.

The paper is divided into three sections. Section I presents a critique of the MPTD, and is divided into three sub-sections: The use and abuse of the proprietorial "of"; Costs and causes; and, The partial derivative as a definition of the marginal product of labour. Section II contains a survey of the MPTD literature, showing how the elements of the above critique have been treated in expositions of the MPTD over the years. Section III draws out a number of implications and conclusions that would follow if the critique can be substantiated.

Section I. The Critique

The use and abuse of the proprietorial “of”

In the MPTD literature the expression “marginal product of labour”¹ appears to be used in two different senses – a monocausal sense and a multicausal sense – related to two different meanings of the preposition “of”. In many references to the “marginal product of labour”, “of” seems to carry a proprietorial significance, with the concept of proprietorship being derived from two sources: (a) the belief or assumption that the marginal product of labour is produced by the marginal unit of labour; and (b) an implicit or explicit invoking of a Lockean or contributor theory of property rights according to which property rights in an object are said to belong to the person who has produced it. In standard expositions of the MPTD it is often either argued or implied that since the marginal unit of labour produces the marginal product, the marginal unit of labour has a moral right to be the exclusive proprietor of the marginal product.

However, in accordance with some past critics of the MPTD, the thesis being presented here is that the use of “of” in an exclusive sole-proprietor sense in the expression “marginal product of labour” is neither logically, nor linguistically, nor ethically justified, because the marginal product that occurs *after* the employment of an extra unit of labour (with other factors held constant) does not occur *solely because* of that extra unit of labour. It occurs because of the combined effect of the extra unit

¹ For simplicity, the discussion will be conducted in terms of the marginal product of labour, i.e. assuming labour is the variable factor and the other factors are held constant, but the arguments would be equally applicable to the marginal products of the other factors.

of the variable factor acting together with the factor or factors held constant. It is a multicausal phenomenon rather than a monocausal phenomenon.²

It follows that if the marginal unit of labour is not the sole cause of the marginal product, a Lockean theory of property rights can not be used to support a claim that the marginal unit of labour has exclusive proprietorship over the marginal product. It also follows that, if the marginal product consists not only of what J.B. Clark called the “specific” product of the marginal unit of labour, but also of a productive contribution from the factors held constant, then the *use* of “of” in an exclusive, sole-proprietor sense in the expression “marginal product of labour” should be more correctly described as an *abuse* of language.

J.B. Clark and others believed that the specific product of the marginal unit of labour could be disentangled from the marginal product, and separately identified and measured, by observing the change in the total product that occurs after adding an additional unit of the variable factor while holding all other factors constant. By this process, it was thought that the marginal product could be monocausally related to the marginal unit of labour.

As shown in the literature survey in Section II below, this disentanglement claim has been vigorously disputed. The claim involves a denial of the causative contribution of the constant factors; or in other words, an assertion that when the other factors are held constant, the only causative influence at work is that of the variable factor. But the *appearance* of monocausality is deceptive, for the marginal unit of labour does not

² See Robinson (1971, 57): “The marginal product of an additional man employed provides the wage per man-year *and* the profit on the capital required to employ him. It is far from being the case that each ‘factor’ separately receives its marginal product. Man-plus-capital earns the marginal product”.

act alone. It acts in combination, not only with other variable factors, but also with the factors that are held constant or assumed to be constant. These constant factors contribute causally to the change in the total product, even when they are being held constant. Holding them constant does not eliminate their effectuality. It is an illusion to think that, when labour is the only thing that changes, labour is the only cause and the only cost. *Ceteris paribus* is not the same as *ceteris inefficacibus*. An active, causal factor is not magically transformed into a passive, non-causal factor simply by intoning *ceteris paribus*. In calculating the marginal product of labour, *ceteris paribus* eliminates the effect of any *changes* to capital, but does not eliminate the ongoing causal influence of *existing* capital.

To regard the marginal product of labour as being caused by the marginal unit of labour alone, is to argue *post hoc ergo propter hoc*, or to confuse correlations and causes. The longevity and popularity of the MPTD could be seen as a tribute to the awesome influence of that fallacy in economics. The change in the total product that *occurs after* the application of a marginal unit of labour is not *caused by* that marginal unit of labour alone. The marginal product is labour-correlated, but not, in a monocausal sense, labour-created.

To avoid confusion resulting from the abuse of the proprietorial “of”, it would be preferable to refer to the (multicausal) “marginal product of labour” by an alternative expression such as the “marginal product after labour”. In the ensuing discussion, the terms “marginal product after labour” (MPAL) and “specific marginal product of labour” (SMPL) will be used. The MPAL refers to the non-specific or multicausal product resulting from the combined effect of the constant capital and the marginal unit of labour. It is the entity normally referred to as the “marginal product of labour”, i.e. the change in output that occurs after the employment of an extra unit of

labour. SMPL refers to the specific or monocausal product actually contributed by the marginal unit of labour, i.e. the labour-created portion of the MPAL, as distinct from the portion created by the constant factors. It is the SMPL that can lead (in a Lockean philosophy of property rights) to sole-proprietor claims.

As will be seen in the literature survey in Section II, critics have sought to illustrate the impossibility of disentangling the SMPL from the MPAL by making use of a variety of analogies from everyday life, some more appropriate than others. Thus, for example, Fraser (1947, 352-53) referred to the impossibility of unscrambling eggs, or of deciding “how much of a Beethoven symphony is due to the violins and how much to the trumpets and flutes”. Hobson (1910) employed a physiological metaphor – the separation of the operation of the hand or foot from the activity of the human body. Roscher made use of a metaphor from genetics – what part of the calf comes from the cow and what part from the bull? Davenport (1929) illustrated the point with a transport analogy – how much of the movement is due to the horse and how much to the wagon; and with a furniture analogy – which leg of a three-legged stool supports the stool? Bernard Shaw (1928) opted for an agricultural analogy – how much of the wheat is due to the farmer, how much to the labourers, how much to the seed, how much to the farming equipment? Taussig (1921) asked how the specific product of the tool could be separated from the specific product of the labourer using it.

A particularly appropriate analogy might be drawn from the process of climbing a ladder where each successive movement taken by the climber is “the marginal unit of labour” and each extra step above the ground is “the marginal product”. To say that the marginal product is created monocausally by the marginal unit of labour is equivalent to arguing that in the process of climbing a ladder, one’s upward progress is entirely attributable to (or caused by) one’s labour alone, and that the ladder (the

constant factor) provides no causative contribution; or, in other words, that in climbing a ladder one does not need a ladder.

The fact that labour and capital cooperate in the production process is indisputably obvious. Samuel Whitbread writing in 1807 said: “Almost all Capital is rendered productive by the Operation of Labour” (Letter to Malthus, 5 April, 1807)). And the inextricability of the respective causal contributions of the various factors to the multicausal product is a fact of common observation. It is clearly evident in the example of a man with a spade digging a ditch. How much of the ditch is causally due to the man and how much to the spade? It could possibly be argued that the causal contribution of the spade can be identified by comparing the amount produced by the combination of man and spade with the amount produced by the man alone, clawing at the ground with his bare hands. If it is found that the man alone can dig ten metres of ditch in a day while the man-and-spade can dig 200 metres, does it follow that the causal contribution of the spade is 95% and the causal contribution of the man is 5%, and that the relative causal contribution factors are not therefore inextricable? The argument founders when the two causes – the man and the spade – are treated in reverse order. If, instead of asking how much can the man produce without the spade, we ask how much can the spade produce without the man, their causal inextricability is reasserted.

J.B. Clark did not argue that it is possible to separate the *total product* into the specific causal contributions of the various factors. But he argued that this separation is possible in the case of a *marginal product* – by varying one factor and holding the others constant. The reply of critics has been to say that disentanglement is just as impossible for a marginal product as for the total product, because each marginal product is also a multicausal product. In this paper it is argued that the

disentanglement of both the total product and the marginal product cannot be substantiated once the causative role of the constant factors is admitted.

The matter is further complicated by the fact that, when labour is the variable factor, the constant factors can consist, not only of capital and land, but also of previous units of labour. The MPTD does not explain how the contribution of the marginal unit of labour can be separated from the contribution of the earlier units of labour. For example, suppose a man is employed to roll a log, unaided by capital equipment, and is to receive a wage of \$10 for every 20 metres it rolls. By himself, he fails to budge it, but when a second unit of labour is added, they succeed in pushing it 20 metres. According to the MPTD, the \$10 wage should go entirely to the second man! A partial derivative of the production function would confirm that outcome, as the effort of the first man remains unchanged. But such a distribution of rewards could hardly qualify as a norm of distributive justice or as a condition for efficiency and equilibrium.

Costs and Causes

The preceding discussion has focussed on the question of causation. But supporters of the MPTD have argued that, although some early versions of the MPTD might have been concerned with causes, modern versions are concerned with revenues and costs, irrespective of their causes; and that the validity of the MPTD is neither dependent on the identification of causal influences, nor in any way affected by the problems of multicausality and disentanglement.

If we accept the impossibility of disentanglement of the causative contributions, and/or leave aside completely the problem of causation, considering the MPTD in

terms of costs rather than causes, is it true to say that factor rewards tend to be distributed in proportion to factor costs?

It is the contention of this paper that the validity of the MPTD is in fact damaged by the fact of multicausality, i.e. by the causative role of the constant factors, even when the MPTD abstracts from the problem of identifying causal inputs, and is considered purely in terms of revenues and costs. The reason for this contention is that, if the marginal product that occurs after the employment of the marginal unit of labour is not produced by that unit of labour alone and contains more than the specific product of that unit of labour – i.e. if the SMPL is not the sole component of the MPAL – then it follows that the cost of that marginal unit of labour is not the only cost of the marginal product. Multiple causes imply multiple costs. In a productive process requiring more factors than labour – i.e. if the causes of the MPAL are multiple – then the costs of production will also be multiple. If the MPAL is equated with the wages of the marginal unit of labour, the other costs remain uncovered; the fixed factors that act in combination with labour receive no revenue.

If you cannot climb a ladder without the help of the ladder, i.e. that your upward progress has more than one cause, then it would seem obvious that the cost of the ladder should be taken into account, along with the cost of your labour, in estimating the marginal cost of the process. If the ladder contributes to the climber's upward progress, if capital (though held constant) contributes to the marginal product (MPAL) that occurs after the employment of the marginal unit of labour, then it would seem to be economically inappropriate (as well as being morally inappropriate, on Lockean terms) to allocate the entire MPAL to wages, and not to reserve some portion of the MPAL as a reward for, or to cover the costs of, the contributions made by the constant capital.

A concern with costs rather than causes does not eliminate the causative role of the constant factors. If we accept that capital continues to play a productive role even while being held constant – i.e. if we accept that to climb a ladder we need a ladder – then it would seem logical for a due proportion of the cost of capital to be added to the cost of labour in estimating the cost of producing the (non-specific and multicausal) marginal product of labour (MPAL). If in accordance with the MPTD, the entire MPAL is paid to the variable factor, then no return is available to cover the cost of the constant factor that has contributed to the production of the MPAL.

According to the MPTD, the equality of the marginal product of a factor and the return to the factor is a condition of profit maximisation and equilibrium. The above argument suggests the contrary. Common sense and proper accounting practice would seem to demand that a proportionate share of the cost of the constant factors be taken into account when estimating the costs incurred in employing the marginal unit of labour; and that it would be commercial folly to maintain that, as a condition of profit maximisation and equilibrium, the entire marginal product (MPAL) associated with the employment of the marginal unit of labour should be paid as wages to labour. It makes no commercial sense to give to labour the entire value of something that has been produced by both labour and (constant) capital. Such a procedure would be a criterion, not of profit maximisation and equilibrium, but of imminent bankruptcy and disequilibrium. Any business operators who had been beguiled by the MPTD into thinking that they would achieve maximum profits and equilibrium by equating MPAL and wages would soon come to regard their studies of neoclassical economics as regrettable.

The partial derivative as a definition of the marginal product of labour

Many textbooks define the marginal product of labour in calculus terms as the first partial derivative of output with respect to labour ($\partial Q/\partial L$), or in words equivalent to the calculus expression.³ This conveniently leaves aside the problems (addressed by Alfred Marshall but often ignored subsequently) of calculating the *net* marginal product, i.e. of measuring and subtracting the increase in output that is due to any other variable factors that have to be employed along with the marginal unit of labour because of the nature of the productive process – for example, extra lighting, power and paper for an extra printing employee to use; or, in Marshall’s example, an extra crook for an extra shepherd. A partial derivative is calculated by assuming no changes in any other variables when the extra unit of labour is added.

But, once we accept that the constant factors continue to exert a causative, productive role, then the partial derivative of output with respect to labour cannot logically be regarded as the marginal product of labour, if “of” is to be used in a specific monocausal sense (i.e. the SMPL). The taking of a partial derivative of output with respect to labour does not magically disentangle that part of the output specifically attributable to labour from that part specifically attributable to the constant capital. It tells us how output has changed *after* an extra unit of labour is employed; it does not tell us that the extra labour is the sole cause of the extra output. $\partial Q/\partial L$ is a measure of MPAL, not SMPL.⁴ To say that “the first partial derivative of output with respect to

³ For example: “the ratio of the movement in output to a small increment in labor, with capital fixed” (Lancaster 1989, p. 82).

⁴ To calculate $\partial Q/\partial L$ it is not necessary to assume that labour has any *causal* influence on output. A derivative is merely a measure of the change in one variable compared to a (small) change in

labour” is “the marginal product of labour” would not be correct if “of” in the latter expression is used in a monocausal and sole-proprietor sense. To use “of” in that expression in another, undefined sense, without explicitly advertent to the different sense, is to say the least very confusing, and could even be described as an abuse rather than a use, of language.

The introduction of the partial derivative has facilitated mathematical treatment of the MPTD, and has thereby enhanced its status as a scientific principle, as well as enhancing the status of economics as a hard, positive science. The abandonment of the partial derivative as a definition of the marginal product of labour would have the reverse effect.

J.B. Clark’s belief that the MPTD provides a solution to the problem of distributive justice has been described as a fanciful fairytale. But it is just as fanciful to believe that the first partial derivative of output with respect to labour is the marginal product of labour – i.e. the specific product actually contributed by labour, or the SMPL – or that an equality between the first derivative and the wages paid to labour constitutes an equilibrium and profit-maximising situation.

The mistaken impression that a partial derivative ($\partial y/\partial x$) somehow establishes a causal connection between x and y is perhaps a carry-over from the misuse of the terms “determining variable” and “determined variable” in referring to the coordinates of a graph. The variable on the vertical axis (y) is commonly described as the determined variable, and the variable on the horizontal axis (x) as the determining variable; whereas of course either the reverse could be the case, or each of the two

another variable. It does not require or imply any causal relationship between the two variables.

It expresses a correlation, which may or may not be also a causal relation.

variables could be both causally determining and causally determined, or there could be merely a correlation and no causal connection whatsoever between the two variables. The ongoing causative influence of capital cannot be eliminated simply by holding capital constant, or by assuming it is constant. To think otherwise would be a triumph of methodology over reality; or, worse still, a subordination of economics to calculus.

Section II. A Survey of the MPTD Literature

The aim of this Section is to consider the history of the MPTD in relation to the themes raised in Section I. A comprehensive treatment of the history of the MPTD would be beyond the scope of this paper, but an attempt has been made to include at least the most significant contributions.

The MPTD literature to 1941

There are a number of possible claimants to the title of founder of the MPTD, including William Petty, T.R. Malthus and Mountifort Longfield. But as Collison Black (1971, 13) has argued in the case of Longfield their contributions should more accurately be described as productivity theories, rather than marginal productivity theories.

The first truly marginal productivity theory appears to have come from J.H. von Thünen in *The Isolated State*, 1850; for example, in the chapter entitled “The wage is equal to the extra product of the last labourer who is employed in a large enterprise” (1966, 235). But if von Thünen is credited with being a founder of the MPTD, he must also be debited with being a founder of the failure to recognise that the marginal product that occurs after the employment of an additional unit of labour is not the

marginal product that can be causally and specifically attributed to that unit of labour, i.e. the failure to recognise that MPAL is not the same as SMPL.

W.S. Jevons made use of marginal concepts in his theory of value, but he did not extend that concept to a general theory of distribution. However, Dobb (1973, 185) believed that Jevons' theory of capital, in which the return of capital is increased by lengthening the period of production, contains "essentially the notion of marginal productivity".

The theory of imputation of F. von Wieser (*Natural Value*, 1888) was an attempt to solve the problem of distribution by the use of simultaneous equations. But it is doubtful whether the imputation method provides an answer to the disentanglement problem. And even if the imputation method is theoretically sound, its complexity renders it impracticable.

Alfred Marshall acknowledged von Thünen's influence – "I loved von Thünen above all my other masters" (Marshall 1925, 360) – but whether he followed von Thünen in thinking that the MPAL is the same as the SMPL depends on the meaning that Marshall intended to convey by the term "net product" – a term which he preferred to "marginal product" but which he unfortunately left unclear. It is not clear whether, by "net product", he meant fully net or partially net. The fully-net marginal product of labour would be found by deducting from the marginal product the cost of all the other factors used in conjunction with the marginal unit of labour, including not only other variable factors such as extra fuel and materials, but also a proportionate part of the cost of the fixed capital, such as land and buildings and existing machinery. The partially-net marginal product of labour would include only the cost of the extra variable factors used in conjunction with the marginal unit of labour. If by "net product" Marshall intended only partially-net, his argument suffers from the same

defect as that of his master (von Thünen), viz. it would fail to recognise the causative role and the cost of the fixed factors. Textual evidence can be found supporting both the partially-net and the fully-net interpretations. The balance of probabilities leads, in my estimation, to the partially-net interpretation.

The partially-net interpretation appears to coincide with the meaning given to “net product” by Joan Robinson (Robinson and Eatwell (1973, 41).

But, whichever interpretation is correct, Marshall’s treatment of the MPTD did not explicitly or systematically address the issue raised later by Joan Robinson, Geoff Harcourt, and others in the Cambridge capital controversy. His argument requires the cost of capital items (variable or fixed) to be deducted from the marginal product of labour (MPAL) in order to arrive at the “net product” of labour. But the cost of capital items can be calculated only if the rate of profits is already known. And the rate of profits can be found only if the rate of wages is known; or, in other words, only if the distribution of income between profits and wages is known. Marshall’s MPTD sets out to solve the problem of distribution via the calculation of net products, but the calculation of net products is possible only if the problem of distribution is already solved.

This circularity of argument means that the MPTD is incapable of presenting a determinate solution to the problem of distribution, or a solution that is an acceptable alternative to a pre-scientific solution by means of bargaining power and market conflict.

Stigler (1941) and Schumpeter (1954) believe that Marshall finally accepted the MPTD, although Schumpeter described Marshall’s “net product” as a “dangerous concept” because it prevented the use of partial differentials (1043). He believed that

Marshall had a “justifiable aversion to assigning a ‘causal’ role to the partial coefficients of the production function” (1033n.).

Phillip Wicksteed’s main contribution to the MPTD debate (*An Essay on the Co-ordination of the Laws of Distribution*, 1894) was his attempt to solve the adding-up problem (or the exhaustion-of-the-product problem) – i.e. to prove mathematically that if each factor receives a reward equal to its marginal product, the sum of the rewards will exactly equal the total product. He later withdrew, or appeared to withdraw, his solution following criticism from Pareto and Edgeworth, but, as Steedman (1987, IV, 917) notes, there has been considerable discussion of the significance of his actual or apparent withdrawal. Robbins (1933, xi) was of the opinion that Wicksteed merely withdrew his mathematical proof of the adding-up theorem, and did not abandon the MPTD as such; but a 1916 letter from Wicksteed to J.M. Clark (Dorfman 1964, 295) suggests a deeper disillusionment. However, Wicksteed in his public writings appears never to have doubted the basic principle of the MPTD, viz. that the MPAL (or the first derivative of the product with respect to labour) represents the specific product of the marginal unit of labour.

The most famous and most persuasive advocate of the MPTD has undoubtedly been J.B. Clark. In his *Distribution of Wealth* (1899) he argued that distribution is just, and economic exploitation is non-existent, if each factor receives what it produces, i.e. if each factor receives its “specific product”. If this does not occur, there is “institutional robbery” and a “legally established violation” of the rights of property. The MPTD is the “natural law of distribution” (Clark, J.B. 1956, 4, 8). He believed that the specific product of a factor is equal to the increase in the total product that results from adding one extra unit of the factor. According to his son (J.M. Clark),

J.B. Clark thought that “his most basically-important contribution” was the ethical implication of the notion of specific productivity (Clark, J.M. 1968, 306).

The validity of J.B. Clark’s normative version of the MPTD depends on his claim that the specific product of each factor can be disentangled and identified. This claim was subsequently subjected to intense criticism, and – as argued above in Section I – is difficult to sustain. One of the earliest and most vigorous opponents of the idea that the productivity of the marginal unit of a factor can be separately identified was J.A. Hobson.⁵ In his *Economics of Distribution* (1900), he accused defenders of the MPTD of “a false separatism ... which ignores the organic unity in a business” (1972, 144). Hobson argued that a marginal dose of one factor usually requires additional doses of other factors, and that the individual contributions of the factors in this “composite dose” can not be separated. Marshall replied – in what Blaug (1968, 442) has described as a “long unsatisfactory footnote” designed to give Hobson “a lesson in differential calculus” – that the contributions of the other factors are “of the second order of small” and can be neglected. Blaug has argued that Marshall’s introduction of the concept of net product was an attempt to refute Hobson, but that the concept was “illegitimate” and was in effect a capitulation to Hobson (Blaug 1968, 443).

Hobson’s critique was based, as already stated, on the argument that disentanglement is impossible because the variable factor usually requires an accompanying change in other factors. But he did not address the more fundamental point that disentanglement is impossible because the fixed factors exert a causative influence, even when they are unchanged.

Perhaps Hobson’s most lasting contribution to the anti-MPTD argument was his forceful statement of the political and ideological implications of the MPTD. In his

Work and Wealth (1914) he stated that the MPTD demonstrates “the final futility of all attempts of the labouring classes to get higher wages” (1968, 174-75).

In reviewing Wicksteed’s *Essay on the Laws of Distribution*, A.W. Flux (1894) offered a more mathematically-precise version of the adding-up theorem, but, like Wicksteed, did not question the fundamental notion that the marginal product of a factor can be isolated and measured by taking the first differential of the total product with respect to the factor.

Knut Wicksell in his *Lectures on Political Economy* (1901; first English translation, 1934) endorsed the MPTD as a general principle:

the share of the product going to any particular factor of production is determined by its marginal productivity (1934, I, 147)

and made a major contribution to the attempt to solve the adding-up problem. He also recognised the political and ideological significance of the MPTD. He saw it as a refutation of the theory of value of the socialists, which he regarded as “a terrible weapon against the existing order” (Wicksell 1934, I, 28). But he struck a major blow against the MPTD in arguing that it could not be applied when capital is the variable factor, because of the impossibility of measuring capital in units other than market prices – an argument that was to figure prominently in the Cambridge Capital Controversy.

Herbert Joseph Davenport argued vigorously against the idea that marginal products can be separately identified. In his *Value and Distribution* (1908), he emphasised the “togetherness” of the co-operating factors and the impossibility of determining the “separate and specific productivity” of each factor (1964, 471). It follows therefore,

⁵ For a more detailed analysis of Hobson’s criticisms of the MPTD, see Schneider 1996.

given a Lockean theory of property rights, that J.B. Clark erred in attaching moral sanction to what each factor happens to receive.

What under the ordinances of competition one gets, gives no safe report of what under the ordinances of God he deserves (Davenport 1918-19, 283).

These arguments were re-asserted and strengthened in his *Economics of Enterprise* (1913), when they were supported by the analogies of the horse and wagon (each is useless without the other) and the three-legged stool (“which leg of a three-legged stool supports the stool?”) (Davenport 1929, 147).

A similar criticism of the concept of specific productivity was made by F.W. Taussig – “We can disengage no concretely separable product of labor and capital (1911, II, 213-14).

An even more forceful attack on the MPTD was mounted by W.M. Adriance in the *Quarterly Journal of Economics*, 1914-15. He argued that it is a “verbal absurdity” to ascribe the joint product of labour and capital to either factor alone (157). In cutting down a tree, “what fraction of the work is done by the man and what fraction by the ax [sic]” (157)? And, referring to the productivity of a group of fishermen, he declared that the “mathematical error” of the MPTD lies in “*not attributing to the coöperation of the rest of the group any part of the so-called ‘marginal product’*” (159-60).

Adriance also provided interesting sociological and psychological reasons for the popularity of the MPTD – reasons that might be just as true now as then. With its “parade of scientific accuracy”, it gives a “scientific sanction” to the “deep-seated belief that ... a man gets out of his productive activities what he deserves”. It establishes a “soothing correlation between reward and productive contribution”

(1914-15, 169, 174, 175). He added that the MPTD “has tended to make us, as economists, more conservative than we have any right to be” (175-76).

In his *Studies in the Economics of Overhead Costs* (1923), J.M. Clark, although not explicitly criticising his father’s MPTD, raised some serious doubts about its validity. He argued, for example, that if additional labourers are paid their marginal products, “the whole value of the product would be absorbed before all the operating expenses were covered, leaving nothing for the owners but a deficit” (1923, 468-9). Such statements appear to recognise, as argued in Section I above), that the marginal product after labour (MPAL) is a multicausal phenomenon, consisting not only of what labour has specifically produced (the SMPL), but also of what has been produced by capital. This would appear to provide cogent grounds for rejecting the MPTD, as cogent as any that were later advanced during the Cambridge Capital Controversy.

The thesis advanced in this paper differs from J.M. Clark’s insofar as he implies that the productivity of capital needs to be considered only in the long run, i.e. when capital is variable, whereas it is argued in Section I above that capital exercises a productive function even in the short run, when it is fixed.

A comprehensive analysis and critique of the MPTD was made in 1928 by the Dutch economist, Dr Willem L. Valk, in his book *The Principles of Wages*, which had received an Honorable Mention in a 1926 competition for the best original treatise on the theory of wages. The adjudication committee included Laurence Laughlin, John Bates Clark and Wesley C. Mitchell.

Valk’s conclusion was that there does exist an economic law that determines distribution, and that therefore there is “no need to examine the Bargain Theory of

Wages” (134). However, he argued that this law of distribution is to be found not in the MPTD, but by combining the theories of Clark and Cassel. His objection to the MPTD was based on a rejection of the adding-up theorem. He argued that “when the prices of the means of production are equal to marginal productivity, the sum of these prices would exceed the sum available for distribution” (134).

An important practical implication of Valk’s Cassel-Clark law of distribution was that no “artificial change” in distribution could be successful, by which he apparently meant no change imposed by political policy; but that social improvement can be achieved by a change in circumstances – such as the supply and education of labour – and above all by increasing the total amount of produce available for distribution.

In his *Wages* (1928), Maurie Dobb gave recognition to the multicausality of marginal products (85) but apparently did not regard it as a serious defect in the MPTD. However, he later published two objections to the MPTD – the first based on the circular reasoning involved in the measurement of capital, and the second on possibility of the switching of techniques. The latter, he argued, gives “the coup de grâce to the whole notion of a production function, and hence to the very idea of marginal productivity as a determinant of profit” ([1970], 1988, 117).

Counter-arguments have been made to both of these criticisms. Circular reasoning, it is said, is a common occurrence in economics, and merely reflects the reciprocity of economic processes. And although the switching of techniques prevents the formation of a production function that is smooth and differentiable over its entire length, it does not preclude differentiation over the range between two successive switch points where one particular technique is operative.

Dennis H. Robertson, in his chapter on “Wage Grumbles” in *Economic Fragments* 1931, defended the MPTD against a number of criticisms. In particular, he attacked those who asserted the impossibility of disentangling specific products, even at the margin. He referred (1931, 224-25) to Bernard Shaw’s statement that, when a farmer and his labourers produce a crop of wheat, “nobody on earth can say how much of the wheat each of them has grown”; and to Bertrand Russell’s question: When a railway employee shunts good trains, “what proportion of the goods carried can be said to represent the produce of his labour?”. Robertson accused such “popular writers” of “ignorance of the elements of mathematical economics” and “sheer ignorance of the existence of the weapons [the marginal principle ?] forged by economic science for performing the process of disentanglement” (1931, 224-25).

The contention of this present paper is that Robertson attributed too much power to the “weapons forged by economic science”. They can isolate the “marginal product after labor” (MPAL), but not the “specific marginal product of labour” (SMPL). They cannot be expected to achieve the impossible. The MPTD can claim to be a scientific law of economics, and an alternative to bargaining-power as an explanation of distribution, only if the impossible can be achieved.

Support for the marginal productivity principle also came from J.R. Hicks (1932). He emphasised “the extremely abstract assumptions on which alone it is rigorously true to say that wages equal the marginal product of labour” (9-10), but he did not question the fundamental validity of the MPTD.

The validity and importance of this principle we shall see no reason to question. (v)

This ‘Law of Marginal Productivity is regarded by most modern economists as the most fundamental principle of the theory of wages. Nothing will be said here to contradict that view. (9)

the most ordinary non-mathematical analysis shows that every factor must get its marginal product. (234)

In an Appendix, Hicks considered the adding-up problem and concluded that the improvements made by Walras and Wicksell to Wicksteed’s unsatisfactory suggestions had provided a convincing solution.

Paul Douglas in *The Theory of Wages* (1934) responded to some of the criticisms made by Hobson against the MPTD —despite at the same time describing Hobson as “one of the finest spirits in modern life” (1957, 61). To Hobson’s view that the MPTD was invalid because of the impossibility of identifying marginal products, Douglas replied, following Wieser, that it is not necessary to identify marginal products: the MPTD requires only that the extra product be “imputed” to the appropriate factor. He also claimed, quoting Edgeworth, that the error that “lies behind much of Hobson’s criticisms” was his confusion of x and Δx (1957, 65) – a claim that Hobson quite rightly rejected.

George J. Stigler’s *Production and Distribution Theories*, 1941, provided a detailed discussion of the MPTD and of other theories of distribution, including a 68-page chapter on Euler’s theorem. As noted earlier in this Section II, Stigler praised the attempts of Wicksteed and others to prove the adding-up theorem, and with some minor reservations appears to have believed that the adding-up theorem and the

MPTD in general are valid.⁶ The analysis of the MPTD is, however, rather selective, concentrating mainly on like-minded defenders of the MPTD and suggesting a progressive line of theoretical development. No reference is made, for example, to the views of Hobson, Davenport, Adriance, Taussig, or J.M. Clark.

This survey of contributors to the MPTD up to 1941 has necessarily been very selective. Many others would have to be included in a more general survey less constrained by time and space; for example, F.Y. Edgeworth, D. Davidson, L. Walras, V. Pareto, E. Barone, H.K.E. von Mangoldt, I. Butt, C. Menger, C.J. Bullock, F.H. Knight, A. Berry, S.J. Chapman, Stuart Wood, E.M. Burns, H.M. Thompson, J.W.F. Roue, A.C. Pigou.

The recent MPTD literature

Despite the many vigorous attacks on the possibility of a Clarkean or normative MPTD, recent literature reveals frequent instances where the language used in presenting the MPTD lends itself to a monocausal and normative interpretation, even if that was not the intention of the writers; and where the equalization of marginal products and factor rewards is regarded as a profit maximization and equilibrium criteria.

⁶ Despite Sigler's confidence in the validity of the adding-up theorem, others remain less convinced. Thus, G.L.S. Shackle, while fully agreeing that it is profitable to employ labour up to the point where wages and the value of the marginal product of labour are equal, nevertheless admits that the adding-up problem is "still lacking a tidy and complete solution after nearly seventy years" (1959, 115). This is a strange admission, given that it is generally agreed that, unless the adding-up theorem can be rigorously proved, the MPTD cannot stand.

In the following examples, the use of the proprietorial “of”, and possessive pronouns, and other expressions indicating a monocausal or sole-proprietor connection has been highlighted:

factors are paid according to the value of *their* marginal products.
(Chamberlin 1942, 188)

The marginal-product of a productive factor is the extra product or output *added by* one extra unit of that factor, *while other factors are being held constant*. (Samuelson 1958, 504)

each of the three factors will be paid *its* marginal product. (Samuelson 1958, 598)

each factor will be paid the value of *its* marginal product. (Paish 1964)

Marginal-productivity theory contends that in equilibrium each productive agent will be rewarded in accordance with *its* marginal productivity as measured by the effect of the addition or withdrawal of a unit *of that agent* on the total product, *the quantity of the other agents being held constant*.
(Blaug 1968, 432)

a firm in a competitive industry will hire workers up to the point where the value of the marginal product ... just equals the cost of the factor.
(Dernburg & McDougall 1968, 192)

a factor of production receives as much income as corresponds to *its* contribution to production. (Pen 1971, 76)

[The marginal product of labour is] the additional amount of product ∂Q which *results from* one additional unit of labour ∂L . (Pen 1971, 431)

wholesale denial of marginal productivity theory would mean that, among other things, we could never raise questions about the *contributions of individual workers* to output. (Blaug 1972, 206n)

the value of each factor in production is determined by *its* marginal contribution to total output. (Ekelund and Hebert 1975, 384)

[a firm will hire factors] up to the point where *each factor's marginal product* is just equal to its income payment. (Ekelund and Hebert 1975, 384)

the marginal revenue product [of a factor is] the addition to revenue resulting from the sale of the product *contributed by an additional unit* of the variable factor. (Lipsey 1975, 347)

the firm will increase production up to the point at which *the last unit of the variable factor employed adds* just as much to revenue as it does to cost. (Lipsey 1975, 347)

[the MPTD enables us to determine] the productive factor's *specific* contribution to the making of the final product. (Waud 1986, 679)

[the MPTD] holds that the payment for any factor of production tends to be about equal to the value of *its* marginal product. (Dorfman 1987, III, 323)

if the wage of any factor is less than the value of the output that *an additional unit could produce*, successive units of that factor will be employed until the inequality vanishes. (Dorfman 1987, III, 323)

[the marginal product of labour is] the ratio of the increment in output to a small increment in labor, with capital fixed. (Lancaster 1989, 82)

[the marginal product of labour is] the increase in total product that *results from* a one-unit increase in the quantity of labour employed. (McTaggart, Findlay and Parkin 1999, 105)

a one-unit increase in labour input [in a hamburger shop] from two to three workers, increases output from 60 to 80 hamburgers. (McTaggart, Findlay and Parkin 1999, 106)

Section III. Conclusion

1. Vestigial remnants of the Clarkean, normative MPTD appear to have survived in the language of modern versions of the MPTD. This suggests that at least some modern expositors still believe that the MPTD carries legitimate moral implications. If these normative implications are not intended, less confusion would occur if the normative language were either discontinued or accompanied by explicit warnings to that effect. In the absence of such warnings, modern renditions of the MPTD send both normative and non-normative messages.
2. Defenders of the MPTD present it as a purely positive law that substitutes a commercial imperative for a moral imperative. They claim for it all the status and respect due to a law of nature, or a law of the hard, physical sciences. But this paper contends that the scientific pretensions of the MPTD are no more justified than its moral pretensions and that the MPTD can claim neither normative nor non-normative status.
3. The use of the differential calculus adds a scientific aura to the MPTD, but the aura is pseudo-scientific if partial differentials are not a valid measure of specific marginal products. If the equalisation of partial differentials and factor

rewards does not represent a profit maximisation and equilibrium condition, economic models incorporating that equilibrium condition would have to be reassessed.

4. The misuse of the *ceteris paribus* clause perpetuates the myth that the specific marginal products of the various factors can be disentangled. *Ceteris paribus* does not mean *ceteris inefficacibus*.
5. The MPTD endeavours to formulate a scientific law of income distribution to replace the bargaining power or conflict view of distribution. The ongoing causative role of the constant factors, and the problem of the disentanglement of specific marginal products, would appear to raise considerable obstacles to the success of the endeavour.
6. Marshall's concept of *net* marginal product raised the possibility of identifying specific marginal products, or using net products as substitutes for marginal products, but its calculation presupposes a determination of the wage-profit relationship, and hence does not provide a theory of distribution that is independent of buyer-seller, employer-employee conflict.
7. It has long been acknowledged that the MPTD is not a complete theory of distribution because it is concerned only with demand-side aspects of distribution. This paper contends that, even in conjunction with adequate supply-side considerations, the MPTD still does not provide a complete theory of distribution, because even as a demand-side theory it is deficient.
8. Some writers have argued that unless wages are equal to the value of the marginal product of labour (MPAL), labour is being exploited by capital. But in fact, if the argument of this paper is sustainable, labour is exploiting capital if

wages are equal to the MPAL. Capital is exploiting labour if profits are equal to the “marginal product after capital”.

9. The “ $MP_L = \text{wages}$ ” rule is sometimes defended as a necessary corollary of the “ $MR = MC$ ” rule; but this would be true only if “marginal cost” includes a proportionate share of the cost of the constant factors.
10. This paper also discounts the argument that the MPTD would be a valid and useful theory of income distribution if capital were minutely divisible (for example, as in the sowing of seed grain). The divisibility of capital would not overcome the problem of disentanglement.
11. The incautious use of the proprietorial “of” (and other monocausal and proprietorial expressions) is not peculiar to the MPTD. It is also evident in the language of the labour theory of value and in the law of diminishing returns, both of which fail to distinguish between “the product of labour” and the “product of labour alone”.
12. The history of the MPTD could be seen as an unsuccessful attempt to introduce scientific certainty and determinacy into an area of human affairs that does not admit of certainty and determinacy. It has been a fruitless search for a theoretical alternative to the harsh reality of bargaining-power and market-place conflict. In the absence of a satisfactory alternative, the only realistic theory or law of income distribution would appear to be: labour gets what capital doesn't, or vice versa.

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