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No. 2005-13

Working Paper Series in Economics

ISSN 1442 2980

http://www.une.edu.au/febl/EconStud/wps.htm

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Abstract

The phenomenon of population aging is now an established demographic characteristic of many economies. Public policy makers are thus increasingly concerned about the economic consequences of large numbers of retired citizens. Economists working in the endogenous growth theory tradition have sought to model the relationship between public pensions, financed on a 'Pay-As-You-Go' (hereafter PAYG) basis, and the growth in per capita incomes. It appears that the resultant intergenerational wealth redistribution from young to older people seems to decrease private savings, diminish capital accumulation, and lower the growth of per capita incomes. The underlying transmission mechanism appears to be a crowding out effect in private capital markets contingent upon the introduction of public pension systems. A growing literature exists on the interrelationships between public pension schemes, fertility rates and endogenous growth. Following Wigger's (1999) pioneering overlapping generations endogenous growth model, we examine the effects of a savings subsidization system on the rate of per capita income growth, fertility and voluntary intrafamily wealth transfers, where parents view children both as an insurance good and a consumption good. Moreover, children care about the consumption levels of their parents. An increase in contributions to a savings subsidized public pension scheme will crowd out private intergenerational transfers from the young to the old and negate the usefulness of children as an insurance good.

Key Words: Endogenous growth; fertility; savings subsidization; public pension scheme

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INTRODUCTION

The Phenomenon of population aging is now an established demographic characteristic of many economies. Public policy makers are thus increasingly concerned about the economic consequences of large numbers of retired citizens. Economists working in the endogenous growth theory tradition have sought to model the relationship between public pensions, financed on a 'Pay-As-You-Go' (hereafter PAYG) basis, and the growth in per capita incomes. It appears that the resultant intergenerational wealth redistribution from young to older people seems to decrease private savings, diminish capital accumulation, and lower the growth of per capita incomes (see, for instance, King and Ferguson (1993)). The underlying transmission mechanism appears to be a crowding out effect in private capital markets contingent upon the introduction of public pension systems.

But pension systems are not their only means of securing financial viability in old age. In many societies throughout recorded history, the institution of the family has also served to transfer wealth intergenerationally between the young and the old (Hanson and Stuart, 1989). Indeed, the prospect of intergenerational transfers can help explain the formation of families and child rearing, with children perceived as insurance goods (Cigno, 1993). Accordingly, compulsory public pension schemes might reduce the significance of children as insurance goods and thus lower fertility rates (Zhang and Zhang, 1995). Where the sole motive for having children derives from an insurance motivation, it seems that unfounded public pension systems result in lower fertility levels and concomitantly higher rates of per capita income growth (Zhang, 1995). The economic explanation for this outcome appears to be that lower fertility levels reduce the crowding out effect and thus boost the growth in per capita incomes.

However, the proposition that people bear and raise children solely as insurance against old age does not seem intuitively plausible. It would appear much more likely that several different motives exist simultaneously for parenthood. In order to enhance the realism of modelling relationships between fertility, pension systems and economic growth, Berthold Wigger (1999) investigated the case where parents view children as both an insurance good and a consumption good. He demonstrated that the impact of unfounded public pension systems on fertility and per capita income growth 'essentially depend(s) on the extent of the intergenerational redistribution caused by the public pension scheme' (Wigger, 1999, p.636).

Following the Wigger (1999) model, we argue that if a government runs a public pension system as a savings subsidization (hereafter SS) scheme, where the fraction of income τ from a young person is used to subsidize the savings of an old person, then participation individuals will have a stronger incentive to save *ceteris paribus* than would otherwise be the case. Moreover, we demonstrate that this results in lower fertility levels and higher growth in per capita incomes.

The paper itself is divided into three main sections. Section 2 develops the general model we employ. Section 3 investigates the effects of the introduction of an SS scheme on fertility, voluntary intrafamily wealth transfers and the rate of per capita income growth. The paper concludes with some brief comments on its policy relevance in the context of developing economies in section 4.

THE BASIC STRUCTURE OF THE MODEL

Production of Final Output

At any given time, the economy has a certain amount of capital K_t, labor L_t, and labor productivity A_t, and these are combined to produce an output A_t. The production function takes the form

$$Y_t = F(K_t, A_t L_t).$$

Thus, the economy grows because of increases in the amount of labours and/or capital employed and through increases in the productivity of labor. We assume the following:

(i) Production occurs accordingly to a constant returns to scale technology.

(ii) A fraction δ of capital depreciates in each period, where $\delta \in [0,1]$.

(ii) The labor force is equal to the size of the young generation N_t.

(iii) A_t represents labor productivity with an Arrow(1962)-Romer(1986) type externality that depends on the accumulated investments per worker expressed in the form

$$\overline{A_t} = \frac{\overline{K_t}}{a\overline{L_t}} = \frac{\overline{K_t}}{aN_t}, \tag{1}$$

where a is a positive technological parameter, $\overline{K_t}$ and $\overline{L_t}$ are the aggregate amounts of capital and labor.

Labor and capital are paid their marginal products. Thus by differentiating $F(K_t, A_t L_t) - \delta K_t$ with respect to L_t and K_t respectively, by using equation (1) the real wage ω_t and the gross interest rate R_t in the period t are

$$\omega_t = \varpi \frac{\overline{K_t}}{N_t}, \tag{2}$$

$$R_t = 1 + f'(a) - \delta = R$$

where $\varpi = \frac{f(a) - af'(a)}{a}$ and $f\left(\frac{K_t}{AL_t}\right) = F\left(\frac{K_t}{AL_t}, 1\right)$. We note that the wage rate is linear in the capital stock per worker of the same period. Moreover, the gross interest rate R is constant over time and is the gross private return on capital. We also note that the product market equilibrium obtains when aggregate investment equals aggregate saving:

$$\overline{K_{t+1}} = N_t s_t \tag{3}$$

where s_t is an agent's saving during the first period at time *t*.

Thus from equation (2) and equation (3), we can obtain

$$s_t = \frac{\omega_{t+1}}{\varpi}\left(1+n_t\right) \tag{4}$$

where n_t is population growth at time *t,* i.e., $1+n_t = \frac{N_{t+1}}{N_t}$.

Household Behavior

We base our analysis on a conventional overlapping-generations models (Samuelson, 1958; Diamond, 1965) in which individuals live for two periods. In the first period of life, young (individuals) work and give birth to $1+n_t$ children, and use their income ω_t for consumption, investment in children $\lambda(1+n_t)^{\mu}\omega_t$, where $\lambda > 0$ and $\mu \geq 1$ (see Zhang and Zhang, 1995), making gifts to their parents $q_t\omega_t$ saving for second period s_t and contributing to a public pension system $\tau_t\omega_t$. In the second period, we assume that the (old) parents do not work any more, and that there are three sources of support: a return from a child $q_{t+1}\omega_{t+1}$, and a public return from the SS $\sigma_{t+1}s_t$, and the use of resources acquired during working life $R_t s_t$, where σ_t is the subsidy saving rate at time t and R_t is the gross interest rate at time t. Suppose that the intertemporal utility function of the individuals born in period t depends on their first period consumption c_t^t, second period consumption c_{t+1}^t, the number of children $1+n_t$ and the parents' consumption c_t^{t-1} respectively. Thus utility is given by

$$U\left(c_t^t, c_{t+1}^t, 1+n_t, c_t^{t-1}\right) = \ln c_t^t + \rho \ln c_{t+1}^t + \eta \ln(1+n_t) + \theta \ln c_t^{t-1} \tag{5}$$

where ρ is an intertemporal discount factor measuring the felicity of one's own consumption when old and θ is an intergenerational discount factor measuring the strength of altruistic concerns towards the parent. The parameter η is associated with the consumption aspect of children, and measures the joys of raising them. Thus the individual faces the following budget constrain in the first and second periods respectively,

$$c_t^t = \left[1 - \tau_t - q_t - \lambda(1+n_t)^{\mu}\right]\omega_t - s_t$$

$$c_{t+1}^t = \left(R_{t+1} + \sigma_{t+1}\right)s_t + \left(1+n_t\right)q_{t+1}\omega_{t+1}$$

$$c_t^{t-1} = \left(R_t + \sigma_t\right)s_{t-1} + \left(1+n_{t-1}\right)q_t\omega_t$$

The Public System

The government runs the public system as a SS scheme; the contributions of the young are used to subsidize the savings of the old during the first period. A balance budget public pension system then requires:

$$\sigma_{t+1}s_t = \left(1+n_t\right)\tau_{t+1}\omega_{t+1}, \text{ for all } t. \tag{6}$$

where from a social point of view, n_t is the average fertility rate at time t. We can prove that $\sigma_{t+1} = \tau_{t+1}\varpi$.

We assume that the taxes of labor earnings are at a constant rate over time, i.e. $\tau_t = \tau$ and τ is restricted to the interval $[0,1)$.

Optimization

In this section, we determine the solution of the above maximization problem. We employ the standard Nash assumption. An individual maximizes utility (5), subject to the budget constraint with equation (6) respectively, and equation (4), non-negativity constraints concerning first and second period consumption, the number of children, and gift and the subject to the given transfers to his children. Note, however, that the particular form of the utility function rules out corner solutions with respect to the number of children and first and second period consumption. Thus only a non-negativity constraint concerning the gift rate has to be taken into account. The first order conditions can then be written as:

$$U_1 = (R + \sigma_{t+1})U_2, \tag{7}$$

$$\lambda\mu(1+n_t)^{\mu-1}\omega_t U_1 = q_{t+1}\omega_{t+1}U_2 + U_3 \tag{8}$$

$$U_1 \geq (1+n_{t-1})U_4, \text{ with equality if } q_t > 0 \tag{9}$$

where,

$$U_1 = \frac{\varpi / \omega_t}{(1-\tau-q_t-\lambda(1+n_t)^{\mu})\varpi-(1+g_t)(1+n_t)},$$

$$U_2 = \frac{\rho\varpi / \omega_t}{(1+g_t)(1+n_t)[(\tau + q_{t+1})\varpi + R]},$$

$$U_3 = \frac{\eta}{1+\eta_t},$$

$$U_4 = \frac{\theta\varpi / \omega_t}{(1+n_{t-1})[(\tau + q_t)\varpi + R]}.$$

Note that U_i is partial derivative of U with the respect to its $i-th$ argument and $g_t = \frac{\omega_{t+1}}{\omega_t} - 1$ is the growth rate per capita income at time *t*. Equation (7) describes the second period consumption with first period consumption assumed to be constant. Then an increase of the first period consumption will lead to an additional in order to keep the ratio constant. Equation (8) describes the tradeoff between having an extra child and its gift in the next period, and forgone first period consumption because of additional child rearing costs. Finally, equation (9) describes the compensation for additional transfers to parents to first period consumption.

Equilibrium Analysis

A balanced growth equilibrium is a competitive equilibrium in which the fraction of income q_t used as gifts to old parents and the fertility rate n_t are constant, denoted by *q* and *n* respectively in what follows, and in which young and old age consumption, c_t^t and c_t^{t+1}, grow at the same endogenously determined and constant growth rate of per capita income and saving, denoted by g and ξ. Therefore, we can rewrite the above equations in the following form after some simple computation:

$$\{[1-\tau-q-\lambda(1+n)^{\mu}]\varpi-(1-g)(1+n)\}(\tau\varpi+R)\rho=(1+g)(1+n)[(\tau+q)\varpi+R]$$

$$=(1+g)(1+n)[(\tau+q)\varpi+R]\ , \tag{10}$$

$$(1+n)(1+g)\rho q\varpi+\eta(1+g)(1+n)[(\tau+q)\varpi+R$$

$$=\lambda\mu(1+n)^{\mu}\varpi\rho(\tau\varpi+R)\,, \tag{11}$$

$$(1+g)(1+n)\leq\frac{\rho(\varpi\tau+R)}{\theta}\text{, with equal sign if } q>0 \tag{12}$$

$$1+g=1+\xi \tag{13}$$

From equation (10) and equation (11) we have,

$$q=\frac{(\tau\varpi+R)\{\mu\rho\varpi(1+\tau)-[\eta+(1+\rho)\mu](1+g)(1+n)\}}{[(\mu+\eta+\rho)(1+g)(1+n)+\mu\rho(\tau\varpi+R)]\varpi}\,. \tag{14}$$

It is clear from equations (12) and (13) that the growth rate for SS system depends positively on the gross saving rate but negatively on fertility for $q>0$. Moreover, from equations (14), the increase of the factor $(1+g)(1+n)$ will decrease the gifts from the young individuals to their parents.

EFFECT OF INCREASING CONTRIBUTIONS TO A PAYG SCHEME AND AN SS SCHEME

In this section, we examine the effect of increasing the contribution of SS System on the returns to parents, fertility, per capita income growth and saving.

The following lemma shows that whether young individuals in fact make gifts to their parents or not depends on the magnitude of intergenerational transfers caused by the public pension system.

Lemma

Suppose that the gift q_t from the individuals to their parents is strictly positive. Then we can obtain the following results:

For $\theta \geq \underline{\theta} \equiv \dfrac{[(1+\rho)\mu+\eta]R}{\mu\varpi}$, $q \; > 0$ if and only if

$0 < \tau < \hat{\tau}$ such that $\hat{\tau} = \dfrac{\theta\mu\varpi - [\eta + (1+\rho)\mu]R}{[(1+\theta+\rho)\mu+\eta]\varpi}$.

Also $\hat{\tau}$ is smaller than one. Thus there is always feasible public pension system completely crowding out private transfers. The value of θ will determine whether there is a strictly positive private intergeneration transfer in the absence of the public pension scheme. On other words, if $\theta > \underline{\theta}$ altruism of young individuals towards their parents are strong enough to tender the gift motive operative if there are no public intergenerational transfers.

Moreover, if $\tau \in [0, \hat{\tau})$, we have $\frac{dq}{d\tau} < 0$. On other words, a rise in τ will induce young individuals to transfer smaller q_t to their parents. As a consequence, public system SS tends to crowd out private intergeneration transfers from the young to the old and negate the usefulness of children as an insurance good.

Proof: See the Appendix

The following propositions, which the consequences of the above lemma, show us the interrelation between fertility, economy growth and the size of SS systems.

Proposition

For all contribution rates $\tau \in [0, \hat{\tau})$, a rise in τ increases the per capita income growth rate g and decreases the fertility rate *n*.

Proof: See the Appendix

We shall now turn our attention to the interrelation between fertility, economic growth and the size of he public systems in the interval $[\hat{\tau}, 1)$.

Proposition

Let $\tau \in [\hat{\tau},1)$. Then (i) $\frac{dg}{d\tau} \leq 0$, with equal sign if $\mu = 1$ and (ii) $\frac{dn}{d\tau} < 0$.

Proof: See the Appendix

We explain these proposition by using figures 1 and 2. As illustrated in Figure 1, an increase in τ will reduce $1+n$ and *vice versa*. Figure 2 shows that $1+g$ is maximum, where $\tau = \hat{\tau}$.

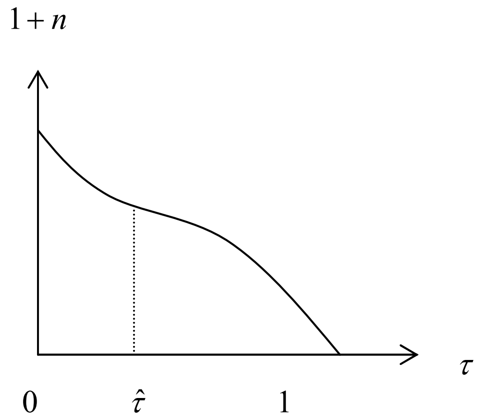

Figure 1

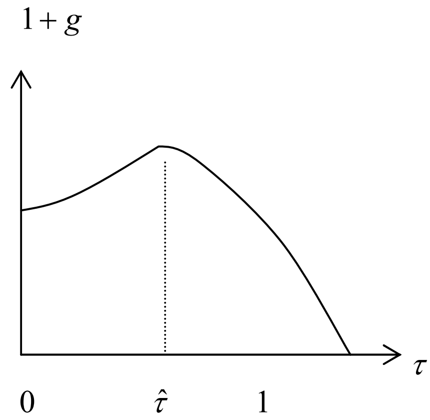

Figure 2

We should note that all the effects of SS public pensions system on growth and fertility derived in this paper not only occur in the long run, but immediately after the change in public pension policy (see Wigger (1999), proposition).

CONCLUDING REMARKS

The model developed in this paper has some interesting policy implications for the design of public pension systems. In general, intergenerational wealth redistribution from young to older people would appear to decrease the aggregate magnitude of private savings. We have demonstrated that although SS schemes represent a form of intergenerational wealth redistribution in favor of older people, they nevertheless will stimulate per capita income growth, provided the transfers involved are relatively small. However, fertility growth will be simultaneously depressed. By contrast, if the wealth transfers in question are large, then both fertility and per capita income growth will be lower than would otherwise be the case *ceteris paribus.* It is thus crucial that policy makers carefully consider the magnitude of wealth redistribution implicit in SS schemes.

In this sense our model differs from Wigger's (1999) PAYG model. In his proposition 2, as qualified by lemma 3, Wigger's (1999, p.636 and Figure 2) observes that 'in an intermediate region, on the other hand, a further increase in public intergenerational redistribution may have the potential to increase fertility'. By contrast, in our SS model, fertility steadily decreases as the tax contribution to the public system increases, as evident in Figure 1. This difference has significance from the perspective of public policy making in developed and developed and developing countries. For example, in rapidly aging advanced economies the increase in fertility evident in the 'intermediate region' in Wigger's (1999) PAYG model may not be problematical. However, in comparatively youthful developing societies, public policy that induces an increase in fertility may have substantial costs. In our SS model, however, there is no 'intermediate region'. Accordingly, it may thus represent a more optimal approach to the design of public pension schemes in developing countries.

APPENDIX

A Lemma 3.1

Suppose that $q > 0$. Then from (12) and (14) we can write

$$q = \frac{\mu\theta\varpi - [\eta + (1+\rho)\mu]R}{[\mu(1+\theta) + \eta + \rho]\varpi} - \frac{[\eta + (1+\theta+\rho)\mu]}{[\mu(1+\theta) + \eta + \rho]}\tau . \tag{15}$$

Moreover $q > 0$ implies

$$\tau < \hat{\tau} = \frac{\theta\mu\varpi - [\eta + (1+\rho)\mu]R}{[(1+\theta+\rho)\mu + \eta]\varpi} . \tag{16}$$

If $\tau > \hat{\tau}$ and $q > 0$, then (14) implies that

$$\tau = 1 - \frac{[\eta + (1+\rho)\mu](1+g)(1+n)}{\mu\rho\varpi} \tag{17}$$

It is obvious that τ is decreasing in $(1+g)(1+n)$. Using (12) and (17), we can show that $\tau \geq \hat{\tau}$, which contradicts our hypothesis.

Since the numerator is always smaller than the denominator in (16), there are always feasible public pension systems completely crowding out private transfers. Moreover, from (16), a necessary and sufficient condition for $\hat{\tau}$ to be positive is that

$$\theta \geq \underline{\theta} \equiv \frac{[(1+\rho)\mu + \eta]R}{\mu\varpi} .$$

Differentiating (15) with respect to τ leads to $\frac{dq}{d\tau} < 0$.

B Proposition 3.2

Replacing q by (15) and $(1+n)(1+g)$ by (12). In (10) and (11), it follows after some manipulations that

$$(1+n)^{\mu} = \frac{\eta R}{\theta \lambda \varpi \mu} + \frac{(\eta+\rho)A}{\theta\lambda\mu} + \frac{\eta B - (\eta+\rho)[\mu(1+\theta+\rho)+\eta]}{B\theta\lambda\mu}\tau .$$

Where

$$A = \frac{\mu\theta\varpi - [\mu(1+\rho)+\eta]R}{[\mu(1+\theta)+\eta+\rho]\varpi} \text{ and } B = \mu(1+\theta)+\eta+\rho .$$

Since $\eta B - (\eta+\rho)[\mu(1+\theta+\rho)+\eta] = -\mu\rho[1+\eta+\theta+\rho]$,

Therefore if we differentiate the above equations, we will obtain the results.

C Proposition 3.3

From Lemma 1, it is known that $q = 0$ for $\tau > \hat{\tau}_{SS}$. Then from (10) and (11), it follows that

$$\rho\mu\lambda(1+n)^{\mu}\varpi - \eta(1+g)(1+n) = 0,$$

$$\rho(1-\tau)\varpi - \rho\lambda(1+n)^{\mu}\varpi - (1+\rho)(1+g)(1+n) = 0$$

Employing the implicit function theorem one gets:

$$\begin{pmatrix} \eta(1+g)-\mu^2\lambda\rho(1+n)^{\mu-1}\varpi & \eta(1+n) \\ -\left\{\lambda\mu\rho(1+n)^{\mu-1}\varpi+(1+\rho)(1+g)\right\} & -(1+\rho)(1+n) \end{pmatrix} \begin{pmatrix} dn/d\tau \\ dg/d\tau \end{pmatrix} = \begin{pmatrix} 0 \\ \varpi\rho \end{pmatrix}$$

The Jacobian determinants is given by:

$$\Delta = \lambda\mu\rho(\mu+\eta)(1+n)^{\mu}\varpi > 0$$

Now, if we replace the first column of the Jacobian determinant by the column vector on the right hand side, we obtain,

$$\Delta_n = -\rho\eta(1+n)\varpi < 0 .$$

Similarly, if we replace the second column of the Jacobian determinant by the column vector on the right hand side, we obtain,

$\Delta_g = -\rho\eta(1+g)(\mu-1)\varpi \le 0$, with equality if $\mu = 1$

Then Cramer's rule gives the following: $\frac{dn}{d\tau} < 0$ and $\frac{dg}{d\tau} \le 0$ with equality if $\mu = 1$.

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