STABILITY CONDITIONS IN THE USE OF FIXED REQUIREMENT APPROACH TO MANPOWER PLANNING MODELS

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STABILITY CONDITIONS IN THE USE OF FIXED REQUIREMENT APPROACH TO MANPOWER PLANNING MODELS*

1. Introduction

One important approach to Educational and Manpower Planning is usually referred to in economics literature as the fixed requirement approach. Models proposed by Parnes (1962), Tinbergen and Correa (1962), Tinbergen and Bos (1965), Eckaus (1964) and others are essentially variants of the same underlying approach. The OECD (1965) Mediterranean project was the first attempt to project educational and manpower needs with a view to planning their educational subsectors. The attempt was far from successful, and economic reasons for the failure of these models to predict accurately have been described in detail by Sen (1966 and 1969).

In this paper we examine the dynamic properties of the models, and show that a very important question related to the ability of a model to predict, namely its stability, appears to have been neglected. All the models examined by the authors were highly unstable, and their lack of success in projections is therefore not surprising.

The fixed requirement approach treats the requirement of educated manpower of different types at a given point of time as functions of the level and sectoral distribution of output. When this requirement is related to the stock of manpower already in the work-force, the result is a system of linear difference equations expressing the interrelationships between the different requirements and the level of output.

The model is then used to forecast the requirements of educated manpower as follows: The level of total output of the country is assumed to follow a certain pattern (for example, a constant growth rate path), and this projection is used to solve the difference equations to obtain projected requirements.

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1. Countries included in this project were Greece, Italy, Portugal, Spain, Turkey and Yugoslavia.
In order to illustrate the way in which systems of linear difference equations have been used in manpower planning exercises, we cite a simple six-equation model used by Blum (1965).

The model was specified as follows:

\[
\begin{align*}
N_t^2 &= v_t^2 v_t', \\
N_t^2 &= \mu^2 N_{t-1}^2 + m_t^2, \\
m_t^2 &= n_{t-1}^2 - n_t^3, \\
m_t^3 &= n_{t-1}^3, \\
N_t^3 &= \mu^3 N_{t-1}^3 + m_t^3, \\
N_t^3 &= \nu^3 N_t^3 + \pi^2 n_t^2 + \pi^3 n_t^3.
\end{align*}
\]

All variables are endogenous except for \( v_t \) and were defined as follows:

- \( v_t \), the total volume of production;
- \( N_t^2 \), the labour force with a secondary education;
- \( N_t^3 \), the labour force with a third-level education;
- \( m_t^2 \), those who have entered the labour force \( N_t^2 \) within the previous 6 years;
- \( m_t^3 \), those who have entered the labour force \( N_t^3 \) within the previous 6 years;
- \( n_t^2 \), the number of students in secondary education;
- \( n_t^3 \), the number of students in third-level education.

The technical coefficients were assigned the values \( \nu^2 = 0.039, \nu^3 = 0.016, \pi^2 = 0.03, \pi^3 = 0.07, \mu^2 = 0.85 \) and \( \mu^3 = 0.835 \).

If we now define the endogenous vector \( y_t \) by

\[
y_t = [N_t^2, m_t^2, N_t^3, m_t^3, n_t^2, n_t^3].
\]

and the exogenous vector \( x_t \) by
the system of difference equations (1.1) to (1.6) can be written in the matrix form

\[ A_0 Y_t = A_1 Y_{t-1} + B_0 x_t \]  

(1.8)

where

\[
A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & -\frac{3}{2}
\end{bmatrix}, \quad
A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad
B_0 = 0
\]

The model (1.8) was used to project each of the endogenous variables (1.7) by assuming \( x_t \) would grow at a constant rate. Thus

\[ x_t = x_0 \omega^t, \]  

(1.9)

where in this particular case \( \omega = 1.5 \) and \( x_0 \) is the (known) initial value of \( x_t \).

Assuming now that each of the endogenous variables grow at the same rate \( \omega \), the vector \( y_t \) is given the form

\[ y_t = y_0 \omega^t, \]  

(1.10)

where \( y_0 \) is the unknown vector of initial values of \( y_t \). Substituting (1.9) and (1.10) into (1.8), the solution is found in the form

\[ y_0 = \left[ I - \frac{A}{\omega} \right]^{-1} A_0^{-1} B_0 x_0 \]

(1.11)

where \( A = A_0^{-1} A_1 \). Equation (1.10) now completely determines the time-paths of the endogenous variables.
In section 2 we examine the dynamic properties of models of this type and show that the above procedure is only valid if the model is stable. Section 3 returns to this particular example and the stability of the model is investigated.

2. The Dynamic Properties of a System of Difference Equations

Consider a system of g linear difference equations in the g endogenous variables \( y_{1t}, y_{2t}, ..., y_{gt} \) and r exogenous variables \( x_{1t}, x_{2t}, ..., x_{rt} \). For simplicity it is assumed that no endogenous variable appears with higher order lag than one.\(^{(2)}\)

We will define vectors of the endogenous and exogenous variables by

\[
\begin{align*}
    y_t &= (y_{1t}, y_{2t}, ..., y_{gt})', \\
    x_t &= (x_{1t}, x_{2t}, ..., x_{rt})'.
\end{align*}
\]

respectively. Then, any system of linear difference equations can be written in the matrix form

\[
A_0 y_t = A_1 y_{t-1} + B_0 x_t, \quad (2.1)
\]

where \( A_0 \) and \( A_1 \) are \( g \times g \) matrices and \( B \) is \( g \times r \).

In order to facilitate later manipulations, we introduce here the lag operator \( L^i \), defined by the relation

\[
L^i z_t = z_{t-i}, \quad (2.2)
\]

where \( z_t \) is any variable. We will also use the notation

\[
I z_t \equiv L^0 z_t = z_t, \quad L z_t \equiv L^1 z_t = z_{t-1}.
\]

2. The theory that follows can easily be extended to cases where there are lags of higher orders, but all the essential dynamic features of such systems can be seen by considering the simple case.
It can be shown that the lag operator $L$ may be treated in exactly the same way as an ordinary algebraic symbol. (See, for example Dhrymes (1970), p.509-517).

An alternative form for equation (2.1) would then be

$$\begin{align*}
[I - AL]y_t &= Bx_t, \\
\end{align*}$$

where $A = A_0^{-1} A_1$ and $B = A_0^{-1} B_0$.

The solution of (2.1) and (2.3) is readily obtained as

$$y_t = A^t y_0 + (I + AL + A^2L^2 + \ldots + A^{t-1}L^{t-1})Bx_t. \quad (2.4)$$

If we now regard $L$ as an ordinary algebraic symbol, the term in brackets above is simply the sum of a finite geometrical progression with common ratio $AL$, and (2.4) reduces to

$$y_t = A^t[y_0 - (I - AL)^{-1}Bx_0] + (I - AL)^{-1}Bx_t. \quad (2.5)$$

The time-path of the endogenous vector $y_t$ is thus seen to have two components:

(a) The term $(I - AL)^{-1}Bx_t$, which depends only on the time-path of the exogenous vector, and the way the lagged endogenous variables enter the system (characterised by the matrix $A$). This component of $y_t$ is often called the "equilibrium" or "steady-state" time-path.

(b) The term $A^t[y_0 - (I - AL)^{-1}Bx_0]$, which gives the movement of $y_t$ about the equilibrium time-path, and is often called the "disequilibrium component". The relative importance of this component is determined by the factor $A^t$. If $A^t \rightarrow 0$ as $t \rightarrow \infty$, then no matter how the system is disturbed it will always eventually return to the equilibrium path. Furthermore, if $A^t \rightarrow 0$
rapidly, then following any exogenous shock to the system, it will quickly adjust back to the equilibrium. Such a system is said to be stable. On the other hand, if \( A^t \to \infty \) as \( t \to \infty \), then no matter how small a shock the system may receive, the system will diverge from the equilibrium time-path, and never return to it. In this case the system is unstable.

Clearly, if the model is to be used to predict levels of the endogenous variables based on projected time-paths for the exogenous variables, the question of the stability of the model is of crucial importance. If the system is unstable, any departure of the exogenous vector from its projected path will result in the disequilibrium component progressively dominating the endogenous vector, until the exogenous variables become effectively irrelevant in the determination of the paths of the endogenous variables.

The Stability of the System

The system (2.1) is stable if and only if \( A^t \to 0 \) as \( t \to \infty \).

It is a well-known result from matrix algebra that for any square matrix \( A \) there exists a non-singular matrix \( P \) such that
\[
A = PDP^{-1},
\]
where
\[
D = \text{diag} \left( \lambda_1, \lambda_2, \ldots, \lambda_g \right), \quad \text{and the } \lambda_i \text{ are the eigenvalues of the matrix } A.
\]
Thus,
\[
A^t = (PDP^{-1})^t = PDP^{-1},
\]
where
\[
D^t = \text{diag} \left( \lambda_1^t, \lambda_2^t, \ldots, \lambda_g^t \right).
\]

It is clear then that a necessary and sufficient condition for the system to be stable is that \( \lambda_i^t \to 0 \) as \( t \to \infty \) for all \( i = 1, 2, \ldots, g \). That is, \( |\lambda_i| < 1 \) for all \( i \). If only one eigenvalue exceeds unity in modulus, the system will be unstable.
The Autoregressive Form

The system in the form (2.1) or (2.3) expresses each endogenous variable \( y_{it} \) in terms of lagged values of itself and other endogenous variables, together with exogenous variables.

If we set

\[
C(L) = I - AL,
\]

the elements of \( C(L) \) are polynomials of degree one in the lag operator \( L \).

Now the matrix \( C(L) \) may be written as

\[
C(L) = \det(C(L))[C^*(L)]^{-1},
\]

where \( \det(C(L)) \) and \( C^*(L) \) are the determinant and adjoint respectively of \( C(L) \).

Thus, multiplying both sides of (2.3) by \( C^*(L) \), the system can be expressed in the 'autoregressive form'

\[
\det(C(L)) y_t = C^*(L) B x_t.
\]

As \( \det(C(L)) \) is a scalar, and not a matrix, (2.7) differs from (2.3) in that each endogenous variable \( y_{it} \) is given in terms of lagged values of itself alone, together with exogenous variables. Furthermore, since the same lag operator \( \det(C(L)) \) acts on each \( y_{it} \), the way in which lagged values of \( y_{it} \) affect the current value of \( y_{it} \) is the same for all \( i \).

It has been shown that the dynamic properties of the system are governed by the eigenvalues of the matrix \( A \). By definition, these eigenvalues are the solutions of the determinental equation

\[
\det(A - \lambda I) = 0,
\]

where here \( I \) is the unit matrix. Now

\[
\det(A - \lambda I) = (-\lambda)^g \det(I - A\lambda^{-1}) = (-\lambda)^g \det(C(\lambda^{-1})�
\]

and so the eigenvalues are just the roots of the 'characteristic function' \( f(\lambda) \),
defined by

$$f(\lambda) = |C(\lambda^{-1})|.$$  \hspace{1cm} (2.8)

As each endogenous variable is characterised by the same operator $|C(L)|$ in the autoregressive form, it follows that each endogenous variable will have the same characteristic roots, and hence, the same dynamic properties.

**Recursive Systems**

We have seen that in general all the endogenous variables have the same dynamic properties. There is, however, a special case for which this is not so.

Let us suppose that the endogenous vector $y_t$ is partitioned as

$$y_t = [y_t^{(1)'}, y_t^{(2)'}, ...]'$$  \hspace{1cm} (2.9)

and that, with respect to this partitioning, the matrix $C(L)$ takes the form

$$C(L) = \begin{bmatrix}
C_{11}(L) & 0 \\
C_{21}(L) & C_{22}(L)
\end{bmatrix}.$$  \hspace{1cm} (2.10)

Such a system is recursive in the sense that while the variables $y_t^{(1)}$ do not depend on past or present values of $y_t^{(2)}$, the variables $y_t^{(2)}$ do depend on past and/or current values of $y_t^{(1)}$.

Using well-known results for partitioned matrices on (2.10), the autoregressive form (2.7) can be shown to be

$$|C_{11}(L)| y_t^{(1)} = \left( C_{11}^*(L) B_1 x_t \right)$$  \hspace{1cm} (2.11)

$$|C_{11}(L)| |C_{22}(L)| y_t^{(2)} = \left( -C_{22}^*(L) C_{21}(L) C_{11}^*(L) B_1 + |C_{11}(L)| C_{22}^*(L) B_2 \right) x_t$$  \hspace{1cm} (2.12)
where

\[ B = (B_1', B_2'). \]

Thus, when \( C(L) \) has the submatrix \( C_{12}(L) = 0 \), the subset \( y_t^{(1)} \) has its dynamic properties determined by the roots of \(|C_{11}(L)|\), while \( y_t^{(2)} \) has its properties determined by \(|C_{11}(L)| \cdot |C_{22}(L)|\). These two subsets will have different dynamic properties. It may happen, for example, that the endogenous variables \( y_t^{(1)} \) are stable, while \( y_t^{(2)} \) are unstable.

In a recent article McElroy (1978) has shown how the recursive nature of a system may be discovered, that is, how the partitioning (2.9) may be carried out in order that \( C_{12}(L) = 0 \).

3. Stability of the six-equation model

We return now to the simple model outlined in section 1. The procedure for obtaining time-paths for the endogenous vector of manpower requirements was seen (from equation (1.11)) to be exactly equivalent to solving the system

\[ y_t = (I - AL)^{-1} B x_t, \]

when \( x_t \equiv v_t = v_0 (1.5)^t \), and so from (2.5) is equivalent to a solution which ignores the disequilibrium component \( A^t [y_0 - (I - AL)^{-1} B x_t] \). Such a procedure will only be reasonable if at least one of the following conditions hold:

(a) The exogenous variable \( v_t \) never departs from the projected growth path. If this were to happen the disequilibrium component would always be zero. These authors believe such an assumption is totally unrealistic, and that the best that could happen is that the actual growth path consists of small random fluctuations around the projected growth path.
(b) The matrix $A^t$ is so small that the disequilibrium component is relatively insignificant. As we have seen, this depends on the stability or otherwise of the system. The authors have been unable to find a single case where the stability of a model has been investigated.

We will now investigate the question of stability in the context of the above example.

If we choose

$$y_t^{(1)} = [N^2_t, n^2_t]'$$

$$y_t^{(2)} = [N^3_t, n^3_t, n^2_t]'$$

the system can be written in the recursive form, where

$$C_{21}(L) = \begin{bmatrix} I & 0 \\ -(I - \mu L) & -I \end{bmatrix}$$

and $C_{21}(L)$ is a $4 \times 2$ matrix, zero everywhere except for $I$ in the $(1,2)$ position.

Now $|C_{11}(L)| = I$, and so the characteristic equation associated with the set $y_t^{(1)}$ does not involve $\lambda$. The disequilibrium component for this set is identically zero.

It is easily shown that

$$|C_{11}(L)| |C_{22}(L)| = \pi^2 + (\pi^3 - \pi^2 \mu^3) - (1 + \mu^3 \pi^2)L^2$$
and so the characteristic equation for the set $y_{t}^{(2)}$ is
\[ \lambda^2 + \left( \frac{\pi^3 - \pi^2 \mu^3}{\pi^2} \right) \lambda - \left( \frac{1 + \mu^3 \pi^3}{\pi^2} \right) = 0. \]

After substituting the assigned values for the coefficients and solving for the characteristic roots, we obtain
\[ \lambda_1 = 5.238 \quad \text{and} \quad \lambda_2 = -6.736. \]

The set $y_{t}^{(2)}$ is thus highly unstable. This means, for example, that if there is a small exogenous shock to the system, say $v_t$, deviates slightly from the constant growth path in one period, the demand for tertiary educated man-power ($N^3$) will leave its constant growth path and never return. This conclusion is so completely unacceptable that the validity of the whole model must be very dubious.

The implications of instability are two-fold:

(a) Projection from the model of future manpower needs becomes impossible. The very nature of random shocks to a system is that they are unpredictable. In an unstable system, the time-paths of the endogenous variables depend on the magnitude and timing of the shocks.

(b) As pointed out above instability is unrealistic and therefore points to some fundamental model mis-specification. The characteristics of the matrix $A$ must be incorrect. It is however this matrix which is used in obtaining the projections, which can therefore have little value.
Those who have used this methodology for projecting manpower requirements have used much more refined models than the simple six-equation model we have investigated here. However, the basic criticism remains: no attempt appears to have been made to establish whether the models are stable. In the check the authors have made of some of the larger models used in the Mediterranean project, and also a recent application to Sri Lanka by Deen (1977) (by no means an exhaustive check), they have been found without exception to be highly unstable.

4. Conclusion

Models of the Tinbergen-Correa-Bos variety have been widely used in manpower planning exercises. These models relate the endogenously determined manpower requirements to the exogenous movements in the economy through a series of linear difference equations.

An important part of the modelling process is the validation of the model before it is put to use, and in this context, an examination of the dynamic properties should form part of this validation. Where instability is discovered, a critical re-evaluation of the relationships expressed in the model is required. Structural instability virtually guarantees that a model cannot project well.

These authors are unable to find evidence that this model validation has been carried out in the field of manpower planning, and in the cases in which the dynamic properties have been examined, instability has invariably been the result.

In view of the above, it is not surprising that OECD (1965) experiments in projecting requirements of educated manpower (the Mediterranean project) have been in each of the six cases off the mark.

We conclude by quoting Dhrymes [(1970), p.542]: "...it is not enough to estimate the parameters of a model, casually inspect them, and
References


