INVESTMENT AND TRADE SANCTIONS AGAINST SOUTH AFRICA
IN A MODEL OF APARTHEID

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by JONATHAN BALDRY AND BRIAN DOLLERY

This paper formulates a stylised dualistic model of the South African economy using both a rigid wage and a competitive wage mechanism for unskilled (black) wage determination. The model is used to conceptually evaluate the allocative and distributional consequences of investment and trade sanctions against South Africa. The potential impact of investment sanctions is more clearcut than its trade counterpart.

Despite recent political developments in South Africa, scholarly interest in South African political economy remains strong, especially in the unsettled debate surrounding economic sanctions. Attention focused on the question of economic sanctions is bound to continue for at least two reasons. Firstly, various forms of economic sanctions have become increasingly evident in the postwar era as an instrument of international diplomacy (Gary Hufbauer and Jeffrey Schott, 1985), and consequently the South African episode will continue to be examined as an important case study. And secondly, both political activists and scholars alike will persist in debating the role sanctions may have played in resolving the South African crisis.

The economic analysis of economic sanctions against South Africa has been problematic for a number of reasons. At the conceptual level, the subtle and complex nature of institutional bias in the South African economy arising from apartheid regulation has proved difficult to model in a theoretically satisfactory manner without the addition of arbitrary ad hoc assumptions. These problems have been further compounded by institutional change in South Africa, particularly during the Botha administration from 1978 onwards with its reformist policies. Moreover, the form and content of economic embargoes and restrictions adopted by initiating nations have tended to evolve through time, exacerbating difficulties in accurate model building. The early specific measures, like the arms and oil boycotts characteristic of the 1960s and 1970s, have now been supplemented by more blanket instruments represented by financial, investment and trade sanctions. Similarly, the empirical analysis of economic sanctions against South Africa has been problematic, not least as a result of the lack of an acceptable theoretical framework. Furthermore, artificially-engineered data paucities have aggravated problems facing researchers. Quite apart from the suppression of disaggregative trade and investment
statistics by the South African authorities, nations, firms and individuals engaging in economic relationships with South Africa tend to disguise or understate the extent of their transactions. In addition, counterfactual analysis is rendered even more difficult by an historical pattern of relatively large and unpredictable movements in the South African business cycle.

The purpose of the present paper is to provide a stylised dual-economy model of South Africa in order to conceptually examine the allocative and distributive consequences of investment and trade sanctions. It appears to us that the distributional impact of sanctions is especially important in examining their impact. Sanctions are usually intended to alter, in some way, the behaviour of the government of the target country; in the case of sanctions against South Africa, to pressure the Republic's government to abandon all racially-discriminating policies and institutions. However, there is probably only a weak direct link between sanctions and government policy, a more important link being provided by the effects of sanctions on individuals and groups in the domestic community. In particular, economic sanctions are likely to have a differential impact on the real incomes of particular groups, and those groups which suffer the greater costs would be expected to exert pressure on the government (within or without the existing political structure) for a change of policy.

In the South African case we can conceive of two main possibilities along these lines. First, if sanctions harm whites and either benefit or do not significantly affect blacks, it could be expected that whites would pressure the government, through the ballot box, to alter its policies. Alternatively, if sanctions harm blacks and not whites, pressure is likely to come from the black population, in the form of riots, strikes, and so on. Hence the impact of economic sanctions on the relative incomes of blacks and whites is one of the most important factors in assessing its likelihood of success. For example, if peaceful change is the objective, the most desirable types of sanctions are those which harm whites and not blacks. We also note that different types of sanctions could have countervailing effects. For example, trade sanctions might harm whites and investment sanctions benefit them, and if this is the case, packages of economic sanctions need to be designed with care.

To examine these issues, we expand on the earlier pathbreaking efforts of Richard Porter (1978) and Mats Lundahl (1982) in modelling a southern African type economy. We extend their work to include inter alia a three-sector modern economy, two distinct assumptions about black (unskilled) wage determination, and an incorporation of recent reforms of apartheid in South Africa.

The paper itself is divided into four main sections. Section I briefly reviews the massive and growing literature on South African political economy generally, and the question of economic sanctions specifically, and seeks to place the present paper within the context of this literature. Section II outlines the differences and similarities between our model and those of Porter (1978) and Lundahl (1982), sets out the formal model and its two (competitive and Harris-Todaro) variants, and details various observations on the model. The effects of economic sanctions are examined in Section III, which outlines the
method of analysis and general results, and then investigates investment and trade sanctions separately. (Detailed derivation of the results is contained in the Appendix I to this paper.). Special emphasis is placed on the distributional effects of investment and trade sanctions for the various interest groups in South Africa, since this issue is obviously important for the potential success or otherwise of economic sanctions. Section IV summarizes the results and offers some concluding observations.

I. The Economics of Apartheid

The causes, mechanisms and consequences of racial discrimination in South Africa have long attracted the attention of scholars from a wide range of disciplines (Rodney Davenport, 1977; Jacqueline Kalley, 1987). In the postwar era emphasis has fallen on the relationship between capitalism, as it is manifested historically in the South African milieu, and apartheid. The ensuing debate has been characterised by the development and refinement of two broad schools of thought. In crude terms, an orthodox or "liberal" perspective attempted to provide a coherent account of South African political economy by construing the "irrational" racist policies of apartheid as dysfunctional to the rational forces of South African capitalism (Jeffrey Butler, Richard Elphick and David Welsh, 1987). A competing "revisionist" or marxian body of opinion tried to explain the historical evolution of events on the premise that the institutions of apartheid facilitated and enhanced the expropriative power of South African capitalism (Martin Murray, 1988).

In an effort to investigate the efficiency and equity repercussions of apartheid legislation in post-1948 South Africa, economists operating in the liberal tradition have attempted to formally model the complexities of modern South African political economy. Work in this area has evolved in two broad directions. First, some writers have examined the allocative and distributional consequences of the purported contradictions between apartheid and economic efficiency on the assumption that the panoply of apartheid laws are exogenously determined by political considerations (Stephen Enke, 1962; J.B. Knight, 1964; Richard Porter, 1978; Mats Lundahl and N.B. Ndlela, 1980; Mats Lundahl, 1982; Ronald Findlay and Mats Lundahl, 1987; Brian Dollery, 1989). And secondly, other economists have treated the plethora of apartheid regulation as endogenously determined through the complex interplay of various interest groups in South Africa, and tried to explain the pattern of apartheid legislation accordingly (J.B. Knight and Michael McGrath, 1977; Anton Lowenberg, 1984; 1989; Merle Lipton, 1985; Mats Lundahl, 1989; Brian Dollery, 1990). A similar dual line of investigation has emerged in the emotive debate over the question of trade and investment sanctions against South Africa. One school has focused largely on the effectiveness of economic sanctions in terms of the consequent induced economic damage inflicted on the South African economy, and examined the allocative and distributional results of various kinds and intensities of economic embargoes.

The present paper falls in the first of the two broad approaches to South African political economy generally, and trade and investment sanctions specifically. In particular, the formal model employed falls within the same general equilibrium genre as Porter (1978) and Lundahl (1982), although it is rather different in several significant ways.

II. Structure of the Model

A. Preliminary Comments

The model of an apartheid-type economy described in this section is closely related to the earlier Porter (1978) and Lundahl (1982) models listed above. In common with these contributions, the model developed here adopts a dual-economy approach: the economic system as a whole is divided into a distinct traditional or "homelands" economy and an advanced or "modern" economy. This type of dualistic characterisation of the South African economy is both descriptively accurate (Desmond Hobart Houghton, 1964) and a deliberate outcome of the institutions of apartheid in the post-1948 era (David Yudelman, 1984).

The stylised homelands economy acts as a reservoir of unskilled labour and produces only an agricultural commodity. However, the specification of the modern economy differs significantly from Porter (1978) and Lundahl (1982). Whereas both Porter and Lundahl aggregated mining and manufacturing in a single "industry" sector, the present model identifies three production sectors in the modern economy, namely agriculture, manufacturing, and mining. The rationale underlying this disaggregation is based on observed differences in production technologies, the pattern of trade, and a desire to distinguish between important classes of factor suppliers. Thus, consistent with empirical evidence, mining is classified as a capital-intensive industry employing relatively little skilled as compared with unskilled labour, and producing an intermediate product which is exported (Frank Biggs, 1982; Merle Lipton, 1985; Peet Strydom, 1987). Similarly, manufacturing produces importables, and utilises capital and both skilled and unskilled labour in significant proportions (Frank Biggs, 1982; Colin McCarthy, 1988; Peet
Strydom, 1987). And finally, agriculture employs an abundance of land in conjunction with relatively low proportions of capital and skilled labour, but large quantities of unskilled labour, and produces an exportable commodity (Frank Biggs, 1982; Peet Strydom, 1987; T.I. Fenyes, Johan van Zyl and Nick Fink, 1988).

If we were to follow the Porter/Lundahl two-sector classification of the modern economy, then several identification problems would arise in an analysis of the effects of economic sanctions. In the first place, if agriculture is deemed to produce an exportable commodity, then the other (industry) sector must be producing an importable commodity, but with mining included in the industry sector, this is quite inconsistent with the observed statistical importance of mining exports to the South African economy (Strydom, 1987). Moreover, since mining is a capital-intensive exporting industry (Biggs, 1982), trade sanctions which lower its export price would reduce its profitability. However, if mining is included as part of the importing "industry" sector, then trade sanctions would raise the price of its output and hence increase its profitability. In short, aggregating mining and manufacturing into a single sector has implications which are quite inconsistent with the empirical evidence.

An alternative two-sector aggregation involving the summation of agriculture and mining into a single "primary" sector poses rather different problems. In particular, doubtful results arise consequent upon strong differences in factor intensities between agriculture and mining, which are masked by aggregation. For example, an investigation into the impact of investment sanctions should intuitively demonstrate powerful effects on capital-intensive mining, and minimal effects on labour- and land-intensive agriculture. But any aggregation would obscure these important differences.

For these and other similar reasons, it is desirable to utilise an alternative three sector level of aggregation for the modern economy. Increased disaggregation imposes obvious costs both in terms of analytical complexity and in terms of the strength of derived results, but these costs can be reduced to some extent by imposing some special conditions on the structure of the model. Even in their two-sector, modern economy framework, Porter and Lundahl employed restrictions of this kind. Lundahl, for instance, assumed intersectoral capital immobility, while Porter adopted a zero elasticity of substitution between the two types of labour in his "industry" sector. The Porter restriction was justified by reference to the job reservation ratios which applied in South Africa at the time, but the Lundahl restriction was not related to any empirical or other evidence. Our additional restrictive conditions are based on stylised facts broadly in accord with empirical evidence on the South African economy (Jill Nattrass, 1981).

We assume that the only purely general factor is unskilled labour, while all land is allocated to agriculture, all skilled labour to manufacturing, and capital to both manufacturing and mining. However, each factor is taken to be perfectly mobile between the sectors in which it is utilised, and on the assumption that all market participants are price-takers, factor rewards are consequently equalised between sectors in the modern
Our assumptions about the productivity of labour closely follow those adopted by Lundahl (1982). We assume that all unskilled labour is black labour, and all skilled labour is white. In Lundahl's model, some skilled labour is black (and all unskilled labour is black), but his assumption about the interracial composition of skilled labour can easily be incorporated into our model if we assume that all skilled blacks are located in the modern economy. Finally, skilled and unskilled (or white and black) labour are assumed to be imperfect substitutes in the manufacturing sector.

The justification for these assumptions relates to the racially discriminatory institutions of apartheid. In common with Lundahl, but unlike Porter, apartheid in the labour market is not evidenced by racial job reservation ratios, but rather emerges in the form of a concentration of black workers in the unskilled category, and a preponderance of white workers in the skilled category. This presumed correlation between skill and race is explained as a result of "the process by which white labour becomes skilled and black labour remains unskilled" (Porter, 1978, p.744, fn.). Discrimination thus arises from the limited access to educational and training facilities enjoyed by blacks, rather than from statutory colour bars and other discriminatory legislation which have now been repealed in South Africa (Lundahl, 1989).

Across the dualistic divide, the traditional or homelands economy is treated in much the way as in Lundahl (1982). Using only land and unskilled labour, this sector produces an exclusively agricultural commodity identical to that produced by the agricultural sector in the modern economy. Moreover, all land in the homelands or national states is black-owned (while all other land is white-owned) and its product is freely tradeable in the modern economy, and hence competes equally with agriculture in the modern economy. In common with all sectors in the modern economy, the homelands production function exhibits constant returns to scale. However, the treatment of homeland labour differs significantly from both Porter (1978) and Lundahl (1982). The Porter/Lundahl models place restrictions on the migration of unskilled labour from the homelands, and justify this on the basis of the notorious pass laws which restricted the influx of (unskilled) black workers into the modern economy (Lipton, 1985). In contrast, we assume no restrictions on the migration of homeland labour. This accords with the recent relaxation of statutory migration controls (including the pass laws), and the abolition of urban residential restrictions (especially section 10 rights) in South Africa (Lundahl, 1989).

Two alternative approaches to modelling the process of free labour migration from the homelands to the modern economy are used in this analysis. The first analyses migration in frictionless, strictly competitive terms: unskilled wages in both homelands and modern economies are perfectly flexible, and there are no costs to migration, hence the black (unskilled) wage will be equalised between the homelands and the modern economy and there is zero unemployment, as in a standard dual economy model. The second assumes that the black (unskilled) wage is exogenously determined (by union or government action). Migration takes place whenever the expected unskilled wage in the modern economy (discounted for the probability of being unemployed there) is different from the
homelands wage. In equilibrium, the expected black (unskilled) wage in the modern economy is equated with homeland wages, as in the basic Harris-Todaro model (John Harris and Michael Todaro, 1970). These two extreme assumptions about black (unskilled) wage determination define two basic variants of the general model developed here. The implications of the two contrasting assumptions about unskilled wage determination in the modern economy are examined in some detail since, while the Harris/Todaro model may be criticised over the rigidity of its assumptions, inter alia the exogeneity of the urban unskilled wage, it nevertheless explains significant observed phenomena within the South African milieu, especially the persistence of significant urban unemployment (Trevor Bell, 1985), which are difficult to account for within a perfectly flexible competitive wage system.

The assumed exogenous determination of the black (unskilled) wage is embodied in variant I of the model which in all other respects is perfectly competitive. In this it contrasts sharply with Porter (1978) and, to a lesser extent, with Lundahl (1982). For instance, Porter invokes a wide range of discriminatory labour legislation to substantiate some inflexibility in the black (unskilled) labour market, and assumes that both white and black wages in the modern economy are set exogenously. Similarly, Lundahl defends the rigidities in black wage determination in his model on the basis of apartheid regulation of the horizontal mobility of black labour. In our model, exogenous determination of black wages (in the modern economy) is assumed in variant I, but there are no other rigidities in the system.

B. The Formal Model

The model as outlined above is formally specified using standard duality notation and results. [See, for example, Allan Woodland (1982, pp. 42-43), Anthony Atkinson and Joseph Stiglitz (1980, pp. 165-68)]. Noting that constant returns to scale are assumed for all production functions, we define $c^i(\omega^i)$ as the unit cost of production in sector $i$, where $\omega^i$ is the vector of prices of those factors used in sector $i$. Functions $c^i(\omega^i)$ are continuous, increasing, and concave (Hal Varian, 1984, pp. 44-46). Derivatives of unit-cost functions are defined as $c^i_j(\omega^i) = \partial c^i / \partial \omega_j$, where $\omega_j$ is the price of factor $j$. Using Shephard's Lemma, demand for factor $j$ in sector $i$ is given by $c^i_j x_i$, where $x_i$ is output in sector $i$.

Using this notation, we first define equilibrium conditions in the modern economy's factor markets as:

1. $c^2 x_2 + c^3 x_3 = K$
2. $c^3 x_3 = E$
3. $c^1 x_1 = A_E$
4. $c^1 x_1 + c^2 x_2 + c^3 x_3 = B_M$
where factor supplies are $K$ (capital), $E$ (white labour), $A_E$ (white-owned land) and $B_M$ (black labour). Factor prices are $r$ (capital rental), $w$ (white wage), $s_E$ (rent on white-owned land), and $v$ (black wage). Total factor supplies $K$, $E$ and $A_E$ are exogenously given, but the supply of black labour to the modern economy ($B_M$) depends on the black wage ($v$), as evidenced below.

To these factor-market equilibrium conditions, we add the output market (cost = price, or zero-profit conditions):

\begin{align}
  (5) & \quad c_1^1 (v, s_E) = 1 \\
  (6) & \quad c_2^2 (v, r) = p_2 \\
  (7) & \quad c_3^3 (w, v, r) = p_3
\end{align}

Variables $p_2$ and $p_3$ are prices of mining and manufacturing commodities respectively, set by world conditions (all commodities are tradeable). The agricultural commodity is used as the numeraire. We note that the full specification of the unit cost functions as given in (5) - (7) defines the factors of production utilized in each sector.

If $B_M$ were given, then the system (1) - (7) would constitute a complete competitive equilibrium system for the modern economy, to be solved for the three output levels and the four factor prices. Alternatively, if we were to fix the black wage ($v$), as in the Harris-Todaro (H-T) variant I of our model, then we can regard $B_M$ as variable, and still solve the system. Hence in variant I, the equilibrium of the modern economy can be examined independently of the equilibrium of the homelands. However, in the (competitive) variant II, a solution requires the simultaneous consideration of both the homelands and the modern economy.

Equilibrium in the homelands economy is specified by

\begin{align}
  (8) & \quad c_4^H x_H = A_H \\
  (9) & \quad c_5^H x_H = B_H \\
  (10) & \quad c^H (q, s_H) = 1
\end{align}

where $A_H$ is the supply of land, and $B_H$ the supply of black labour in the homelands. Variable $q$ is the (black) wage in the homelands and $s_H$ is the land rental. Clearly (8) and (9) are the factor market equilibrium conditions, and (10) is the zero-profit condition for equilibrium (where we note that the price of the commodity produced is the price of the identical commodity produced by modern agricultural sector 1). If $B_H$ (along with $A_H$) were fixed, (8) - (10) could be solved for output ($x_H$) and factor prices ($q$ and $s_H$). However, in (H-T) variant I, $B_H$ depends on $B_M$ and on equilibrium in the migration process (considered below), while in (competitive) variant II, $B_H$ is related to $B_M$ by the zero unemployment condition. Hence in both variants, equilibrium in the homelands is dependent on the nature of equilibrium in the modern economy.

The final section of the model consists of the Harris-Todaro equations:

\begin{align}
  (11) & \quad v (1 - u) = q \\
  (12) & \quad u = U \left( U + B_M \right) \\
  (13) & \quad L = U + B_M + B_H
\end{align}
where $U$ is the number of unemployed blacks (in the modern economy), $u$ is the rate of unemployment, and $L$ is aggregate black labour supply. Hence (12) defines the rate of unemployment, and (13) is a factor exhaustion definition, while equation (11) is the equilibrium migration condition. This requires the expected modern-economy black wage, $v(1-u)$, to equal the homelands wage ($q$). Note that $(1 - u)$ is the probability of securing employment in the modern sector if (as is implicitly assumed in models of the H-T type) employment placement is a completely random process, and contracts are of infinitesimally short duration.

C. Observations on the Model

As set out in equations (1) - (13), this model of an apartheid-style economy is complex by the normal standards applied in this type of analysis, and it might be expected that little in the way of useful results can be derived. However, the assumption that all factors except for black labour are more-or-less specialized in their productive role, simplifies the analysis to some extent. Further, the special assumptions used for the two variants also lead to some simplification. As noted, in variant I the black wage $v$ is fixed, so the modern economy described by (1) - (7) can be considered independently of the rest of the model: the model thus becomes one with recursive features. In variant II, this is not the case, but with competitive determination of the black wage, $v = q$, so one variable is dropped from (8) - (10), and (11) - (13) become redundant (since $u = 0$ in this case). Nevertheless, even though these special assumptions help simplify each of the variants of the general model, the structures are still fairly complex, especially those of variant II.

The analytical process involved in assessing the impact of sanctions formally requires the total differentiation of the equation system of the relevant variant with respect to all endogenous variables, and with respect to the exogenous variables which are directly affected by sanctions. In order to do this, we define sanctions in the following way.

Investment sanctions are characterized as a reduction in the capital stock at any given capital rental. To justify this, we first suppose the (domestic and overseas) supply of capital services to the modern economy is less than perfectly elastic: domestic and overseas investors have preferences about where to invest, perhaps partly based on cultural ties, or on confidence in the political and economic futures of the alternatives, and hence will not be indifferent between investing in different countries for the same return (as would be required for perfectly elastic supply). Investment sanctions imposed by overseas countries specify penalties for their residents who invest in the apartheid economy. The risks associated with the existence of these penalties and attempted enforcement of the sanctions mean that a higher gross rental is required as an incentive to invest, or if the gross rental is fixed, a lower supply will be forthcoming.

Trade sanctions enforced by foreign countries are assumed to imply penalties for those
overseas residents exporting to or importing from the apartheid economy. Accordingly, overseas importers will only buy at a discount, in order to offset the penalty risk, and overseas exporters will demand a higher price to cover the risk. Thus the terms of trade move against the apartheid economy: the price received for exports falls, and the price paid for imports increases. Using our notation, \( p_2 \) falls and \( p_3 \) increases, while the price of agricultural exports (used as the numeraire) remains constant.

The process of deriving comparative static results from either variant of the model is inevitably complex, though this is somewhat easier in variant I because of the possibilities for partitioning the model, noted above. Nevertheless, strong and unconditional results relating to the effects of investment sanctions can be derived for variant I, and these can be utilized for variant II. Somewhat weaker, but still useful and interesting results can be derived for the trade sanctions case using variant I, but unfortunately very little can be said about the effects of trade sanctions in variant II.

III. The Effects of Sanctions

A. Analytical Procedure

The flavour of the following analysis may be given using a diagram (Figure 1) adapted from a paper by Max Corden and Ronald Findlay (1975). In this diagram, the modern economy's demand for black labour, \( D_M \), is measured rightwards from 0, and the homelands demand \( (D_H) \) leftwards from \( L \), where \( 0L \) measures total black labour supply. The left-hand vertical axis measures the black wage in the modern economy \( (v) \), while the right-hand axis measures the homelands wage. Demand for black labour in the modern sector is \( D_M (\bar{K}, \bar{p}) \) defined by the given capital stock \( \bar{K} \) and given commodity price vector \( \bar{p} \), while homelands demand is \( D_H \). From the specification of the model, \( D_H \) is inversely related to \( q \), and \( D_M \) is inversely related to \( v \).7

Now suppose that in variant I, the fixed modern-economy wage is measured by \( 0\bar{v} \). Employment in the modern economy will then be \( 0B_{M}^1 \). Drawing the rectangular hyperbola \( ab \) through \( d = (B_{M}^1, \bar{v}) \), the intersection of \( ab \) with \( D_H \) defines equilibrium in \( H \): labour usage in \( H \) is measured by \( B_{H}^1 L \), and the wage by \( Lq_1 \). This is because, from the definition of \( u \) in (12), \( 1 - u = B_M(U + B_M) \), hence using (11), \( vB_M = q (U + B_M) \). But since from (13), \( U + B_M = L - B_H \), then \( vB_M = q (L - B_H) \). In the equilibrium illustrated in Figure 1, area \( 0\bar{v}dB_{M}^1 = 0ceB_{H}^1 \) (since \( ab \) is a rectangular hyperbola), so this condition is satisfied at \( (B_{M}^1, \bar{v}) \) and \( (B_{H}^1, q_1) \), noting that \( U = L - B_{H}^1 - B_{M}^1 \) is measured by distance \( B_{M}^1B_{H}^1 \).
The main problem in the analysis of variant I is to determine how $D_M$ is affected by a change in $\bar{K}$ (investment sanctions) or a change in $\bar{p}$ (trade sanctions). Given that the $(v, D_M)$ relationship is a single reduced-form equation derived from (1) - (7), if sanctions shift $D_M$ to the left, then (as can be verified from the diagram) $B_M$ will fall, $B_H$ will increase, $U$ will increase and $q$ will fall. The implications for black incomes are easily derived, and as will be seen, the implications for white incomes can be derived from manipulation of (5) - (8). The modifications required to these results if $D_M$ shifts to the right due to sanctions are then readily perceived.

In (competitive) variant II, equilibrium requires equality between $v$ and $q$, and zero unemployment. In Figure 1, this equilibrium is illustrated at $(B^*, v^*)$, where equilibrium $v$ is measured by $Ov^*$, $B_M$ by $OB^*$ and $B_H$ by $B^*L$. Again, if $D_M$ shifts leftwards due to sanctions, the black wage will fall, and it is easy to establish that total black income will fall. The effects on the various categories of white incomes are once more readily assessed from (5)-(8).

To summarize, what is crucial for the analysis is whether the modern economy's demand for unskilled labour increases or decreases (at a given wage) when sanctions of either type are imposed. A decrease in demand will reduce the "traditional" wage $q$ in either variant, and decrease the "modern" wage $v$ in variant II, and these conclusions enable us to derive further results concerning the effect of sanctions on black incomes. The major problem is to establish whether (or under what conditions) unskilled labour demand will decrease with sanctions. Derivation of results for white incomes is, as we shall see,
fairly straightforward once this is accomplished.

To establish whether unskilled labour demand decreases or increases when sanctions are imposed, we differentiate the modern-economy sub-model while holding \( v \) constant. For variant I, the resulting expressions will enable us to deduce the effect on white incomes and (if unambiguous) allow us to then utilize the H-T equations (11) - (13) and the homelands equations (8) - (10) to derive the effects on black employment and homelands incomes. For variant II, these results provide the starting point for further analysis. That is, if (say) with \( v \) constant, sanctions reduce the demand for unskilled labour, we infer that \( v \) will decrease as a result; also, \( B_H \) will increase. Other effects are found by varying \( v \) accordingly, with \( K \) or \( p \) held constant (at their new values). These are second-round effects: they result from a movement along the new \( D_M \) curve, whereas the first-round effects are the result of the shift in the curve. The second-round effects are found first using (8) and (10) (with \( v = q \)) to find the effect of the decrease in \( v \) on \( s_H \). We then use (5) - (7) to infer the effect of \( dv < 0 \) on \( w, r \) and \( s_E \), and output effects can be derived using (1) - (3). These procedures used in relation to variant II, we emphasise, are merely a means of subdividing a long and complex model into easily-manageable portions.

B. General Results

As described above, derivation of comparative static results for variant I requires total differentiation of the complete model, with either \( dK < 0 \) (investment sanctions) or \( dp_2 < 0, dp_3 > 0 \) (trade sanctions), and with \( dv = 0 \). However, as noted, we may first concentrate just on the modern economy equations (1) - (7), and then substitute the relevant expressions into the rest of the model to derive a complete set of results. For variant II, formally we disregard (11) - (13) and set \( dv = dq \), but otherwise follow the same procedure. In practice though, we follow a two-stage process: first infer the effect of sanctions on \( v \), following the method outlined above, and then utilize the relevant sign of the change in \( v \) to infer the effects of sanctions on other variables of interest. In either case, it is convenient first to set out the structure of the complete differential system, from which the special cases required for the specific analyses can be derived. The differential results are all stated here in proportional form, and utilize some standard notation and standard duality results. (See, for example, Woodland (1982; pp. 42-43), Atkinson and Stiglitz (1980; pp. 167-70)). We define for example the proportional change in \( x_i \) as \( \dot{x}_i \equiv dx_i/x_i \); the share of factor \( j \) in income generated in sector \( i \) is \( \theta_i^j \equiv (\omega_j F_i^j)/(p_i x_i) \), etc., where \( F_i^j \) is the allocation of factor \( j \) to sector \( i \). The proportion of the total modern-economy employment of factor \( j \) used in sector \( i \) is denoted by \( \lambda_i^j \), for instance. \( \lambda_2^K \equiv K_2/K \). Note that \( \sum_{i=1}^{3} \lambda_i^j = 1 \) and \( \sum_{j=1}^{4} \theta_i^j = 1 \). Detailed derivation of the expressions given here is contained in Appendix I (and references to the appropriate equations in Appendix I are specified where helpful).
First, using (5) - (7), we derive the relationships between factor prices and commodity prices as:

\[ (14) \quad \hat{S}_E = - (\theta_1^B / \theta_1^F) \gamma \]

\[ (15) \quad \hat{r} = (\hat{G}_2 / \theta_2^K) - (\theta_2^B / \theta_2^F) \gamma \]

\[ (16) \quad \hat{w} = (\hat{G}_3 / \theta_3^F - [\theta_3^K / (\theta_3^F \theta_2^F)]) \hat{r}_2 + [(\theta_3^B \theta_2^B - \theta_3^F \theta_2^F) / (\theta_3^F \theta_2^F)] \gamma \]

(see A1 - A3).

Some clear results are immediately apparent here. Both the (white) land and capital rentals are \textit{ceteris paribus} inversely related to the black wage \( v \), which follows quite simply because in both sectors 1 and 2, black labour is an unambiguous substitute for one other input (land and capital respectively). The relationship between \( w \) and \( v \) is not so clear though. The underlying problem (which will recur in a number of contexts) is that white labour is substitutable for both black labour and capital (in sector 3). If, say, \( v \) were to increase (with commodity prices constant), \( r \) would fall; the fall in \( r \) will decrease demand for white labour (hence decrease \( w \)), while the rise in \( v \) will increase demand (hence increase \( w \)). The net effect depends on the relative income shares of the various factors, as seen in the expression \( \theta_3^K \theta_2^B - \theta_3^B \theta_2^K \). Clearly, if we are considering the effects of trade sanctions \( \hat{G}_3 > 0, \hat{G}_2 < 0 \), the overall effect on \( r \) and \( w \) is more difficult to unravel, because the change in the terms of trade will tend to increase \( w \) and decrease \( r \) (by respectively decreasing and increasing the two factors' marginal revenue products) as a direct effect, but there is also an indirect effect operating via \( v \). However, if we hold \( v \) constant, the influence of commodity prices on factor prices is quite apparent, and is loosely consistent with the Stolper-Samuelson relationships: sector 2 is (by definition) capital intensive relative to 1, so \( r \) increases as \( p_2 \) (the relative price of 2) increases; sector 3 is (again by definition) relatively white-labour intensive, so \( w \) increases as \( p_3 \) increases.

In terms of the overall analytical procedure to be followed, differential equations (14) - (16) will be useful in two ways. First, they can be used to reduce the size of the rest of the differential system by eliminating from it other factor prices. Secondly, when the effect of sanctions on \( v \) is inferred in variant II, these equations may be used to determine the impact of this change in \( v \) on white incomes.

Turning now to the other modern-economy equations (1) - (4), with the aim of evaluating the effects of sanctions on black labour demand (in the modern economy) at a given black wage, we first set \( \hat{v} = 0 \). Then noting from (14) that this implies \( \hat{S}_E = 0 \), hence \( \hat{x}_3 = 0 \) (see A6), the differential of (3) disappears, and the differential of (1)-(4) may be summarized as:

\[ (17) \quad \hat{x}_3 = -R_E \hat{r} + W_E \hat{w} \]

\[ (18) \quad \lambda_2^K \hat{r}_2 + \lambda_3^K \hat{x}_3 = R_E \hat{r} - W_K \hat{w} + \hat{K} \]

\[ (19) \quad \lambda_2^B \hat{r}_2 + \lambda_3^B \hat{x}_3 - \hat{E}_M = -R_B \hat{r} - W_B \hat{w} \]

where:
and $\sigma_{jm}^3$ is the (Allen-Uzawa) elasticity of substitution between factors $j$ and $m$ in sector 3, while $\sigma_i^j (i = 1, 2)$ is the elasticity of substitution between the only two factors used in sector $i$. We note that all terms $R_j$ and $W_j$ are positive.

Interpreting (17) - (19), if we set $v = \text{constant and change } K (\tilde{K} < 0 \text{ for investment sanctions})$ then since all other factor prices are constant [from (14) - (16)], output changes for sectors 2 and 3 can be calculated from (17) and (18), and the resulting change in black labour demand from (19). This then feeds into the homelands sector and the H-T equations. For trade sanctions (with $v$ constant), $\tilde{p}_2 < 0$ and $\tilde{p}_3 > 0$, so other factor prices change [in (14)-(16)]; these are then fed into (17)-(19) as before (but with $\tilde{K} = 0$).

Differentiating the homelands sector equations (8) - (10), we have:

\begin{align*}
\frac{\partial \tilde{h}}{\partial \tilde{q}} + (\theta_{hi}^B / \theta_{hi}^A) \sigma_{hi}^q xh &= 0 \\
\frac{\partial \tilde{h}}{\partial \sigma_{hi}^q} &= \tilde{b}_h \\
\theta_{hi}^B q + \theta_{hi}^A \tilde{s}_h &= 0
\end{align*}

These equations cannot be solved separately from the rest of the model for variant I. In this case, the H-T equations are needed to provide the link between $\tilde{B}_H$ and $\tilde{B}_M$ (as solved from the modern-economy equations). However, in variant II, $\tilde{q} = \hat{\nu} = 0$, so (20) - (22) can be solved independently.

From the H-T equations (8) - (10), only relevant for variant I, substitution of (8) and (9) into (10) gives:

\begin{equation}
L = (v / q) B_M + B_H
\end{equation}

and differentiation of this gives:

\begin{equation}
b_M (\tilde{B}_M - \hat{q}) + h \tilde{B}_H = 0
\end{equation}

where $b_M = (v / \tilde{B}_M) / (qL)$ is the share of black labour income generated in the modern economy, and $h = B_M / L$ is the proportion of black labour employed in the modern economy. This equation (24) provides the link between the modern economy and homelands sector equations.

Using the relevant restrictions, solutions can be obtained to (14) - (24), if $v$ is constant. If $v$ is not constant, solution is still in principle possible, but in practice the expressions which result are of little use in deriving strong results. These can be derived...
only if we impose additional restrictions, but it is convenient and simpler to follow the analytical procedure outlined, rather than to add algebraic restrictions to the complete model.

C. Distributional Indicators

For reasons outlined earlier, the economic impact and effects of sanctions should be evaluated largely in terms of their distributional implications, especially in terms of the relative impact on white and black incomes. In this model, a number of distributional indicators are obviously apparent in the solution to the differential equation system, but others of interest can be specified.

In the white population, we may identify three income classes - (capital) rentiers with aggregate income $M^K$, white landlords (income $M^A$) and white (skilled) workers (income $M^E$). Since $M^A = s_E A_E$ and $M^E = w E$, the effects of sanctions on the incomes of white workers and landlords is immediately seen from the signs of $w$ and $s_E$ respectively. For the rentier class, we assume that any reduction in the capital stock due to sanctions originates from overseas-owned capital, so the effect on domestic rentiers is simply indicated by the sign of $r$.

Overall domestic white income is defined as:

$$M = w E + s_E A_E + r (1 - f) K$$

where $f$ is the share of foreign ownership in the domestic capital stock. Hence:

$$\hat{M} = e \hat{w} + a \hat{s}_E + k \hat{r}$$

where $e$, $a$ and $k$ are the relevant (domestic) factor shares in white income.

In respect of black incomes, use of (23) gives total black labour income as $N^B = q L$, while income of black landlords is $N^A = S_H A_H$. Hence effects on these classes are evaluated by the signs of $s_H$ and $q$. Total black income is $\hat{N} = N^A + N^B$, so:

$$\hat{N} = g \hat{q} + (1 - g) \hat{s}_H$$

where $g$ is labour's share in black income.

D. Investment Sanctions

With $\nu$ fixed, the impact of investment sanctions on demand for labour in the modern economy is found by solving (14) - (19), with $\hat{\nu} = \hat{p}_2 = \hat{p}_3 = 0$ and $\hat{K} < 0$.

From (14) - (16), it is seen that $\hat{w} = \hat{r} = \hat{s}_E = 0$ in this case, and also, from (17), $\hat{x}_3 = 0$. Substituting these values into (18) and (19), we derive:

$$\hat{x}_2 = \hat{K}/\lambda_{2}^K < 0$$

$$\hat{b}_M = (\lambda_{2}^B/\lambda_{2}^K) \hat{K} < 0.$$
Using Figure 1, we can immediately conclude that for the H-T model (variant I) \( q \) will fall and \( B_H \) increase (since demand curve \( D_M \) shifts leftwards with a reduction in the capital stock). Formally, from (20) - (22), we derive
\[
\hat{q} = -\left(\frac{\partial M}{\partial M} / \sigma^H\right) \hat{B}_H \quad \text{and using (24)}:
\]
\[
\hat{q} = \left[ (b_M \theta^H)/(h \sigma^H + b_M \theta^H) \right] \hat{B}_M < 0
\]
Other results follow simply; in particular
\[
\hat{s}_H = -\left(\frac{\partial B}{\partial B} \hat{B} \right) \hat{q}
\]
while it can be easily verified that \( U \) and \( u \) increase.

The overall distributional impact of investment sanctions in variant I is that white incomes (\( M^A, M^K, M^E \), and hence \( M \)) are totally unaffected; black labour income (\( N^B \)) decreases and black landlords' income (\( N^A \)) increases. Less obviously, total black income decreases, since
\[
\hat{N} = \left( g - \frac{\theta^B}{\theta^H} \right) \hat{q} < 0
\]
where the share of labour in black income (\( g \)) is necessarily greater than labour's share in the homelands, given that \( B_M > 0 \).

Turning to (competitive) variant II, we utilize the procedures outlined earlier to conclude that investment sanctions will reduce \( v \). Hence we set \( \hat{v} < 0 \) in equations from (14) onwards to derive results.

Using (22) (and noting that \( \hat{v} = \hat{q} \) in variant II) we have
\[
\hat{s}_H = -\left(\frac{\partial B}{\partial B} \hat{B} \right) \hat{v} > 0
\]
As before, it is easy to see that while black landlords gain (\( \hat{N}^A > 0 \)) in this case, black labour, and blacks in general, again suffer income losses (\( \hat{N}^B, \hat{N} < 0 \)).

For white income earners, (14) and (15) show that both landlords and capital owners gain (\( \hat{g}_E, \hat{r} > 0 \)) while the effect on white workers is uncertain. As is apparent from (16), the sign of \( \hat{w} \) depends on the sign of the term \( \theta_3^K \theta_2^B - \theta_3^H \theta_2^B \), about which nothing useful can be said. For example, “sensible restrictions” might be \( \theta_3^K < \theta_2^H \) and \( \theta_2^B > \theta_3^B \)
but this is not of much help.

Strong output implications can be derived to some extent. The fall in \( v \) increases labour employment in both agricultural sectors (1 and \( H \)) hence, with a given usage of land, increases output levels \( x_1 \) and \( x_H \). However, for mining and manufacturing (sectors 2 and 3) the outcome is not clear. For sector 3 (manufacturing) we use (17) and the expressions for \( \hat{w} \) and \( \hat{r} \) to derive:
\[
\hat{x}_3 = \left[ 1/(\theta_3^E \theta_2^K) \right] \left[ \theta_3^E \theta_2^B R_E + (\theta_3^K \theta_2^B - \theta_3^H \theta_2^K) W_E \right] \hat{v}
\]
whose sign again depends on the sign of the term \( \theta_3^K \theta_2^B - \theta_3^H \theta_2^K \). Further, it can be shown (see Appendix I) that substitution of the definitions for \( R_E \) and \( W_E \) does not help. Similar considerations apply to the sign of \( \hat{x}_2 \); we cannot specify the sign of this term \( a \) priori.
For white landlords and rentiers then, investment sanctions increase incomes in variant II, but for white workers, the effects are uncertain. This uncertainty then translates into uncertainty about the effect on aggregate white income.

For the intuition behind these results, consider first variant I (with $v$ constant). A reduction in the capital stock leads to excess demand for capital (by sectors 2 and 3) at the given rental. If the rental were to increase, unit costs in mining (2) would increase and become greater than the (given) commodity price. Hence the mining sector would cease production. The result would be a negative excess demand for capital (since sector 3 would also be reducing production and changing factor proportions as its costs rise), and downward pressure on $r$. Only if $r$ returns to its original level would both sectors continue to produce. Both sectors 2 and 3 must hence face unchanged factor prices, and use unchanged factor proportions. However, with full employment of white workers (required with flexibility of $w$) this necessitates an unchanged output in manufacturing, hence the burden of the sanctions falls totally on the mining sector. It is the contraction of this which leads to the fall in black incomes.

With $v$ variable (variant II), the downward pressure on the black wage which would follow the above sequence, leads to a reduction in mining costs, hence a desire to increase output, and an increased capital demand (hence $\hat{r} > 0$). However, in sector 3 (manufacturing) the increase in $r$ and decrease in $v$ leads to input substitution between all three relevant factors, and the signs of these depend on the relevant elasticities of substitution; hence, for example, we cannot determine whether demand for capital in sector 3 will also increase (adding to the increased demand by sector 2, so leading to the overall effect $\hat{r} > 0$), or whether it will decrease (so the overall effect on $r$ will be uncertain). It is clearly impossible to determine the overall effect on demand for white labour (hence an $w$) from all this, even if we could specify the direction of change in the capital rental.

In general then, investment sanctions will benefit capital owners and white landlords if the black wage is flexible and competitively-determined, while the effect on white workers is uncertain. If the black wage is fixed, (domestic) white incomes will be unaffected. However, whether the black wage is fixed or flexible, black workers and all blacks will, on average, be harmed.

E. Trade Sanctions

For the analysis of trade sanctions, we proceed as above, but keeping $\hat{K} = 0$, while $\hat{p}_2 < 0$ and $\hat{p}_3 > 0$.

For variant I, we hold $\hat{v} = 0$. From (14) - (16), we see immediately that $\hat{s}_F = 0$, $\hat{r} < 0$ and $\hat{w} > 0$. With $v$ fixed, the intuition behind these results is quite clear: importing sector 3, the only user of white labour, becomes more profitable, while exporting sector 2 becomes less profitable with trade sanctions. Accordingly, demand for white labour
increases, so \( w \) increases relative to \( v \), while demand for capital relative to black labour falls, so \( r \) falls.

From (17), we then conclude that \( \hat{x}_3 > 0 \) and, less obviously, that \( \hat{x}_2 < 0 \) (while \( \hat{x}_1 = 0 \)). Again, this is quite expected: the more profitable sector expands.

The effect on black labour demand however, is more complex. A glance at (19) shows that knowledge of the signs of \( \hat{w} \), \( \hat{r} \), \( \hat{x}_2 \) and \( \hat{x}_3 \) is, by itself, insufficient to derive any conclusions. As shown in Appendix I, the effect on the modern economy's demand for black labour again depends both on elasticities of substitution and on measures of factor intensities in sectors 2 and 3. If, for example, we assume zero elasticity of substitution between black labour and other factors in sectors 2 and 3 (\( \sigma^2 = \sigma^3_{BE} = \sigma^3_{BK} = 0 \)), then the effect on demand for black labour will clearly depend solely on the input-output coefficients for black labour, and on the allocation of black labour between sectors 2 and 3. However, if we allow substitution between black labour and other factors, then the fall in \( r \) will lead to substitution of capital for black labour in sector 2, with an increasing demand for black labour as a consequence, but in sector 3, the changes in both \( w \) and \( r \) affect the black labour:output ratio in a way which cannot be determined \textit{a priori}, and even if it could, it would not solve the problem.\(^8\)

For these reasons we are unable to determine the overall effect on \( B_M \), and hence on \( B_H \) and the homelands wage (\( q \)). Accordingly, the aggregate effect on black incomes and unemployment cannot be determined.

As far as white incomes are concerned, the increase in \( w \) and fall in \( r \) clearly increase \( M_E \) and reduce \( M_K \), but the aggregate effect (on \( M \)) is indeterminate. Substituting from (15) and (16) into (26), we have:

\[
\hat{M} = (e/\theta^E_3) \hat{p}_3 + [(k \theta^E_3 - e\theta^K_3)/(\theta^E_2 \theta^K_2)] \hat{p}_2
\]

The sign of the bracketed expression \((k \theta^E_3 - e\theta^K_3)\) appears uncertain, but by substituting the relevant definitions, it can be written

\[
\left\{[wr(1-f)EK]/M\right\}[(\lambda^E_3 - \lambda^K_3]
\]

and since \( \lambda^E_3 = 1, \lambda^K_3 < 1 \), it is positive. Hence, while \( \hat{p}_3 > 0 \) increases white incomes, \( \hat{p}_2 < 0 \) decreases them, and the overall effect of trade sanctions is uncertain.

These conclusions reached for the effect of trade sanctions on the H-T variant (I) of the model are clearly much weaker than the conclusions reached in respect of investment sanctions. It is not surprising that this pattern carries over to variant II which, as was evident when analysing the impact of investment sanctions, yields fewer strong results than variant I. Indeed, virtually nothing of interest can be said about the effects of trade sanctions in variant II. Being unable to assess whether or not trade sanctions reduce black labour demand in the modern economy, we cannot say how the black wage will be affected, nor how other factor prices are affected by the changing black wage. Moreover, nothing can be said about the effects on the output mix.

None of this is really surprising. An unconstrained, competitive, multi-factor and
multi-commodity model can, if sufficient factor specialization or immobility assumptions are made, lead to results of the Rybczynski type regarding the effects of factor supply changes (eg. investment sanctions), but it is always more difficult to derive results for commodity price changes (eg. trade sanctions). Certainly, if detailed knowledge of the values of key parameters is available, more can be done, but at the level of generality considered here, the effects of trade sanctions on income distribution in a fully competitive economy of the type described here must remain a matter for conjecture. About all we can be certain of is that sanctions would lead to increased output of the manufactured importable, and decreased output of the (mining) exportable \((\hat{x}_2 < 0, \hat{x}_3 > 0)\), with consequent changes in the pattern of trade. However, the effect on output of agricultural commodity 1 - which might be an importable or exportable - is uncertain; if the unskilled wage increases \(x_1\) will fall (because costs rise) and vice versa. It follows that the effect on output of the homelands is uncertain.

IV Concluding Observations

The results of this analysis are summarized in Table 1 below, where the relative difficulty of assessing the effects of trade sanctions, as compared with the effects of investment sanctions, is quite apparent. Also evident is the relative difficulty of assessing the effects of either type of sanction in an unconstrained, competitive model (variant II) as compared with a more constrained, rigid wage model (variant I).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variant I Investment Sanctions</th>
<th>Variant I Trade Sanctions</th>
<th>Variant II Investment Sanctions</th>
<th>Variant II Trade Sanctions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MK)</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>(ME)</td>
<td>0</td>
<td>+</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>(MA)</td>
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<td>0</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>(M)</td>
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<td>?</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>(NB)</td>
<td>-</td>
<td>?</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>(NA)</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>(N)</td>
<td>-</td>
<td>?</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>(U)</td>
<td>+</td>
<td>?</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>(u)</td>
<td>+</td>
<td>?</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>(x_H)</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>(x_1)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>(x_2)</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>(x_3)</td>
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<td>+</td>
<td>?</td>
<td>+</td>
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</tbody>
</table>
What is clear from this table is that investment sanctions will reduce black incomes, whether or not the black labour market is flexible, and this seems a plausible result (Keith Ovenden and Tony Cole, 1989). The reduction in the capital stock has reduced the output possibilities of the modern economy, and in particular of mining and manufacturing. Demand for black labour in mining will fall, and insofar as mining is a major user of black labour, we might expect aggregate demand for black labour to fall. In any event, if the aim of sanctions is to make blacks attack the (apartheid) status quo, the analysis suggests that investment sanctions will always be effective in this aim. However, if the aim is to harm whites, the analysis suggests that the aim is unlikely to be achieved via investment sanctions. These may harm white workers (if the black wage is flexible), but will benefit white capital rentiers and landlords under flexible wages and have zero impact on any white factor prices if the black wage is rigid. In short, some whites may be harmed by investment sanctions, while others may benefit or be unaffected.

By contrast, a perusal of Table 1 indicates that the qualitative impact of trade sanctions is much less clearcut. Consequently, whether or not a particular distributional aim will be achieved by means of trade sanctions depends on the values of a large number of key parameters relating to the production process. Without prior knowledge of the values of these key parameters, trade sanctions should be regarded as a high-risk method of policy intervention in South Africa.

However, in order to provide at least some idea of the qualitative impact of trade sanctions, and of the quantitative effects of both investment and trade sanctions, values of key parameters involved in the determination of income shares and factor allocations in South Africa were estimated from various sources and, together with plausible values of substitution elasticities, were employed to simulate the effects of economic sanctions on incomes and employment. (Details of parameter values and sources of estimates are given in Appendix II). Trade sanctions were assumed to increase the price of importables by 10 per cent and decrease the price of exportables by 10 per cent, whereas investment sanctions decreased the capital stock by 10 per cent. These assumed figures correspond to other efforts at simulating the impact of investment and trade sanctions against South Africa (Charles Becker and Haidi Ali Khan, 1990). The results of the simulation exercise are reported in Table 2 below.

Whilst obviously a high degree of accuracy cannot be ascribed to these results, they nevertheless do suggest that both investment and trade sanctions are likely to benefit whites and harm blacks whatever the extent of wage flexibility, though the less flexible the black urban wage the more this harm will be manifest in unemployment increases rather than wage reductions. Given the speculative nature of the parametric estimates used to derive these results, the magnitudes contained in Table 2 can be regarded as nothing more than illustrative. However, the outcomes of investment sanctions for both variants of the model do seem quite
plausible. To proceed any further would necessitate better parametric estimates, and more
precise estimates of the impact of investment sanctions on the capital stock, and the effects of
trade sanctions on the terms of trade.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Investment Sanctions</th>
<th>Trade Sanctions</th>
<th>Investment Sanctions</th>
<th>Trade Sanctions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{\nu})</td>
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<td>-0.09</td>
<td>-1.16</td>
</tr>
<tr>
<td>(\hat{q})</td>
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<td>-0.19</td>
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<td>-1.16</td>
</tr>
<tr>
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<td>0.76</td>
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<td>4.6</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.06</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.11</td>
<td>1.68</td>
</tr>
<tr>
<td>(\hat{s}_E)</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0.24</td>
</tr>
</tbody>
</table>
Appendix I

This appendix gives the general form of the differential equations derived from the complete model (1) - (13), and the specific versions relating to the two model variants, hence deriving the expressions and justifying the arguments given in the text.

The techniques used are those found in, for example, Woodland (1982) and Atkinson and Stiglitz (1980), to which the reader is referred for detailed derivation of specific operations.

In the differentiation of the general model, derivatives (all expressed in proportional form for convenience) are taken in respect of all endogenous variables, and for the exogenous variables $K$ (whose value is reduced for investment sanctions), $p_2$ and $p_3$ (which are reduced and increased respectively for trade sanctions).

First differentiating the factor market conditions for the modern economy, (1)-(4), we have:

\[ A_1 \lambda_2 \hat{x}_2 + \lambda_3 \hat{x}_3 + \lambda_2 \theta_2^B \sigma^2 (\hat{v} - \hat{r}) + \lambda_3 \theta_3^B \sigma_{\hat{E}K} (\hat{\nu} - \hat{\sigma}) = \hat{R} \]
\[ A_2 \hat{x}_3 + \theta_3^E \sigma_{\hat{E}K} (\hat{r} - \hat{w}) + \theta_3 \sigma_{\hat{E}K} (\hat{v} - \hat{w}) = 0 \]
\[ A_3 \hat{x}_1 + \theta_1^B \sigma^1 (\hat{\nu} - \hat{s}_E) = 0 \]
\[ A_4 \lambda_1 \hat{x}_1 + \lambda_2 \hat{x}_2 + \lambda_3 \hat{x}_3 + \lambda_1 \theta_1^A \sigma^1 (s_E - \hat{\nu}) + \lambda_2 \theta_2^K \sigma^2 (\hat{r} - \hat{\nu}) + \lambda_3 \theta_3 \sigma_{\hat{E}K} (\hat{\nu} - \hat{\sigma}) = \hat{B}_M \]

where symbols have the meanings defined in the text.

Differentiating the modern economy's zero-profit conditions (5)-(7) gives:

\[ A_5 \theta_1^B \hat{r} + \theta_1^A \hat{s}_E = 0 \]
\[ A_6 \theta_3^B \hat{r} + \theta_3^K \hat{r} = \hat{p}_2 \]
\[ A_7 \theta_3^B \hat{w} + \theta_3^K \hat{r} + \theta_3^E \hat{w} = \hat{p}_3 \]

Solving A5-A7 for changes in factor prices $r, w,$ and $s_E$ in terms of $v$ and commodity prices gives:

\[ A_8 \hat{r} = - (\theta_2^B / \theta_2^K) \hat{\nu} + (1/\theta_2^K) \hat{p}_2 \]
\[ A_9 \hat{\nu} = (\theta_3^E / \theta_3^K) - [(\theta_3^K / (\theta_3^E \theta_2^K)) \hat{p}_2 + [(\theta_3^K \theta_2^B - \theta_3^B \theta_2^K)/(\theta_3^E \theta_2^K))] \hat{\nu} \]
\[ A_{10} \hat{s}_E = - (\theta_1^B / \theta_1^A) \hat{\nu} \]

Substituting from A8-A10 into A1-A4 yields the reduced-form differential equation system for the modern economy:

\[ A_{11} \lambda_2 \hat{x}_2 + \lambda_3 \hat{x}_3 + (\hat{\nu} / \theta_2^K) \hat{C}_v = - \sigma_{\hat{E}K} \hat{p}_3 + (C_p / \theta_2^K) \hat{p}_2 + \hat{K} \]
\[ A_{12} \hat{x}_3 = D_p \hat{p}_3 - (\theta_3^K / \theta_2^K) \sigma_{\hat{E}K} \hat{p}_2 - (D_\nu / (\theta_2^K \theta_3^E)) \hat{\nu} \]
\[ A_{13} \hat{x}_1 = - \sigma^1 (\theta_1^B / \theta_1^A) \hat{\nu} \]
\[ A_{14} \lambda_1 \hat{x}_1 + \lambda_2 \hat{x}_2 + \lambda_3 \hat{x}_3 - E_v \hat{\nu} = \hat{B}_M - E_2 \hat{p}_2 - \lambda_3 \sigma_{\hat{E}K} \hat{p}_3 \]

where:
\[ C_v = \lambda_2^B \sigma^2 + \lambda_3^K \sigma^3_{E \bar{K}} (\theta_2^B \theta_3^E - \theta_2^B \theta_2^E) \]
\[ C_p = \lambda_2^K \theta_2^B \sigma^2 + \theta_3^E \sigma^3_{E \bar{K}} + \lambda_3^B \sigma^3_{E \bar{K}} \]
\[ D_p = \theta_3^K \sigma^3_{E \bar{K}} + \theta_3^3 \sigma^3_{E \bar{K}} \]
\[ D_v = \theta_2^K \theta_3^B (1 - \theta_3^K) \sigma^3_{E \bar{K}} - \theta_3^3 (\theta_2^B \theta_3^E - \theta_3^B \theta_2^E) \sigma^3_{E \bar{K}} \]
\[ E_v = \lambda_1^B \sigma^3 + \lambda_2^B \sigma^2 + \lambda_3^B [(1 - \theta_3^K) \sigma^3_{E \bar{K}} + (\theta_3^K / \theta_2^K) \sigma^3_{E \bar{K}}] \]
\[ E_2 = \lambda_2^B \sigma^2 + \lambda_3^K \sigma^3_{E \bar{K}} (\theta_3^K / \theta_2^K) \]

In the homelands economy, differentiation of the factor market conditions (8) and (9) gives:

A15 \[ \hat{x}_H + \theta_2^B \sigma^H (\hat{q} - \hat{s}_H) = 0 \]
A16 \[ \hat{x}_H - \theta_2^A \sigma^H (\hat{q} - \hat{s}_H) = \hat{B}_H \]

while from the zero-profit condition (10) we derive:

A17 \[ \theta_2^B \hat{q} + \theta_2^A \hat{s}_H = 0 \]

Writing \( \hat{q} - \hat{s}_H = \hat{q} / \theta_2^A \) from A17, and substituting into A15 and A16, we derive two reduced-form equations for the homelands economy:

A18 \[ \hat{x}_H + \sigma^H (\theta_2^B / \theta_2^A) \hat{q} = 0 \]
A19 \[ \hat{x}_H - \sigma^H \hat{q} = \hat{B}_H \]

Finally turning to the H-T equations (11) - (13), it is convenient first to derive, from (12):

\[ U = [u / (1 - u)]B_M \]

and from (11):

\[ 1 - u = q / \nu, \ u = (v - q) / \nu \]

Hence:

A20 \[ U = [(v - q) / q]B_M \]

Substituting this into (13) we derive:

A21 \[ L = (v / q)B_M + B_H \]
as the linkage between \( B_M \) and \( B_H \). If we now differentiate this reduced-form equation, we derive:

A22 \[ b_M \hat{v} - b_M \hat{q} + b_M \hat{B}_M + h \hat{B}_H = 0 \]

where \( b_M = vB_M / qL \) is the share of blacks employed in the modern economy in total black labour income, and \( h = B_H / L \) is the proportion of the black labour force employed in the homelands.

We are left with a reduced-form system of differential equations consisting of A11 - A14 (modern economy), A18 - A19 (homelands economy) and A22 (the H-T conditions). This is a total of seven equations in eight unknowns - the four (proportional) output changes \( (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_H) \), two (proportional) factor price changes \( (\hat{v}, \hat{q}) \), and two (proportional) changes in black employment \( (\hat{B}_H, \hat{B}_M) \), with exogenous variable
changes being either \( K < 0 \) (investment sanctions) or \( \hat{p}_2 < 0, \hat{p}_3 > 0 \) (trade sanctions).

Obviously, the complete system cannot be solved as it stands. However, the specific assumptions used in the two variants of the general model equate the number of equations to the number of unknowns. In variant I, \( \hat{\nu} = 0 \). In variant II, \( \hat{\nu} = \hat{q} \), and each of the variants can generally be solved. Having solved the particular variant, other factor price changes can be derived using A5 - A7 and A17. Further, in variant I, unemployment changes can be inferred by differentiation of A20 and A21:

\[
A23 \quad \hat{U} = (b_M / \hat{u}) \hat{\nu} - (b_M / \hat{u}) \hat{q} + \hat{B}_M
\]

where \( \hat{u} = U / L \) is the aggregate rate of unemployment among blacks (distinct from \( u \), which measures the urban black unemployment rate), and:

\[
A24 \quad \hat{u} = -[(1 - u) / u] \hat{\nu} + [(1 - u) / u] \hat{\nu}
\]

**Variant I Solution**

i. *Investment sanctions*

With \( \hat{\nu} = 0 \), \( \hat{K} < 0 \) and \( \hat{p}_2 = \hat{p}_3 = 0 \), then from A8-A10, we have

\[ \hat{r} = \hat{w} = \hat{s}_E = 0. \]

From A12 and A13, \( \hat{x}_1 = \hat{x}_3 = 0 \), from A11, \( \hat{x}_2 = \hat{K} / \lambda_2 < 0 \), and from A14,

\[ \hat{B}_M = (\lambda_2 / \lambda_2 \hat{K} < 0. \]

Solving A18 and A19, \( \hat{q} = - (\theta^A H / \sigma^H) \hat{B}_H \), substitute this into A22 (with \( \hat{\nu} = 0 \)) to derive \( \hat{B}_H = - (b_M \sigma^H) / (b_M \theta^A H + h \sigma^H) \hat{B}_M > 0. \)

Using A17-A19, we derive from the above, \( \hat{q} < 0, \hat{s}_H, \hat{x}_H > 0. \)

ii. *Trade Sanctions*

Setting \( \hat{\nu} = \hat{K} = 0, \hat{p}_2 < 0, \hat{p} > 0 \), from A8-A10 we have \( \hat{r} < 0, \hat{w} > 0, \hat{s}_E = 0. \) From A13, \( \hat{x}_1 = 0 \), and from A12, \( \hat{x}_3 = D_p \hat{p}_3 - (\theta^K / \theta^K) \sigma^K E \hat{p}_2 > 0. \)

Substituting this expression into A11, and solving, we have

\[ \hat{x}_2 = (1 / \lambda_2) [(\sigma^K E + \lambda_3 D_p) \hat{p}_3 + (\lambda_3 (\theta^K / \theta^K) \sigma^K E + (C_p / \theta^K)) \hat{p}_2] < 0. \]

Substituting these solutions for \( \hat{x}_1, \hat{x}_2 \) and \( \hat{x}_3 \) into A14, we derive:

\[ \hat{B}_M = (1 / \lambda_2) [(\lambda_3 \lambda_2 - \lambda_3 \lambda_2 D_p) + (\lambda_3 \lambda_2 \sigma^3 E - \lambda_3 \lambda_2 \sigma^3 E)] \hat{p}_2 \]

\[ + \lambda_2 [(C_p / (\lambda_3 \lambda_2)] + \sigma^2] \hat{p}_2 \]

The second part of this expression \((\lambda_2 [(C_p / (\lambda_2 \theta^2))] + \sigma^2] \hat{p}_2 \) is negative, but the sign of the first part is ambiguous. While we know that \( D_p, \hat{p}_3 \) and \((1 / \lambda_2)\) are positive, the signs of \( \lambda_2 \lambda_2 - \lambda_3 \lambda_2 \) and \( \lambda_3 \lambda_2 \sigma^3 E - \lambda_3 \lambda_2 \sigma^3 E \) are quite uncertain, and
even if an educated guess were made about the first, some strong assumptions would need to be made about the relative magnitudes of the two elasticities of substitution in the second if any progress is to be made. Hence in general, the sign of $\hat{B}_M$ is indeterminate, and hence we cannot deduce the signs of $\hat{B}_H$, $\hat{q}$, $\hat{u}$ and $\hat{U}$, etc.

**Variant II**

i. *Investment Sanctions*

From variant I we know that $\hat{B}_M < 0$ if $\hat{K} < 0$ and $\hat{v} = 0$. Hence, for the reasons explained in the text, we know that $\hat{v} < 0$. From A8-A10, we then conclude that $\hat{r}$, $\hat{w}$, $\hat{S}_E > 0$. Further, from A13 we derive $x_1 > 0$.

From A12, we derive $x_3 > 0$ as $D_v < 0$. However, $D_v = \theta_2^B \theta_3^E - \theta_2^B \theta_2^K$, and we cannot determine its sign *a priori*. Hence the sign of the change in $x_3$ is ambiguous.

Turning to the change in $x_2$, if all factor prices were constant, $\hat{x}_2 = \hat{K}/\lambda_2^K < 0$ (as in variant I). The second-round effect is the effect of the factor price changes. This effect is defined by setting $\hat{K} = 0$ in A11, with $\hat{v} < 0$ and A12 substituted for $\hat{x}_3$. The second-round effect is specified as:

$$\hat{x}_2 = (1/\lambda_2^K)(\lambda_2^K \lambda_2^E \theta_2^F (1 - \theta_2^K) \sigma_{BE}^3 - \lambda_2^K \theta_3^E \theta_2^B \sigma^2$$

- $\theta_3^E = \theta_3^E (\theta_3^E - \theta_2^B \theta_2^E)\hat{v}$

Quite obviously, nothing useful can be said about the sign of this expression relating to the second-round effect, so nothing of interest can be said about the sign of the total effect on $x_2$. Fortunately though, this does not matter insofar as our focus is on distributional matters. Using the methodology explained in the text, since we know that $B_M$ falls, $B_H$ increases (remembering there is no unemployment in this model). From A17 we infer (noting that $\hat{q} = \hat{v}$ in this variant) that $\hat{S}_H > 0$, and from A18 that $\hat{x}_H > 0$. Effects on black and white incomes, reported in the text, follow straightforwardly from these results.

ii. *Trade Sanctions*

Little need be said about this case. Because we cannot infer the effect of trade sanctions on unskilled labour demand in the modern economy when $v$ is constant (i.e. using variant I), we cannot infer the effect of trade sanctions on $v$. As is clear from A8-A10, this means we cannot infer the effect on white incomes and, self-evidently, if the sign of $\hat{B}_M$ is uncertain, we cannot determine the sign of $\hat{S}_H$. 
APPENDIX II

This Appendix describes the procedures employed in the estimation of parametric values for the simulation results reported in Table 2 in Section IV.

i. **Factor allocation**

Estimates are based on data provided by A. Roukens de Lange (1989), and more particularly his Table 1 "Adjusted 1978 SAM for SAMSIM model" (pp. 26-27). Our category "Black Labour" is taken to comprise all "Non-White labour" in the adjusted social accounting matrix for South Africa provided by de Lange (1989). Similarly, our agricultural sector is equated with "Agriculture" in de Lange's (1989) Table 1, our mining sector with his "Mining" and "Gold", while all other sectors in his table (excluding government) are aggregated to correspond with our manufacturing sector. Labour employment is reported as the number of workers, and capital stock by sector is approximated by value of capital. The resultant derived estimates are:

\[ \lambda_1^B = 0.21; \quad \lambda_2^B = 0.17; \quad \lambda_3^B = 0.63; \quad \lambda_2^K = 0.07; \quad \lambda_3^K = 0.93. \]

ii. **Factor shares in modern sectors**

Estimates of factor shares in the modern sector are also based on de Lange (1989) Table 1. Data on white and non-white wages and salaries by sector are divided by sectoral value added, while capital and land shares in the modern sector are taken as residuals. The resultant estimates are:

\[ \theta_1^B = 0.17; \quad \theta_2^B = 0.17; \quad \theta_3^B = 0.42; \quad \theta_3^E = 0.35; \quad \theta_1^A = 0.83; \quad \theta_2^K = 0.83; \quad \theta_3^K = 0.23. \]

iii. **Elasticities of substitution**

Estimates for manufacturing elasticities were taken as the highest reported by Fallon and Layard (1975):

\[ \sigma_{BE}^3 = \sigma_{BE}^3 = 1.66; \quad \sigma_{KS}^3 = 0.91. \]

The estimate for mining was similarly derived from the same source:

\[ \sigma^2 = 1.45. \]

A literature survey showed very little consistency in the estimates for agriculture, which ranged from approximately 0 to approximately 1. We assumed \( \sigma^A = \sigma^H = 1. \)

iv. **Other parameters**

No direct information was available on factor shares in the homelands or on homelands employed relative to black urban employment. The ratio of \( B_H/L \) was accordingly approximated by the ratio of the non-white homeland population to the total population of the non-white homeland.
non-white population, or 0.366 (Republic of South Africa, 1989, Table 10, p. 88).

In the absence of specific information, income shares were assumed to be the following: $\theta^A_H = 0.2; \theta^B_H = 0.8$.

Reliable black unemployment figures are not available for South Africa. In this regard an official South African publication comments on "the dearth of accurate information on total unemployment, especially jobless people not registered with the department", and notes that "the most recent estimates indicate an unemployment rate of ... more than 23 per cent of the total number of economically active people" (Republic of South Africa, 1989, p. 377). Accordingly we set $u = 0.23$.

From the estimates of $u$ and $B_H/L$, parameters $b_m$ and $h$ could be computed.
FOOTNOTES

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1. Any model larger (in terms of factors or sectors) than the "standard" 2x2 constant-returns model will inevitably lose in predictive power what it gains in "realism". The analyst must either accept this trade-off and present all results in conditional form, or incorporate additional restrictions (most commonly, factor immobilities or price rigidities) and retain predictive power. In pursuit of the latter aim, we attempt to find additional restrictions which are both useful and empirically justifiable.

2. Data on the South African labour force is broadly consistent with these assumptions. For example, in 1985, 85 per cent of labourers, and 66 per cent of "production workers", but only 7 per cent of "artisans" were black. The corresponding proportions for white were 0 per cent, 4 per cent, and 75 per cent respectively (with the balance made up by those defined as "Asian" or "Coloured") (Republic of South Africa, 1989, p. 4). Moreover, in 1978, 6 per cent of the white labour force was employed in agriculture, 5 per cent in mining, and 89 per cent in manufacturing. For blacks, the corresponding percentages were 22, 19, and 59 respectively (de Lange, 1989. Table 10, pp. 26-27).

3. The only problem is that assessing the distributional impact of sanctions will become more complex. By equating "white" with "skilled" and "black" with "unskilled", the problems of assessing the impact of sanctions on the inter-racial distribution of income are considerably reduced, because we simply need to assess their impact on factor prices. If some skilled workers are black, the distributional results derived in section 3 need to be qualified in an obvious way.

4. For simplicity we assume zero costs of migration. We also assume that homeland farms are organized on competitive capitalist lines, so wages are set equal to marginal products. If cooperative or other non-capitalist forms of organization are found in the homelands, the homelands sub-model would need to be modified accordingly, and the results would need to be adjusted. However, it is doubtful that any important modifications to the results would be required. Income distribution between black workers and black landowners is less important than the distribution between (all) blacks and whites, and quite clearly, the total of black income generated in the homelands will be positively related to black employment in the homelands, whatever the method of determining wages there. Moreover, if modern-economy wages rise (or
unemployment falls), migration from the homelands will result under any reasonable assumptions.

5. In saying this, we recognize that there are other approaches (e.g. search models) to the explanation of persistent unemployment.

6. The assumption of constant returns in mining is questionable when capital and labour are the only inputs. A more detailed model would include "ore bodies" as an additional input if the constant returns assumption is retained, with the rentals on these ore bodies as an additional type of income. However, for our present purposes it is convenient to regard "capital" as inclusive of these ore bodies, which are owned by the government, so that the capital rental (r) in our model includes mining rentals and royalties remitted to the government.

7. This relationship is not obvious. To derive it, we solve the reduced-form system of equations A11 - A14 (in Appendix I) for \( \hat{B}_M \), setting \( \hat{p}_2 = \hat{p}_3 = \hat{K} = 0 \). The solution derived is \( \hat{B}_M = -E_v \hat{\nu}, \) where \( E_v (>0) \) is as defined in Appendix I.

8. In this context, note that incorporation of the Fallon and Layard (1975) finding of a high degree of complementarity between skilled labour and capital (or \( \sigma_{3K}^3 > \sigma_{3K}^3 \) in our terms) does not help us here.
REFERENCES


Ovenden, Keith and Cole, Tony, Apartheid and International Finance: a Program for