2 Elementary Algebra

2.1 Introduction

An algebraic symbol, i.e., a letter of an alphabet, is nothing to worry about.

*A letter merely stands for a number*, as we have seen in Topic 1.

Elementary Algebra is thus just a generalization of Elementary Arithmetic.

We can use any letters (or symbols) we like. There is nothing special about using the letter $x$, though it is the most commonly-used letter.

*Question:* What is meant by, say, $3x$?

*Answer:* $3x = 3 \times x = x + x + x = 3$ lots of $x$

If $x = 5$, the *value* of $3x$ is $3 \times 5 = 15$

If $x = -2$, the *value* of $3x$ is $3 \times (-2) = -6$

Notice that $3x$ also means $x \times 3 = x3$ (since the order of multiplication doesn’t matter, as explained in Topic 1, Section 3). However, we usually write the number (called the *coefficient*) before the letter.

2.2 Addition and Subtraction

Examples:

\[
\begin{align*}
3x + 4x &= 7x &= 4x + 3x \\
4x - 3x &= 1x &= x \\
3x - 3x &= 0x &= 0 \\
3y + 4y &= 7y &= 4y + 3y \\
-5x + 7x - 6x &= -4x
\end{align*}
\]

Note that we can only add and subtract to make the expression simpler when the letters are the same, e.g, $3x + 4x = 7x$ and $2y + 3y = 5y$, but we can’t make, say, $3x + 4y$ any simpler algebraically since $x$ and $y$ are different.
Exercises 2.1:

(i) $5x - 3x$

(ii) $4x - 7x$

(iii) $3y - 5y$

(iv) $8t - 7t$

(v) $3x - 7x + x$

(vi) $3x - 7y + 2x$

(vii) $4 - 3x + 2x$

(viii) $3a + 5b - 3a$

(ix) $-4x + 3x - 8x + 9x$

(x) $3x - 5w + 4w - 2x - 7 + 3w + 5 - 2w$

2.3 Multiplication

Explanation

(i) $x^0 = 1$, $x^1 = x$, $x^2 = \underbrace{x \times x}$, $x^3 = \underbrace{x \times x \times x}$, $x^4 = \underbrace{x \times x \times x \times x}$

(ii) $3x^2$ means $3 \times x \times x$ \textbf{but} $(3x)^2 = (3x) \times (3x) = 3 \times 3 \times x \times x = 9x^2$

(iii) $5xy$ means $5 \times x \times y$ (= $5yx$ also)

(iv) $7x \times 4y = 7 \times x \times 4 \times y = 28xy$

(v) $2ab^2 = 2 \times a \times b \times b$ \textbf{but} $(2ab)^2 = 2ab \times 2ab = 2 \times a \times b \times 2 \times a \times b = 4a^2b^2$

(vi) $3x^0 = 3 \times x^0 = 3 \times 1 = 3$ \textbf{but} $(3x)^0 = 1$
Examples:

\[
\begin{align*}
2(3x) &= 6x = 3x + 3x \\
3(-2x) &= -6x \\
-2(-4x) &= 8x \\
-x(-x) &= (-x)^2 = x^2 \\
3x(-2x) &= -6x^2 \\
(-2y) \times 3x &= -6xy \\
3x + 5xy &= \\
4x - 3 &= \text{can't be made any simpler} \\
2ab + 3ab^2 &= \\
\end{align*}
\]

Exercises 2.2: Simplify (= make simpler, if possible)

(i) \(3 \times t \times z\)
(ii) \(a \times 4\)
(iii) \(x \times y \times x \times x\)
(iv) \(2x(-x)\)
(v) \(5xt - xt\)
(vi) \(-4(2x) + (-3x)3\)
(vii) \(3xy + 5 - 2yx\)
(viii) \((-3x)^2\)
(ix) \(4x^2y - 3xy - 7yx^2 + 4\)
(x) \(5x^0\)
(xi) Explain why \(2xy^2 \neq 2x^2y^2 \neq (2xy)^2\)

2.4 Brackets

With brackets proceed here as you did in Elementary Arithmetic, Topic 1, Section 5.
Examples:

\[ 3(x - 1) = 3 \times x - 3 \times 1 = 3x - 3 \]
\[ -(x - 2) = -x + 2 \quad \text{Note that the left hand side implies } -1 \times (x - 2) \]
\[ 3x - 4 - 2(x - 2) = 3x - 4 - 2x + 4 = 1x + 0 = x \]
\[ x^2 + x + 1 - (x^2 - x + 1) = x^2 + x + 1 - x^2 + x - 1 = 2x \]

**Exercises 2.3:**

(i) \( 2(x + 1) \)

(ii) \(- (3 - x^2) \)

(iii) \(-2(x - 3) \)

(iv) \( 4(x - 1) - 3(2x + 5) \)

(v) \( 3(2 - 5x) + 4(3x + 4) \)

(vi) \( 2(3x - 6) - 3(2x - 4) \)

2.5 Division

Examples:

\[ \frac{x}{x} = 1 \quad \text{if } x \neq 0 \]
\[ \frac{6x}{3x} = 2 \quad \frac{-2x}{x} = -2 \quad \frac{-5x}{-x} = 5 \]
\[ \frac{8y}{2} = 4y \quad \frac{4xy}{2yx} = 2 \quad \frac{6x^2}{x^2} = 6 \]

Negative indices

\[ x^{-1} = \frac{1}{x} \quad x^{-2} = \frac{1}{x^2} \quad (\text{See also Topic 1, Section 6}) \]
Examples:

\[(2x)^{-1} = \frac{1}{2x}\]
\[(2x)^{-2} = \frac{1}{(2x) \times (2x)} = \frac{1}{4x^2}\]
\[2x^{-2} = 2 \times x^{-2} = 2 \times \frac{1}{x \times x} = \frac{2}{x^2}\]
\[6x(3x)^{-1} = 6x \times \frac{1}{3x} = \frac{6x}{3x} = 2\]
\[(-3x)^{-1}(2x)^{-1} = \frac{1}{-3x} \times \frac{1}{2x} = \frac{1}{-6x^2} = -\frac{1}{6x^2}\]

**Exercises 2.4:**

(i) \(\frac{8x}{2x}\)

(ii) \(-\frac{15t}{-3t}\)

(iii) \(\frac{8y}{4}\)

(iv) \(10x(-5x)^{-1}\)

(v) \((-2x)^{-1}4x\)

**Topic 2 Revision Exercises 2.5:**

(i) \(-7x + 4x - 4x\)

(ii) \(2x^2 + 3x^2\)

(iii) \(3x + 6 - 4x\)

(iv) \(4x^2 - 2x(-3x) - 5x(2x)\)

(v) \(5(x - 2) - 4(x - 3)\)

(vi) \(-\frac{12}{4x}\)

(vii) \(-20x(-4x)^{-1}\)

(viii) \(-6y(2y)^{-1} + (-4y)^{-1}(-8y)\)
2.6 Answers to Exercises

2.1:

(i) $2x$  (iv) $t$  (vii) $4 - x$  (x) $x - 2$

(ii) $-3x$  (v) $-3x$  (viii) $5b$

(iii) $-2y$  (vi) $5x - 7y$  (ix) $0$

2.2:

(i) $3tz$  (iv) $-2x^2$  (vii) $5 + xy$  (x) $5$

(ii) $4a$  (v) $4xt$  (viii) $9x^2$

(iii) $x^3y$  (vi) $-x$  (ix) $-3x^2y - 3xy + 4$

(xi) We have $2 \times x \times y \times y \neq 2 \times x \times x \times y \times y \neq 4 \times x \times y \times x \times y$

2.3:

(i) $2x + 2$  (iii) $-2x + 6$  (v) $-3x + 22$

(ii) $-3 + x^2 (= x^2 - 3)$  (iv) $-2x - 19$  (vi) $0$

2.4:

(i) $-4$  (iii) $2y$  (v) $-2$

(ii) $5$  (iv) $-2$

2.5:

(i) $-6x$  (iii) $-x + 6$  (v) $x + 2$  (vii) $5$

(ii) $5x^2$  (iv) $0$  (vi) $-\frac{3}{x}$  (viii) $-1$