A brief introduction to Set Theory

Broadly speaking, a *Set* is any collection of objects. The *elements* of a set are the objects in a set and each object in the set is unique.
Usually we denote sets with upper-case letters, elements with lower-case letters. The following notation is used to show set membership.

$x \in A$ means that *x* is a member of the set A

$x \notin A$ means that x is not a member of the set A.

Ways of Describing Sets

List the elements (curly brackets are used for this)

e.g. *A = {0,1,2,3,4,5,6,7,8,9,10}*

Define the set using words

e.g. *A= {the numbers from zero to ten inclusive}*

Give some sort of mathematical rule.

e.g. *A = {x: 0≤x≤10}*

Some Special Sets

The **Empty Set** or **Null Set**.

This set has no elements and is usually symbolized by $\varnothing$

The Universal Set.

This is the set of all elements currently under consideration, and is often symbolized by **Ω**

Membership Relationships

Subsets.

$A \subseteq B$ is read as 'A is a subset of B'

We say that A is a subset of B if x, as an element in A, implies that x is also an element in B. In set notation this is written as $x \in A \Rightarrow x \in B$

Another way of saying this is that all the members of A are also members of B. This definition allows for the situation where A = B.

Proper Subsets.

$A \subset B$ is read as 'A is a proper subset of B'

A is a proper subset of B if all the members of A are also members of B, but there is at least one element in B that is not in A (A≠B).

Combining Sets

Set Union $A \cup B$

'A union B' is the set of all elements that are in A, or B, or both.
This is similar to the logical (Boolean) 'OR' operator.

Set Intersection $A \cap B$
'A intersect B' is the set of all elements that are in both A and B.
This is similar to the logical (Boolean) 'AND' operator

Set Complement $\overline{A}$
'A complement,' or 'not A' is the set of all elements not in A.
The complement operator is similar to the logical not, and is reflexive, that is,
$\overline{\overline{A}} = A$

Set Difference $A - B$
The set difference 'A minus B' is the set of elements that are in A, with those that are in B subtracted out. Therefore, $A - B$ is the set of elements that are in A and not in B. Another way of expressing the same result would be: $A - B$ = $A \cap \overline{B}$

Mutually Exclusive and Exhaustive Sets

Two sets A and B are **mutually exclusive** if $A \cap B = \varnothing$, that is, the sets have no elements in common.

A group of sets is **exhaustive** of another set if their union is equal to that set.
For example, if $A \cup B = C$ we say that A and B are exhaustive with respect to C.

Set Partition

A group of sets (or subsets) is said to **partition** another set if they are mutually exclusive and exhaustive with respect to that set.

An example could be
C={All the students in the class}
A={All the male students in the class}
B={All the female students in the class}

So, $A \cap B = \varnothing$ and $A \cup B = C$.