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Mental computation: More than knowing your tables

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Abstract

In our increasingly technological world, the need for quick and accurate computation is as great as ever. The nature of that computation has changed, however, with most of the routine calculations undertaken by machines. Paradoxically, this fact has increased the need for well-developed mental approaches to computation, and curriculum developments worldwide reflect a changed emphasis from written to mental methods. Recent Australian studies have provided insights into the ways in which mental computation competence can be developed in students by stressing a strategy approach to arithmetic.

Introduction

Adults undertake about 90 percent of their daily calculation mentally (Northcote & McIntosh, 1999). Most of these calculations involve estimation or the use of rule-of-thumb methods applicable in a specific situation. When accuracy is required, or when the computation becomes too large for efficient mental processing, we turn to technology such as the computer spreadsheet, cash registers or petrol pumps. In my childhood, I can remember the accounts clerk in the butcher's shop sitting in her enclosed cubicle calculating each customer's bill. Waiting for the bill was part of the ritual of shopping in a more leisured age. No longer, however, do we expect people to carry out long, repetitive calculations, such as adding a supermarket bill, by pen-and-paper methods (Zevenbergen, 2004). Increasingly, instead, we want people to have a feel for numbers, who can recognise when a computation carried out using some piece of technology is not correct. Developing this "number sense" (McIntosh, 1990) is a far more difficult task for teachers than training students to undertake repetitive calculations using a particular rule or algorithm.

These changes are reflected in the latest curriculum documents. In primary schools, there is an increased emphasis on the development of mental computation and a corresponding delay in the formalisation of written approaches. In high schools, mental computation with decimals, fractions and percents is implicitly expected (NSW Board of Studies, 2002).

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We know that the methods used to calculate mentally are different from those used in written computation, and that students use a variety of methods. Bana and Korbosky (1995), for example, identified 17 different valid ways in which students responded to the question 24 ÷ 6. Little is known, however, about the best ways to teach mental computation. We do know what not to do. Emphasis on daily competitive and timed 'mentals' leads to lower levels of achievement and higher levels of mathematics anxiety. Expecting children to learn their tables with little support other than the daily or weekly test is insufficient for the level of number sense we now require from our students (McIntosh, de Nardi, & Swan, 1994).

The Research

Three recent Australian studies offer some ways forward for teachers and schools. The largest of these involved several thousand students in three school systems in two states. One outcome of this study was a developmental scale of mental computation that provides a basis for curriculum development (Callingham & McIntosh, 2001). This scale came from the analysis of over 5500 mental computation tests completed by students across Years 3 to 10. The research indicated that tables facts, for example, were more difficult for students than anticipated, and that it was not until about Year 8 that students could be expected to have total command of these. On the other hand, many Year 3 students were more competent at computation than current curricula indicate, and Year 4 is a critical period of growth in computation skills (Callingham & McIntosh, 2002).

Recent analysis of errors produced by students in this study indicates that the most commonly occurring errors are those involving inefficient counting strategies such as counting forward and backward by 1 in addition and subtraction problems, or skip counting (e.g., 3, 6, 9, 12, ...) when doing multiplication and division. Of more concern is that these difficulties accounted for a greater proportion of errors in older students, indicating that although the success rate on mental computation problems increased, weaker students tended to continue to use counting approaches rather than more efficient strategies (Watson, Kelly, & Callingham, 2005).

Two other studies suggest some strategies that teachers can use to address this problem. The first of these indicates that it is more effective to delay introducing formal algorithmic approaches in classrooms. By encouraging students to develop personally meaningful methods first they develop deeper conceptual understanding (McIntosh, 2005). The second study (Callingham, 2005a) indicates that schools can work with the framework of the NSW Model of Quality Teaching to bring about effective changes in their mathematics programs through a focus on elements such as Substantive Communication (NSW DET, 2003).

The following suggestions arise from these three studies. They are general in nature and indicate ways in which all teachers, not just those teaching mathematics, can contribute to their students' development of computational skills.

Suggestions for Teaching

Develop conceptual understanding

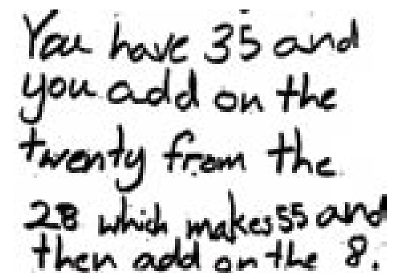
Procedural approaches to mental arithmetic are those in which students rely on the application of a particular process with no explanation that indicates an understanding of the underlying concepts. In contrast, students with conceptual knowledge display understanding of how numbers work, closer to McIntosh's (1990) notion of "number sense". Students relying on procedural knowledge alone do not have a basis for improvement, whereas a clear and deliberate focus on strategy development, building conceptual knowledge, appears to have benefits (McIntosh & Dole, 2004). Such an approach relates to the Deep Understanding element of the Model of Quality Teaching used in NSW schools (NSW DET, 2003). For example, contrast these two Year 6 students' different explanations for 16 + 8. Ben said "I saw 16 as three groups of five plus one, and eight as five plus three and I put the fives together and then the leftovers." Carol, however, answered correctly by using her fingers to count on by ones. Ben was operating conceptually, understanding the relationships among numbers, whereas Carol used an inefficient procedural process, even though in this situation it led to the correct answer. Although students' performances in the class generally improved over time, Carol's achievement fell (Callingham, 2005b).

Encourage informal recording

In the past, in our haste to tell students the way, we have jumped into formal mathematical recording before students have had time to fully understand the meaning of what they are doing. Outside the classroom we tend to use "back of the envelope" recording, jotting down steps along the way rather than the formal algorithm. Informal recording is a helpful support to students who are learning to compute. There is evidence that young children invent their own algorithms for written computation that are more meaningful than the standard processes and equally efficient when they are encouraged to record their calculations informally (McIntosh, 2005).

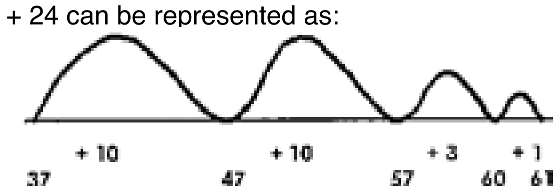
The figure following, shows one child's recording of a correct answer to 35 + 28. The left-to-right nature of the thinking ("add on the twenty") is typical of mental computation. In contrast, the algorithm with which we are familiar adds the units first and "carries" to the tens column. Although informal, the student's work indicates good understanding of the method applied and the underlying numerical structure.

35 + 28 = 63



Focus on strategies and when to use them

Many thinking strategies support mental computation (McIntosh & Dole, 2004). Encourage students to see patterns, for example, or to visualise shapes and number lines as a support for a computation. A problem such as 37 + 24 can be represented as:



This very strong visual representation is widely used in England and it has been suggested that its use has led to an improvement in addition and subtraction skills.

Working with patterns can be a way to improve multiplication and division skills. Knowing that any multiple of nine also adds to nine, or that multiples of five always end in five or zero may be second nature to us but can be missed by students unless it is made explicit to them. "Tricks", like knowing that the last two digits of a leap year, such as 1996, can always be divided by four, and thus any other number that has a similar structure is also divisible by four, are powerful aids to computation, and make connections with other everyday knowledge. It is often amazing how a throwaway line in the classroom about one of these shortcuts can lead to a whole new way of thinking for some students.

Develop estimation skills

Most everyday calculations are based on estimation (Zevenbergen, 2004). We all have personal benchmarks that we draw on, often unconsciously. For example, we recognise that we have about 500g of green beans because it feels about the same as a tub of butter, or about one metre of fabric because it is the length from my nose to my fingertip on an outstretched arm. These can be shared with students, and they should be encouraged to develop their own benchmarks. Without these being made clear, however, students are likely to miss the importance of being able to make these quick estimations.

Conclusion

Mental computation today is far more than reciting tables or counting by two, three or four. It encompasses a wide range of formal and informal skills that must be taught to students; they do not come about on their own. When children spent much of their out of school time helping their parents many of these skills were passed on from older to younger. It is unwise to assume that students know these connections, or will make them by themselves. Changes in our way of life have meant that children may be missing out on valuable life skills that are as important now as they ever were.

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The references marked ** provide good resources for teachers.